Residual Life Prediction of Multistage Manufacturing Processes With Interaction Between Tool Wear and Product Quality Degradation

Li Hao, Linkan Bian, Nagi Gebraeel, and Jianjun Shi

Abstract—**Multistage manufacturing processes (MMPs) usually exhibit an interactive relationship between tool wear and product quality degradation. On one hand, the tool wear in a stage may result in the quality degradation of the products fabricated on that stage. On the other hand, the quality degradation at a preceding stage may lead to the change of the operational condition and thus affect the tool wear in subsequent stages. This interaction needs to be considered to accurately predict the residual life distribution (RLD) of MMPs, which will benefit condition-based maintenance and tool inventory management. In this paper, we propose an interaction model that utilizes a linear model to represent the impact of tool wear on quality degradation and a stochastic differential equation model to capture the impact of quality degradation on the instantaneous rate of tool wear. We then propose a Bayesian framework that incorporates real-time quality measurements to online update the RLD of MMPs. Our methodology is a generalization of an existing "QR-chain model," which is dedicated into a similar research and application area. We conduct numerical studies to test the performance of our methodology and compare with the QR-chain model. The results show that our methodology outperforms the QR-chain model through capturing the impact of quality degradation on the process of tool wear and incorporating real-time quality measurements.**

Note to Practitioners—**MMPs usually exhibit an interactive relationship between tool wear and product quality degradation, which significantly influences the RLD of MMPs. This paper presents a new methodology that utilizes a stochastic model to capture this complex relationship for better prediction of the RLD. In addition, we propose to utilize a Bayesian updating approach that relies on real-time quality measurements to predict the up-to-date RLD of MMPs. Specifically, our methodology is designed to be implemented in a general MMP, in which the tools used to fabricate products are subject to an accelerated wear due to product quality degradation from previous stages. Common examples of such MMPs include multistage machining processes and multistage stamping processes. This method can be potentially applied in the factory floor and improve the planning of condition-based maintenance and tool inventory management by providing more accurate and up-to-date residual life prediction.**

Our methodology is based on the following assumptions: 1) tool wear impacts quality degradation in a linear fashion; 2) quality degradation adjusts the instantaneous rate of tool wear based on the natural rate of tool wear; and 3) the natural rate of tool wear exhibits variability among identical tools. Successful implementation of this methodology requires users to focus on the following aspects:

(a) Identify the sources of process noise, which may affect product quality degradation, and estimate their variance
(b) Understand the physical structure of the MMP, including the layout of the manufacturing system, the locations of quality measurements, etc., in order to estimate the impact of tool wear and process noise on quality degradation.
(c) Acquire historical observations of quality degradation through sensor-based quality measuring system, which are commonly used in the industries.
(d) Acquire historical observations of tool wear through real-time tool-wear monitoring system that identifies tool condition without interrupting the manufacturing process. Such system has also been widely discussed in the literature.
(e) Leverage historical observations of tool wear and quality degradation to estimate model parameters that cannot be determined by engineering knowledge.
(f) Incorporate real-time quality measurements captured by online quality monitoring systems to update the RLD of MMPs.

Index Terms—**Interaction, multistage manufacturing processes (MMPs), prognostics, quality degradation, residual life distribution (RLD), tool wear.**

I. Introduction

MULTISTAGE manufacturing processes (MMPs) are widely known for their high throughput rates and relative flexibility. MMPs produce parts/products through a sequence of manufacturing stages that are equipped with tools for performing specific fabrication tasks. The performance and efficiency of an MMP are usually monitored via the quality of manufactured products. Typically, quality characteristics may be measured on the final products and/or at intermediate stages of the MMP. Deviations from the nominal values of these measurements (hereafter referred to as "quality degradation") are used to assess the efficiency of the MMP, which is deemed operational as long as the quality measurements of the manufactured products are within the engineering specification limits (i.e., parts are classified as conforming). Once these measurements exceed such prespecified thresholds, parts are classified as nonconforming and the MMP is shutdown. Common causes

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of nonconforming products include errors in locating, tool wear, fixture errors, and other random factors [1]–[6].

Tool wear has traditionally been seen as one of the most important factors that affect product quality degradation, and has been studied extensively in the literature. For example, Jin and Shi [3] and Liu *et al.* [7] investigated how the wear of locating pins affects the product quality in sheet metal forming and assembly processes. Another example can be found in metal cutting (i.e., machining) processes, in which tool wear was shown to be a main source of quality variation [4], [6], [8]. However, limited research efforts have been dedicated to investigate the effect of product quality on tool wear.

In MMPs, products are processed on sequential stages. Thus, in addition to the effect of tool wear on the product quality degradation, the quality degradation of outgoing products from a preceding stage also impacts the rate of tool wear in subsequent stages. In other words, there is a two-way interaction between tool wear and quality degradation. Take the cylinder head machining process [8] as an example. This process consists of two stages: a drilling stage and a tapping stage. In the first stage, the tool wear of the drill bit impacts the quality of the hole, such as diameter, depth, straightness, and orientation, etc., whereas, in the second stage, the quality characteristics of the drilled hole impacts the wear rate of the tapping tool. Another example is the doorknob stamping process [9], in which the worn blanking die (tool) in the second stage generates burr (product quality degradation), which can affect not only final product quality but also the tool wear in the subsequent stages, as the burr on the part will accelerate the draw die worn out in the later forming operations.

Although the aforementioned interaction exists widely in MMPs, existing research has been most geared towards monitoring either tool wear or product quality, separately. Very few researchers have addressed the interaction between those two factors. One example is the QR-chain model proposed by Chen and Jin [1], which studied how product quality degradation from preceding stages affects the probability of tool breakage in subsequent stages. In this model, the rate of tool wear is assumed to be independent from product quality degradation. The same model was implemented in the automotive body assembly process (Chen, *et al.* [10]) to investigate how the wear-out of locating pins affects the assembly quality of outgoing products and how quality degradation of incoming products increased the probability of locating pin breakage.

This paper generalizes the notion of the existing QR-chain model to incorporate the impact of quality degradation on the rate of tool wear instead of tool breakage. According to the literature in tool wear [11], the rate of tool wear tends to be higher as the "depth of cut" increases and *vice versa*, and the depth of cut is correlated to the product quality from preceding stages. For example, in a two-stage drilling and tapping process, if the diameter of the hole drilled in the first stage is too small, the tapping tool will cut more material to maintain the final product quality, which accelerates its tool wear.

Our goal is to utilize a stochastic model to predict the residual life distribution (RLD) of the MMP by tracking product quality degradation. Within this context, the failure of the MMP is assumed to be instigated by the occurrence of any nonconforming product. Consequently, nonconforming products will trigger the system shutdown and will not be able to enter subsequent stages. To do this, we develop a two-way interaction model that captures the interaction between tool wear and product quality degradation. The novelty of our work is that it allows us to predict the performance of the MMP, and thus provide ample time to plan for condition-based maintenance while preventing unexpected shutdown of the entire system. In addition, our work will also benefit the inventory management of tools. By monitoring the product quality characteristics and simultaneously accounting for tool wear, we can perform accurate prognostics on the MMP systems.

This paper is organized as follows: Section II reviews the existing literature pertaining to product quality and tool wear, as well as recent findings related to quality reliability interactions. Section III presents the stochastic methodology used to model the interaction between tool wear and quality degradation. Section IV discusses how to estimate the model parameters based on historical data, as well as how to predict and update the RLD of the MMPs. In Section V, we present a series of simulation-based numerical studies. The goal is to study the performance of our methodology and evaluate its sensitivity with respect to model parameters. Section VI discusses the implementation of our proposed methodology in real-world industries. Section VII provides concluding remarks.

II. Literature Review

Research in two important aspects of any manufacturing systems, tool wear and product quality, have traditionally been treated separately. For example, there is a plethora of literature that focuses on the condition monitoring of tool wear [12]–[14]. Some have dealt with machine vision-based methods [15], wear and debris analysis [16], and various types of sensor signals related to force measurements, vibration, acoustic, etc. Techniques such as artificial neural networks and regression analysis proved to be widely popular in tool condition monitoring as noted by [17]. Other methods include wavelet analysis of force signals, acoustic emissions, vibrations, and spindle current with the goal of identifying tool wear, chipping, breakage, and chatter [18].

On the other hand, statistical process control (SPC) methods have been widely used to monitor product quality and capture the root causes of quality degradation, tool wear, and other process variables. Traditional SPC approaches include control charts [19], [20], regression adjustment [21], [22], and the cause-selection chart [23], [24]. SPC in MMPs is particularly complex because of the nature of the manufacturing sequence and the number of process variables involved. However, state-space models have proved to be effective in this area. Apley and Shi [25] and Jin and Shi [3] proposed a state-space model to study the effects of process errors including tool wear on product quality in an automotive assembly process. Huang *et al.* [8] used a similar approach for a cylinder head machining process. Additional work by Ding *et al.* [5] and Huang and Shi [6] was also geared towards using state-space models to detect the potential sources of variation in product quality. See Tsung *et al.* [26] for a comprehensive review of SPC methods involving MMPs.

Very little research has investigated the interaction between tool wear and quality degradation in MMPs. The most relevant studies can be found in Chen *et al.* [10] and Chen and Jin [1], where the authors proposed a QR-chain model. The QR-chain model assumes that tool wear linearly impacts quality degradation, and quality degradation affects the probability of tool breakage. The QR-chain model does not consider the effect of quality degradation on the instantaneous rate of tool wear, which is assumed to be constant at all time in each stage. The scope of the QR-chain model is limited to the survival analysis of the MMP.

This paper is dedicated into a similar research and application area as in the QR-chain model. However, we focus on the scenario where product quality degradation is assumed to impact the instantaneous rate of tool wear rather than the probability of tool breakage. Particularly, this paper builds on the existing methods and proposes a prognostic modeling framework that can online update the RLD of MMPs by incorporating real-time quality measurements, rather than limiting into the survival analysis. To do this, we propose to characterize the process of tool wear using a stochastic differential equation (SDE) model with a Brownian motion error and a drift which is a function of quality degradation as well as the natural rate of tool wear. Stochastic models with a Brownian motion error have been used in many settings to characterize degradation processes for the purpose of predicting the residual life. Examples include models used for accelerated degradation testing [27], [28] and models for predicting and updating RLD of partially degraded components [29], [30]. We assume that the natural rate of tool wear in each stage is stochastic and governed by a prespecified distribution, which is updated using real-time quality measurements.

III. Prognostics Model Considering Tool Wear – Quality Degradation Interaction

In this section, we first propose two individual models for characterizing the effect of tool wear on quality degradation and that of quality degradation on tool wear in MMPs, respectively. We assume that the instantaneous rate of tool wear is linearly dependent on quality degradation, which is modeled as a Brownian motion process with a linear drift that is a function of the level of quality degradation. Subsequently, the two models are synergized into a hybrid modeling framework to capture the interaction between tool wear and quality degradation. The hybrid modeling framework is eventually used to predict the distribution of the remaining time before an MMP produces a nonconforming product, i.e., the RLD of the MMP.

We consider an MMP with M stages, each of which is equipped with a single tool, and N quality measurement points. We let $S_m(t)$ represent the level of tool wear at stage m at time t for $m = 1, 2, \ldots, M$. Also, let $Y_n(t)$ represent the level of quality degradation (i.e., the deviation of the product quality measurement from its nominal value) at measurement point n for $n = 1, 2, \ldots, N$. Thus, $\boldsymbol{S}(t) = (S_1(t), S_2(t), \ldots, S_M(t))'$ and $\boldsymbol{Y}(t) = (Y_1(t), Y_2(t), \ldots, Y_N(t))'$ characterize tool wear and quality degradation at M stages and N quality measurement points, respectively. For example, in the sheet metal assembly process [3], $\boldsymbol{Y}(t)$ corresponds to the deviation of measurement points in the body coordinates and $\boldsymbol{S}(t)$ corresponds to the wear of locating pins. Another example is the stamping process [1], in which $\boldsymbol{Y}(t)$ represents the size of the burr on the edge of the part, and $\boldsymbol{S}(t)$ represents the tool wear of dies.

A. Modeling the Product Quality Degradation

We begin by formally characterizing the effect of tool wear on product quality degradation. We assume that the main assignable cause of part quality degradation is tool wear. All other types of operational errors, such as fixture errors, location errors, and other sources can be lumped and considered as "process noise." The effect of tool wear and process noise on quality degradation is defined by the following linear model:

$$Y_n(t) = \boldsymbol{a}_n' \boldsymbol{S}(t) + \boldsymbol{b}_n' \boldsymbol{Z}(t) \tag{1}$$

for $n = 1, 2, \ldots, N$, where $\boldsymbol{Z}(t) = (Z_1(t), Z_2(t), \ldots, Z_P(t))'$ represents the process noise in MMPs. $\boldsymbol{a}_n = (a_{1,n}, a_{2,n}, \ldots, a_{M,n})'$ and $\boldsymbol{b}_n = (b_{1,n}, b_{2,n}, \ldots, b_{P,n})'$ are the coefficients that characterize how $\boldsymbol{S}(t)$ and $\boldsymbol{Z}(t)$ impact $Y_n(t)$, respectively. Typically, the values of $\boldsymbol{a}_n$ and $\boldsymbol{b}_n$ as well as the functional forms of $\boldsymbol{S}(t)$ and $\boldsymbol{Z}(t)$ can be determined based on specific applications, such as the auto-body assembly process [10]. We assume the following specific forms of $\boldsymbol{S}(t)$ and $\boldsymbol{Z}(t)$:

1) $\boldsymbol{Z}(t)$ follows a P-dimensional Brownian motion process with zero mean and a covariance matrix. Note that in this paper, we use the symbol "$\boldsymbol{\Sigma}$" to represent a covariance matrix in general. Thus, we define the covariance matrix of $\boldsymbol{Z}(t)$ as $\boldsymbol{\Sigma}_{\boldsymbol{Z}} t$.
2) $\boldsymbol{S}(t)$ evolves according to a vector of continuous stochastic processes, the specific form of which is presented in Section III-B.

Hence, the matrix form of (1) can be rewritten as follows:

$$\boldsymbol{Y}(t) = \boldsymbol{A}' \boldsymbol{S}(t) + \boldsymbol{B}' \boldsymbol{Z}(t) \tag{2}$$

where $\boldsymbol{A} = (\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_N)$ and $\boldsymbol{B} = (\boldsymbol{b}_1, \boldsymbol{b}_2, \ldots, \boldsymbol{b}_N)$.

The linear assumption presented in (1) and (2) has been systematically justified and widely used in the literature to character the tool wear on product quality in manufacturing processes [1], [3], [10], [31]. When the physical process knowledge is available, the linear functional form can be used as a reasonably good approximation to characterize the underlying process in many manufacturing systems. Some examples include the sheet metal assembly process [3] and the cylinder head machining process [8]. When the physical process knowledge is not available, Wu and Hamada [31] suggests using this linear model since this linear model represents main effects, which usually have more significant impacts than higher order effects.

B. Modeling Tool Wear

Next, we develop a model that characterizes the effect of quality degradation on tool wear. Specifically, tool wear at stage m is modeled by the following SDE:

$$dS_m(t) = R_m^I(t)\,dt + dW_m(t) \tag{3}$$

for $m = 1, 2, \ldots, M$, where $R_m^I(t)$ represents the instantaneous rate of tool wear that is governed by the level of quality degradation as well as a natural rate of tool wear, and $W_m(t)$ is a Brownian motion process with the diffusion parameter $\sigma_{W,m}$ and is used to capture the randomness of the tool wear process. Thus, $W_m(t)$ is a stochastic process with independent increments that follow a normal distribution with zero mean and variance $\sigma_{W,m}^2 t$. $W_m(t)$ is assumed to be statistically independent from $\boldsymbol{Z}(t)$.

We focus on a base-case stochastic model where the instantaneous wear rate $R_m^I(t)$ takes the linear form expressed below

$$R_m^I(t) = R_m + c_m' \boldsymbol{Y}(t) \tag{4}$$

for $m = 1, 2, \ldots, M$, where R_m represents the natural rate of tool wear at stage m, and the second part captures the rate of tool wear affected by quality degradation with $\boldsymbol{c}_m = (c_{1,m}, c_{2,m}, \ldots, c_{N,m})'$ being an N-dimensional column vector of coefficients describing how quality degradation, $\boldsymbol{Y}(t)$, impacts the instantaneous rate of tool wear. The linear assumption presented in (4) is a special case of the phenomenon that the tool wear at a subsequent stage is positively correlated to the amount of workload the tool needs to perform which is directly affected by the product quality from the preceding stages. In the metal cutting process, the workload can be represented by the "depth of cut" [11]. For example, in a two-stage drilling and tapping process, if the diameter of the hole drilled in the first stage is smaller than its specification, the tapping tool will need to cut more material to maintain the final product quality, which will accelerate its tool wear. Without further information, a linear model, as a case in point, is a reasonable choice to demonstrate our proposed methodology for characterizing the impact of quality degradation on the instantaneous rate of tool wear, which has not been investigated in the existing literature. Depending on the specific real-world applications, different functional forms can be incorporated as the extensions of our proposed model.

Using the expression of the instantaneous rate of tool wear expressed in (4), we can rewrite the SDE model expressed in (3) as

$$dS_m(t) = [R_m + \boldsymbol{c}_m' \boldsymbol{Y}(t)]\, dt + dW_m(t) \tag{5}$$

for $m = 1, 2, \ldots, M$. The matrix-vector form can be expressed as follows:

$$d\boldsymbol{S}(t) = \left[\boldsymbol{R} + \boldsymbol{C}'\boldsymbol{Y}(t)\right] dt + d\boldsymbol{W}(t) \tag{6}$$

where $d\boldsymbol{S}(t) = (dS_1(t), dS_2(t), \ldots, dS_M(t))'$, $\boldsymbol{R} = (R_1, R_2, .., R_M)'$, $\boldsymbol{C} = (\boldsymbol{c}_1, \boldsymbol{c}_2, \ldots, \boldsymbol{c}_N)$, and $d\boldsymbol{W}(t) = (dW_1(t), dW_2(t), \ldots, dW_M(t))'$. $\boldsymbol{W}(t)$ represents an M-dimentional Brownian motion process. Recall that we use the symbol "$\boldsymbol{\Sigma}$" to represent a covariance matrix in general. Thus, we define the covariance matrix of $\boldsymbol{W}(t)$ as $\boldsymbol{\Sigma_W} t$. Specifically, we assume that each individual $W_m(t)$ is statistically independent from others, thus $\boldsymbol{\Sigma_W}$ is as a diagonal matrix with the form $\boldsymbol{\Sigma_W} = \mathrm{diag}(\sigma_{W,1}^2, \sigma_{W,2}^2, \ldots, \sigma_{W,M}^2)$. Since every element of $\boldsymbol{W}(t)$ is assumed to be statistically independent of $\boldsymbol{Z}(t)$, $\boldsymbol{W}(t)$ is also statistically independent of $\boldsymbol{Z}(t)$.

C. Interaction Between Tool Wear and Quality Degradation

Leveraging the two models expressed in (2) and (6), we develop a hybrid model that captures the interaction between tool wear and quality degradation in MMPs. By taking the derivatives of both sides in (2), we have

$$d\boldsymbol{Y}(t) = \boldsymbol{A}' d\boldsymbol{S}(t) + \boldsymbol{B}' d\boldsymbol{Z}(t).$$

Then, we substitute $d\boldsymbol{S}(t)$ in the above equation using $\left[\boldsymbol{R} + \boldsymbol{C}'\boldsymbol{Y}(t)\right] dt + d\boldsymbol{W}(t)$, as illustrated in (6). Consequently, we have

$$d\boldsymbol{Y}(t) = A' \left\{ \left[\boldsymbol{R} + \boldsymbol{C}'\boldsymbol{Y}(t)\right] dt + d\boldsymbol{W}(t) \right\} + \boldsymbol{B}' d\boldsymbol{Z}(t).$$

or

$$d\boldsymbol{Y}(t) = \left[\boldsymbol{A}'\boldsymbol{R} + \boldsymbol{A}'\boldsymbol{C}'\boldsymbol{Y}(t)\right] dt + \boldsymbol{A}' d\boldsymbol{W}(t) + \boldsymbol{B}' d\boldsymbol{Z}(t). \tag{7}$$

Recall that $\boldsymbol{W}(t)$ and $\boldsymbol{Z}(t)$ represent two independent Brownian motion processes. Thus, $\boldsymbol{A}'\boldsymbol{W}(t) + \boldsymbol{B}'\boldsymbol{Z}(t)$ is an N-dimensional Brownian motion process that we denote as $\boldsymbol{W}^*(t)$, which has covariance matrix $\boldsymbol{\Sigma}^* t$ with $\boldsymbol{\Sigma}^* = \boldsymbol{A}'\boldsymbol{\Sigma_W}\boldsymbol{A} + \boldsymbol{B}'\boldsymbol{\Sigma_Z}\boldsymbol{B}$. For notational convenience, we let $\boldsymbol{\Delta} = \boldsymbol{A}'\boldsymbol{C}'$. Thus, (7) can be rewritten as follows:

$$d\boldsymbol{Y}(t) = \left[\boldsymbol{A}'\boldsymbol{R} + \boldsymbol{\Delta}\boldsymbol{Y}(t)\right] dt + d\boldsymbol{W}^*(t). \tag{8}$$

By choosing various forms of $\boldsymbol{A}$ and $\boldsymbol{C}$, (8) can be used to describe different MMP configurations. For example, if an MMP has a series configuration and quality measurements are taken after each stage, quality degradation (tool wear) of a specific stage will only be affected by tool wear (quality degradation) of the current and the preceding stages. Consequently, $\boldsymbol{A}$ and $\boldsymbol{C}$ will be upper-triangular matrices. Similarly, if an MMP has a parallel configuration, the interaction between tool wear and quality degradation will be limited to what occurs within each stage. Thus, $\boldsymbol{A}$ and $\boldsymbol{C}$ will each be a diagonal matrix.

IV. Parameter Estimation and RLD Updating

We discuss in this section how to estimate the model parameters involved in the interaction model, and how to estimate and update RLD of the MMP system using the real-time quality measurements.

A. Estimating and Updating the Model Parameters

Our model is used to describe the interaction between tool wear and quality degradation across a population of similar MMPs. The model consists of deterministic parameters used to characterize features that are relatively constant/fixed across the population of MMPs, and stochastic parameters that capture variability among individual MMPs.

In our model, we assume that $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{\Delta}$ are fixed parameters, since they are typically dependent on the configuration of the MMP and can be determined using the locations of tools and the layout of measurement points. Examples in which such assumptions were used can be found in the two-dimensional assembly process studied in Jin and Shi [3], the three-dimentional auto-body assembly process investigated in Liu *et al.* [7], and the cylinder-head machining process analyzed in Huang *et al.* [8]. In all of these studies, the randomness and variability resulting from $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{\Delta}$ were considered insignificant.

$\boldsymbol{\Sigma}^*$ is also assumed to be constant. $\boldsymbol{\Sigma}^*$ is determined by $\boldsymbol{\Sigma_W}$ and $\boldsymbol{\Sigma_Z}$, where $\boldsymbol{\Sigma_Z}$ is the variance of the process noise and $\boldsymbol{\Sigma_W}$

is the variance of the Brownian motion error of tool wear. Typically, $\boldsymbol{\Sigma_Z}$ can be determined using the physical configuration of the systems and is often assumed to be constant in most of the existing literature. For example, in the auto-body assembly process investigated by Chen and Jin [1], the authors defined process noise as the random orientation of the contacting point between the locating pin and the locating hole on a raw product. They proved that process noise followed a known distribution that was derived through knowledge of the physical system. Other examples include machining processes studied by Zhou *et al.* [4] and Huang and Shi [6], where the variance of the process noise was determined by engineering knowledge. With regards to the other noise component, $\boldsymbol{\Sigma_W}$, it corresponds to noise in the tool wear process. As will be discussed later, the natural rate of tool wear is assumed to be random. In reality, the noise $\boldsymbol{\Sigma_W}$ is usually an insignificant source of variation when compared to the variation in the rate of tool wear [32]–[34]. Therefore, it is reasonable to assume that $\boldsymbol{\Sigma}^*$ is deterministic.

Different from the deterministic parameters discussed above, the natural rate of tool wear is assumed to be stochastic and follow a prior distribution. In other words, each time a specific tool is replaced, its natural rate of tool wear is assumed to be a random draw from its prior distribution. Specifically, we assume that the vector of natural rates of tool wear $\boldsymbol{R}$ follows a prior distribution, which we denote by $\pi(\boldsymbol{R})$. The functional form of $\pi(\cdot)$ can be obtained through subjective information or estimated using historical data. For illustrative purposes, we assume that $\pi(\boldsymbol{R})$ is a multivariate normal distribution with mean vector $\boldsymbol{\mu_R}$ and covariance matrix $\boldsymbol{\Sigma_R}$, where $\boldsymbol{\mu_R} = (\mu_{\boldsymbol{R},1}, \ldots, \mu_{\boldsymbol{R},M})'$ and $\boldsymbol{\Sigma_R} = \operatorname{diag}\left(\sigma^2_{\boldsymbol{R},1}, \ldots, \sigma^2_{\boldsymbol{R},M}\right)$.

Next, we use *in-situ* quality measurements of the MMP to update the prior distribution of $\boldsymbol{R}$. To do this, assume that quality measurements are taken at discrete observation epochs $t_0, t_1, t_2, \ldots, t_k$, where t_0 represents the epoch when the MMP starts to operate and t_k is the most recent observation epoch. Furthermore, we assume that quality measurements are observed at a constant sampling interval, i.e., $t_1 - t_0 = t_2 - t_1 = \cdots = t_k - t_{k-1} = \delta_t$. At t_0, we assume that the initial quality degradation of the MMP is $\mathbf{0}$, which indicates that no quality degradation occurs. Let $\{\boldsymbol{y}(t_0), \ldots, \boldsymbol{y}(t_k)\}$ represent the observed values of quality measurements, where $\boldsymbol{y}(t_0) = 0$. The posterior distribution of $\boldsymbol{R}$ can be expressed as follows:

$$p\left(\boldsymbol{R}|\boldsymbol{y}(t_0), \ldots, \boldsymbol{y}(t_k), \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right) \propto f\left(\boldsymbol{y}(t_0), \ldots, \boldsymbol{y}(t_k) | \boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right) \pi(\boldsymbol{R})$$

where $f\left(\boldsymbol{y}(t_0), \ldots, \boldsymbol{y}(t_k) | \boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right)$ is the likelihood function of quality observations given parameters $(\boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*)$. According to our assumption, $\boldsymbol{y}(t_0)$ is a constant vector with value $\mathbf{0}$. Thus, we have

$$p\left(\boldsymbol{R}|\boldsymbol{y}(t_0), \ldots, \boldsymbol{y}(t_k), \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right) \propto f\left(\boldsymbol{y}(t_1), \ldots, \boldsymbol{y}(t_k) | \boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right) \pi(\boldsymbol{R}). \quad (9)$$

According to (8), $\boldsymbol{y}(t_i)$, for $i = 1, \ldots, k$, follows a Brownian motion with a linear drift. Thus, using the Markovian property $f\left(\boldsymbol{y}(t_1), \ldots, \boldsymbol{y}(t_k) | \boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right)$ can be decomposed as follows:

$$f\left(\boldsymbol{y}(t_1), \ldots, \boldsymbol{y}(t_k) | \boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right) = \prod_{i=1}^{k} f_i\left(\boldsymbol{y}(t_i) | \boldsymbol{y}(t_{i-1}), \boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right) \quad (10)$$

where $f_i\left(\boldsymbol{y}(t_i) | \boldsymbol{y}(t_{i-1}), \boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right)$ represents the p.d.f. of $\boldsymbol{y}(t_i)$ given $\left(\boldsymbol{y}(t_{i-1}), \boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right)$. When $\boldsymbol{\Delta}$ is a full rank matrix, $\boldsymbol{y}(t_i) \,|\, \left(\boldsymbol{y}(t_{i-1}), \boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right)$ follows a multivariate normal distribution with mean vector $\boldsymbol{\mu_Y}(t_i)$ and covariance matrix $\boldsymbol{\Sigma_Y}(t_i)$ [35]. The explicit formulation of $\boldsymbol{\mu_Y}(t_i)$ is

$$\boldsymbol{\mu_Y}(t_i) = e^{\delta_t \boldsymbol{\Delta}} \boldsymbol{y}(t_{i-1}) + \boldsymbol{U}\boldsymbol{R},$$

where $\boldsymbol{U} = \left[e^{\delta_t \boldsymbol{\Delta}} - \boldsymbol{I}\right] \boldsymbol{\Delta}^{-1} \boldsymbol{A}'$. $\boldsymbol{\Sigma_Y}(t_i)$ can be evaluated by solving an ordinary differential equation (ODE) system. In particular, $\boldsymbol{\Sigma_Y}(t_i) = \boldsymbol{X}(t)\,|_{t=\delta_t}$, where $\boldsymbol{X}(t)$ is the solution of the following ODE system:

$$\begin{aligned} \frac{d\boldsymbol{X}(t)}{dt} &= \boldsymbol{\Delta}\boldsymbol{X}(t) + \boldsymbol{X}(t)\boldsymbol{\Delta}' + \boldsymbol{\Sigma}^* \\ \boldsymbol{X}(0) &= \mathbf{0}. \end{aligned} \quad (11)$$

Note that $\boldsymbol{\Sigma_Y}(t_i)$ does not depend on t_i, we have $\boldsymbol{\Sigma_Y}(t_1) = \cdots = \boldsymbol{\Sigma_Y}(t_k) = \boldsymbol{X}(t)\,|_{t=\delta_t}$.

Based on the aforementioned distribution of $\boldsymbol{y}(t_i) \,|\, \left(\boldsymbol{y}(t_{i-1}), \boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right)$, we show in Proposition 1, that the posterior distribution of $\boldsymbol{R}$ also follows a multivariate normal distribution.

Proposition 1: Given real-time quality measurements $\{\boldsymbol{y}(t_0), \ldots, \boldsymbol{y}(t_k)\}$ and model parameters $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*$, the posterior distribution of $\boldsymbol{R}$ follows a multivariate normal distribution with mean $\boldsymbol{\mu}_{\boldsymbol{R},k}$ and covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{R},k}$, where

$$\begin{aligned} \boldsymbol{\mu}_{\boldsymbol{R},k} &= \boldsymbol{\Sigma}_{\boldsymbol{R},k} \left[\sum_{i=1}^{k} \left[\boldsymbol{y}(t_i) - e^{\delta_t \Delta} \boldsymbol{y}(t_{i-1})\right]' \boldsymbol{\Sigma}_{\boldsymbol{Y}}^{-1}(t_i) \boldsymbol{U} \right. \\ &\quad \left. + \boldsymbol{\mu}_{\boldsymbol{R}}' \boldsymbol{\Sigma}_{\boldsymbol{R}}^{-1} \right]', \\ \boldsymbol{\Sigma}_{\boldsymbol{R},k} &= \left(k \boldsymbol{U}' \boldsymbol{\Sigma}_{\boldsymbol{Y}}^{-1}(t_1) \boldsymbol{U} + \boldsymbol{\Sigma}_{\boldsymbol{R}}^{-1}\right)^{-1}. \end{aligned}$$

Proof of Proposition 1: Proposition 1 updates the posterior distribution of $\boldsymbol{R}$ by synergistically leveraging historical knowledge (its prior distribution) as well as current observations (real-time quality measurements). This step can be done using the Bayesian theory in (9). In the following proof, we will show that posterior distribution of $\boldsymbol{R}$ actually follows a multivariate normal distribution. The detailed proof is as follows:

According to (10), $f\left(\boldsymbol{y}(t_1), \ldots, \boldsymbol{y}(t_k) | \boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right)$ can be decomposed as follows:

$$f\left(\boldsymbol{y}(t_1), \ldots, \boldsymbol{y}(t_k) | \boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right) = \prod_{i=1}^{k} f_i\left(\boldsymbol{y}(t_i) | \boldsymbol{y}(t_{i-1}), \boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right)$$

where $f_i\left(\boldsymbol{y}(t_i) | \boldsymbol{y}(t_{i-1}), \boldsymbol{R}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Delta}, \boldsymbol{\Sigma}^*\right)$ represents the p.d.f. of a multivariate normal distribution with mean $\boldsymbol{\mu_Y}(t_i) = e^{\delta_t \boldsymbol{\Delta}} \boldsymbol{y}(t_{i-1}) + \boldsymbol{U}\boldsymbol{R}$ and covariance matrix

$\boldsymbol{\Sigma_Y}(t_i)$, which can be solved using the ODE system expressed in (11). Hence, according to (9), the expression of $p\left(\boldsymbol{R}|\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right),\boldsymbol{A},\boldsymbol{B},\boldsymbol{\Delta},\boldsymbol{\Sigma}^*\right)$ can be expanded as follows:

$$\begin{aligned}&p\left(\boldsymbol{R}|\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right),\boldsymbol{A},\boldsymbol{B},\boldsymbol{\Delta},\boldsymbol{\Sigma}^*\right)\\&\propto \exp\Bigg\{-\frac{1}{2}\Bigg[\sum_{i=1}^{k}\left(\boldsymbol{y}\left(t_i\right)-\boldsymbol{\mu_Y}\left(t_{i-1}\right)\right)'\boldsymbol{\Sigma}_{\boldsymbol{Y}}^{-1}\left(t_i\right)\left(\boldsymbol{y}\left(t_i\right)\right.\\&\quad\left.-\boldsymbol{\mu_Y}\left(t_{i-1}\right)\right)+\left(\boldsymbol{R}-\boldsymbol{\mu_R}\right)'\boldsymbol{\Sigma}_{\boldsymbol{R}}^{-1}\left(\boldsymbol{R}-\boldsymbol{\mu_R}\right)\Bigg]\Bigg\}.\end{aligned}$$

We substitute the expression of $\boldsymbol{\mu_Y}\left(t_i\right)$ using $e^{\delta_t\boldsymbol{\Delta}}\boldsymbol{y}\left(t_{i-1}\right)+\boldsymbol{UR}$ and $\boldsymbol{\Sigma_Y}\left(t_i\right)$ using $\boldsymbol{\Sigma_Y}\left(t_1\right)$, for $i=1,2,\ldots,k$. Thus, we have

$$\begin{aligned}&p\left(\boldsymbol{R}|\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right),\boldsymbol{A},\boldsymbol{B},\boldsymbol{\Delta},\boldsymbol{\Sigma}^*\right)\\&\propto \exp\Bigg\{-\frac{1}{2}\boldsymbol{R}'\left(k\boldsymbol{U}'\boldsymbol{\Sigma}_{\boldsymbol{Y}}^{-1}\left(t_1\right)\boldsymbol{U}+\boldsymbol{\Sigma}_{\boldsymbol{R}}^{-1}\right)\boldsymbol{R}\\&\quad+\left[\sum_{i=1}^{k}\left(\boldsymbol{y}\left(t_i\right)-e^{\delta_t\boldsymbol{\Delta}}\boldsymbol{y}\left(t_{i-1}\right)\right)'\boldsymbol{\Sigma}_{Y}^{-1}\left(t_i\right)\boldsymbol{U}+\boldsymbol{\mu}_{\boldsymbol{R}}'\boldsymbol{\Sigma}_{\boldsymbol{R}}^{-1}\right]\boldsymbol{R}\Bigg\}\\&\propto \exp\left\{-\frac{1}{2}\left(\boldsymbol{R}-\boldsymbol{\mu}_{\boldsymbol{R},k}\right)'\boldsymbol{\Sigma}_{\boldsymbol{R},k}^{-1}\left(\boldsymbol{R}-\boldsymbol{\mu}_{\boldsymbol{R},k}\right)\right\},\end{aligned}$$

where

$$\begin{aligned}&\boldsymbol{\mu}_{\boldsymbol{R},k}\\&=\boldsymbol{\Sigma}_{\boldsymbol{R},k}\left[\sum_{i=1}^{k}\left[\boldsymbol{y}\left(t_i\right)-e^{\delta_t\boldsymbol{\Delta}}\boldsymbol{y}\left(t_{i-1}\right)\right]'\boldsymbol{\Sigma}_{\boldsymbol{Y}}^{-1}\left(t_i\right)\boldsymbol{U}+\boldsymbol{\mu}_{\boldsymbol{R}}'\boldsymbol{\Sigma}_{\boldsymbol{R}}^{-1}\right]'\end{aligned}$$

and $\boldsymbol{\Sigma}_{\boldsymbol{R},k}=\left(k\boldsymbol{U}'\boldsymbol{\Sigma}_{\boldsymbol{Y}}^{-1}\left(t_1\right)\boldsymbol{U}+\boldsymbol{\Sigma}_{\boldsymbol{R}}^{-1}\right)^{-1}$. ■

According to Proposition 1, the expression of $p\left(\boldsymbol{R}|\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right),\boldsymbol{A},\boldsymbol{B},\boldsymbol{\Delta},\boldsymbol{\Sigma}^*\right)$ can be represented by the p.d.f. of a multivariate normal distribution with mean vector $\boldsymbol{\mu}_{\boldsymbol{R},k}$ and covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{R},k}$. Once we obtain the posterior distribution of $\boldsymbol{R}$, we utilize this updated distribution as well as the real-time quality measurements $\{\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right)\}$ to further update the RLD of the MMP functioning in the field. Details are provided in the following section.

B. Updating the RLD of the MMP

The residual life of an MMP functioning in the field is the first time when any quality measurement exceeds a predefined failure threshold. Let T_k represent the residual life of the MMP at time t_k and ℓ_m represent the quality failure threshold at the m^{th} stage. The distribution of T_k can be expressed as follows:

$$\begin{aligned}&P\left(T_k\le t|\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right)\right)\\&\qquad=1-P\left(\boldsymbol{Y}\left(t_k+t\right)\le\boldsymbol{\ell}|\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right)\right)\end{aligned}\tag{12}$$

where $\boldsymbol{\ell}=\left(\ell_1,\ldots,\ell_M\right)'$. Similar definition of system failure has been widely used in the reliability literature, examples include Gebraeel *et al.* [29], Lu and Meeker [36], Wang and Coit [37], and Wang and Pham [38]. Based on (12), calculating the RLD is equivalent to estimating the distribution of $\boldsymbol{Y}\left(t_k+t\right)|\left(\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right)\right)$.

We have proved in Proposition 1 that the posterior distribution of $\boldsymbol{R}$ given $\{\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right)\}$ follows a multivariate normal distribution with mean vector $\boldsymbol{\mu}_{\boldsymbol{R},k}$ and covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{R},k}$. In addition, we have proved in Section IV-A that for all $i=1,\ldots,k$, $\boldsymbol{Y}\left(t_i\right)|\left(\boldsymbol{y}\left(t_{i-1}\right),\boldsymbol{R}\right)$ follows a multivariate normal distribution with mean vector $\boldsymbol{\mu_Y}\left(t_i\right)$ and covariance matrix $\boldsymbol{\Sigma_Y}\left(t_i\right)$.

Similarly, $\boldsymbol{Y}\left(t_k+t\right)|\left(\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right),\boldsymbol{R}\right)$ also follows a multivariate normal distribution with mean vector $\boldsymbol{\mu_Y}(t)=e^{t\boldsymbol{\Delta}}\boldsymbol{y}\left(t_k\right)+\left[e^{t\boldsymbol{\Delta}}-\boldsymbol{I}\right]\boldsymbol{\Delta}^{-1}\boldsymbol{A}'\boldsymbol{R}$ and covariance matrix $\boldsymbol{\Sigma_Y}\left(t\right)=\boldsymbol{X}\left(t\right)|_{t=t}$, where $\boldsymbol{X}(t)$ is the solution to the ODE system in (11). If we define $\boldsymbol{U}_t=\left[e^{t\boldsymbol{\Delta}}-\boldsymbol{I}\right]\boldsymbol{\Delta}^{-1}\boldsymbol{A}'$, according to Bishop [39], the distribution of $\boldsymbol{Y}\left(t_k+t\right)|\left(\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right)\right)$, without conditioning on $\boldsymbol{R}$, still follows a multivariate normal distribution with mean vector $e^{t\boldsymbol{\Delta}}\boldsymbol{y}\left(t_k\right)+\boldsymbol{U}_t\boldsymbol{\mu}_{\boldsymbol{R},k}$ and covariance matrix $\boldsymbol{\Sigma_Y}\left(t\right)+\boldsymbol{U}_t\boldsymbol{\Sigma}_{\boldsymbol{R},k}\boldsymbol{U}_t'$. Consequently, the RLD of the MMP can be calculated based on the distribution of $\boldsymbol{Y}\left(t_k+t\right)|\left(\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right)\right)$. Since $\boldsymbol{Y}\left(t_k+t\right)|\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right)$ is normally distributed, the distribution of $T_k\le t|\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right)$ is skewed. Thus, we utilize the median of $T_k\le t|\boldsymbol{y}\left(t_0\right),\ldots,\boldsymbol{y}\left(t_k\right)$ as the residual life prediction.

V. Numerical Studies

We conduct a series of numerical studies to evaluate the performance of our proposed methodology and utilize the QR-chain model by Chen and Jin [1] as the benchmark to compare the performance. To the best of our knowledge, the QR-chain model is the only existing method that addresses a similar research area — modeling the interactive relationship between tool wear and quality degradation in MMPs with the goal of reliability estimation. Our work is a generalization of the QR-chain model, since we address the impact of quality degradation on the actual process of tool wear rather than the impact on tool breakage, which is a fundamental shift from the previous paradigm. Also, our model incorporates real-time quality measurements rather than only historical data, and thus can be used to obtain real-time estimation of system residual life instead of static offline reliability estimate. Therefore, we use the QR-chain model as a benchmark to investigate how much improvement can be obtained in predicting the system lifetime by utilizing our methodology. Table I explains the connection between our methodology and the QR-chain model.

To implement the QR-chain model in a fair manner, we assume an identical linear model for quality degradation $\boldsymbol{Y}\left(t\right)$ as our proposed quality model described in (2). Note that the QR-chain model only focuses on the impact of quality degradation on the probability of tool breakage rather than the rate of tool wear. Thus, when the impact of quality degradation on tool breakage is ignored, the model of tool wear in the QR-chain model can be expressed as follows:

$$d\boldsymbol{S}\left(t\right)=\boldsymbol{R}dt+d\boldsymbol{W}(t)\tag{13}$$

where the tool wear rate is equal to $\boldsymbol{R}$ but not related to $\boldsymbol{Y}(t)$.

Comparing the equation above to our proposed tool-wear model in (6), we note that the QR-chain model, when it does

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

TABLE I
CONNECTION BETWEEN OUR PROPOSED METHODOLOGY AND THE QR-CHAIN MODEL

	Perspectives	Our Methodology	The QR-chain model
Similarities	System of interest	MMPs	
	Variables modeled	Product quality, tool reliability, and the interaction between them	
	Goal of modeling	To predict the reliability/lifetime distribution of MMPs	
Differences	Tool wear modeling	Modeling the impact of quality degradation on the actual process of tool wear	Modeling the impact of quality degradation on tool breakage only
	Result updating	Online updating of the RLD of MMPs.	Estimating off-line system reliability in a static way
	Type of data	Real-time data stream of the quality measurements.	Historical data related to the tool reliability (maintenance records, warranty information, etc.)
	Form of results	Achieving analytical expressions of the RLD	Using numerical approach to estimate system reliability

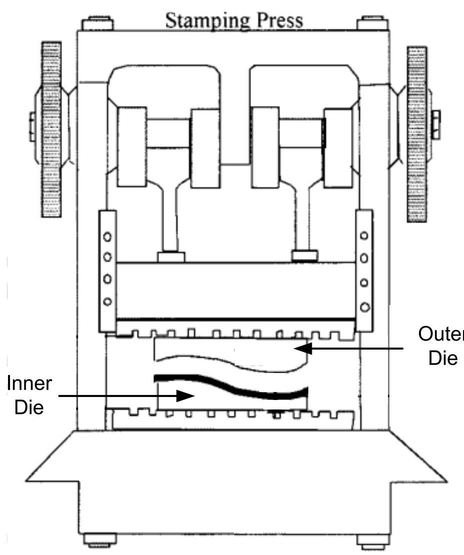

Fig. 1. Stamping process [40].

not consider failures due to tool breakage, is a special case of our proposed model by letting the matrix $\boldsymbol{C}$ equal to $\mathbf{0}$.

In what follows, we first describe the simulation set-up and the procedure we use to simulate the signals of tool wear and quality degradation. Next, we discuss the method for estimating the model parameters and testing the linear assumption in (5) and (6). Finally, we compare the performance of our methodology as well as its sensitivity to different parameter values to the QR-chain model. Model parameters studied in the sensitivity analysis include: 1) the amplitude of process noise; 2) the magnitude of the impact of quality degradation on tool wear; and 3) the number of stages.

A. Simulation Framework and Signal Generation

Our simulation setup is inspired by the single-stage sheet metal stamping process presented by Chen and Jin [40], the same authors as the QR-chain model, who used a similar quality model to study quality-based maintenance. The stamping process (as shown in Fig. 1 [40]) has two dies, an outer die and an inner die, which are subject to tool wear and thus will cause the dimensional deviation of formed products. The quantitative impact of tool wear on quality degradation is estimated using a series of experiments conducted by Jin and Shi [41]. The detailed result with only first-order terms is as follows:

$$\begin{aligned}&\text{Output part quality}\\&\quad= 0.1136 \times \text{tool wear of outer die}\\&\qquad+ 0.0219 \times \text{tool wear of inner die}\\&\qquad+ 0.019 \times \text{incoming part quality} + \text{noise}. \quad (14)\end{aligned}$$

In addition, Chen and Jin [40] characterized the tool wear of both outer and inner dies using the same model in the QR-chain formulation shown in (13). They obtained the tool wear parameters through real stamping processes.

In this paper, we would like to formulate a numerical study framework that resembles real-world applications related to the QR-chain model. In this way, we can compare the performance of our proposed methodology with the QR-chain model on a fair base. Hence, we focus on an MMP system with three consecutive stamping stages, each of which satisfies the quantitative model in (14). According to Chen and Jin [1], a multistage stamping process indeed presents the interaction between product quality and tool wear: The die worn-out at a previous stage generates burr on the edge of the part manufactured at that stage; the burr has an important impact on the draw die wear and breakage rate at subsequent stages. We assume that product quality is monitored after each stage. The process noises consist of the incoming raw part error as well as the measurement noise at each stage. The corresponding simulation setup is illustrated in Fig. 2.

We determine simulation parameters according to (14) and tool wear parameters by Chen and Jin [40]. Parameters that are not provided in (14) or Chen and Jin [40] are determined by subjective knowledge. All simulation parameters are listed in Table II. Note that the unit of tool-wear parameters in Chen and Jin [40] is "per operation." Here we assume sampling is taken every 1000 operations and compute parameter values accordingly.

We simulate tool wear $\boldsymbol{S}(t)$ and quality degradation $\boldsymbol{Y}(t)$ according to the interactive model presented in (2) and (6) in Section III. The details are described as follows:

Step 1) Generate a realization of the nature rate of tool wear $\boldsymbol{R}$ according to its prior distribution, i.e., the multivariate normal distribution with mean vector $\boldsymbol{\mu_R}$ and

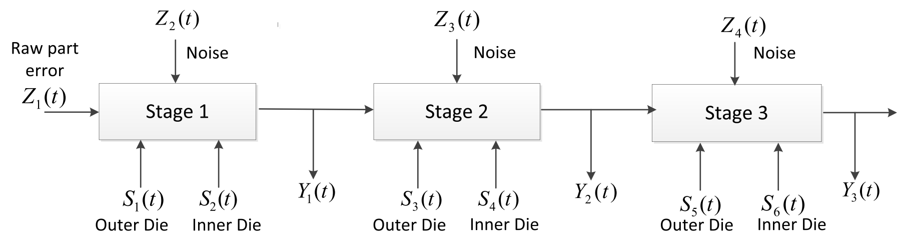

Fig. 2. Simulation setup.

TABLE II
PARAMETER VALUES USED FOR GENERATING DEGRADATION SIGNALS

	Description	Source	True Value	Estimated Value
$\boldsymbol{A}$	Coefficient matrix that characterizes how $\boldsymbol{S}(t)$ affects $\boldsymbol{Y}(t)$	Chen and Jin [40]	$\begin{pmatrix} 0.1136 & 0.0216 & 4.1e^{-5} \\ 0.0219 & 4.16e^{-4} & 7.91e^{-6} \\ 0 & 0.1136 & 0.0216 \\ 0 & 0.0219 & 4.16e^{-4} \\ 0 & 0 & 0.01136 \\ 0 & 0 & 0.0219 \end{pmatrix}$	Obtained via physical knowledge and configuration of the MMP
$\boldsymbol{B}$	Coefficient matrix that characterizes how $\boldsymbol{Z}(t)$ affects $\boldsymbol{Y}(t)$	Chen and Jin [40]	$\begin{pmatrix} 0.019 & 3.61e^{-4} & 6.859e^{-6} \\ 1 & 0.019 & 3.61e^{-4} \\ 0 & 1 & 0.019 \\ 0 & 0 & 1 \end{pmatrix}$	
$\boldsymbol{C}$	Coefficient matrix that characterizes how $\boldsymbol{Y}(t)$ impacts the instantaneous rate of tool wear $\boldsymbol{R} + \boldsymbol{CY}(t)$	The instantaneous rate of tool wear at a stage will only be influenced by the product quality of the current and preceding stages, whereas the highest impact is from the nearest preceding stage.	$\begin{pmatrix} 4 & 4 & 0 & 0 & 0 & 0 \\ 20 & 12 & 4 & 4 & 0 & 0 \\ 0 & 0 & 20 & 12 & 4 & 4 \end{pmatrix} \times 10^{-3}$	$\begin{pmatrix} 4.20 & 4.01 & 0.04 & 0.0 & 0 & 0 \\ 19.7 & 12.0 & 3.78 & 3.93 & 0 & 0 \\ 0 & 0.04 & 19.8 & 12.1 & 3.67 & 3.86 \end{pmatrix} \times 10^{-3}$
$\boldsymbol{\Sigma_Z}$	Covariance matrix of the process noise $\boldsymbol{Z}(t)$	Variance of Z_1 is from Chen and Jin [40] Variances of Z_2 to Z_4 represents sensor noise	$\mathrm{diag}(230,1,1,1) \times 10^{-4}$	Obtained via physical knowledge and configuration of the MMP
$\boldsymbol{\Sigma_W}$	Covariance matrix of the Brownian error $\boldsymbol{W}(t)$	Chen and Jin [40]	$\mathrm{diag}(7.14, 2, 7.14, 2, 7.14, 2) \times 10^{-9} \times h$	$\mathrm{diag}(7.12, 1.99, 7.13, 2, 6.91, 1.95) \times 10^{-6}$
h	Operations per Sampling	1.5 minutes per operation $\approx$ 1000 operations per day	1000	Pre-specified
$\boldsymbol{\mu_R}$	Mean of the prior distribution of the natural rate of tool wear $\boldsymbol{R}$	Chen and Jin [40]	$(6.94, 4.14, 6.94, 4.14, 6.94, 4.14)' \times 10^{-6} \times h$	$(7.15, 4.03, 6.95, 3.98., 7.04, 4.1)' \times 10^{-3}$
$\boldsymbol{\Sigma_R}$	Covariance matrix of the prior distribution of the natural rate of tool wear $\boldsymbol{R}$	The coefficient of variance of R is $\approx$ 1/3	$\mathrm{diag}(5.35, 1.9, 5.35, 1.9, 5.35, 1.9) \times 10^{-6}$	$\mathrm{diag}(5.31, 1.76, 4.95, 1.87, 5.85, 1.74) \times 10^{-6}$
$\boldsymbol{l}$	Failure threshold of quality degradation	Developed using the tool wear threshold in Chen and Jin[40]	$(0.5, 0.5, 0.5)'$	Pre-specified
δ_t	Sampling interval	Sample once each day	1	Pre-specified

covariance matrix $\boldsymbol{\Sigma_R}$. The values of $\boldsymbol{\mu_R}$ and $\boldsymbol{\Sigma_R}$ are specified in Table II.

Step 2) Set $\boldsymbol{S}(t_0) = \boldsymbol{0}$ and $\boldsymbol{Y}(t_0) = \boldsymbol{0}$.

Step 3) At any epoch t_i, for $i \geq 1$, Simulate the values of $\boldsymbol{S}(t_i)$ and $\boldsymbol{Y}(t_i)$ iteratively.

(3a) Given $\boldsymbol{R}$ and $\boldsymbol{Y}(t_{i-1})$, generate $\boldsymbol{S}(t_i)$ according to (6).

(3b) Given $\boldsymbol{S}(t_i)$, generate $\boldsymbol{Y}(t_i)$ according to (2). Stop when $\boldsymbol{Y}(t_i)$ exceeds the failure threshold $\boldsymbol{l}$.

B. Parameter Estimation and Goodness-of-Fit Testing

The parameters involved in our proposed methodology are determined in two ways: (1) We assume that the values of $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{\Sigma_z}$ are determined by the physical configuration of the MMP, and thus can be obtained based on expert knowledge. Similar examples of utilizing expert knowledge can be found in [3], [7], [10]. (2) On the other hand, the prior mean and variance of the nature rate of tool wear $\boldsymbol{\mu_R}$ and $\boldsymbol{\Sigma_R}$, the error variance error $\boldsymbol{\Sigma_W}$, and the coefficient matrix $\boldsymbol{C}$ may not be obtained via expert knowledge. Thus, we estimate these parameters using historical quality degradation and tool wear measurements in a regression framework. The detailed procedure is explained below.

According to the model in (5), given the monitored quality degradation and tool wear measurements for stage m, the difference between two consecutive tool wear levels $S_m(t_{i-1})$ and $S_m(t_i)$ can be expressed as follows:

$$S_m(t_i) - S_m(t_{i-1}) = \int_{t_{i-1}}^{t_i} [R_m + \boldsymbol{c}_m' \boldsymbol{Y}(\omega)]\, d\omega + W_m(\delta_t)$$

for $m = 1, 2, \ldots, M$. The right-hand side of the above expression involves the integration of stochastic process $\boldsymbol{Y}(\omega)$, which is challenging to estimate directly. Instead, we exploit an approximation, which assumes $\boldsymbol{Y}(\omega)$ as constant during (t_{i-1}, t_i) if the sampling interval δ_t is small. That is, $\boldsymbol{Y}(\omega) \approx \boldsymbol{Y}(t_{i-1})$ for $\omega \in (t_{i-1}, t_i)$. Hence, $S_m(t_i) - S_m(t_{i-1})$ can be approximated as follows:

$$S_m(t_i) - S_m(t_{i-1}) \approx [R_m + \boldsymbol{c}_m' \boldsymbol{Y}(t_{i-1})]\, \delta_t + W_m(\delta_t).$$

By virtue of this discretization, $S_m(t_i) - S_m(t_{i-1})$ follows a normal distribution with the mean $[R_m + \boldsymbol{c}_m' \boldsymbol{Y}(t_{i-1})]\, \delta_t$ and the variance $\sigma_{w,m}^2 \delta_t$, where $\sigma_{w,m}^2 \delta_t$ is the variance of the noise term $W_m(\delta_t)$. Consequently, parameters $R_m, \boldsymbol{c}_m'$ and $\sigma_{w,m}^2$ can be estimated through regression.

Suppose the last observation epoch is t_k. Let $\{\boldsymbol{s}(t_0), \ldots, \boldsymbol{s}(t_k)\}$ and $\{\boldsymbol{y}(t_0), \ldots, \boldsymbol{y}(t_k)\}$ represent the observed tool wear and quality degradation, respectively. Thus, the regression-based estimation results for each $m = 1, 2, \ldots, M$ are as follows:

$$\left[\hat{R}_m, \hat{\boldsymbol{c}}_m'\right]' = \left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1} \boldsymbol{X}' \delta \boldsymbol{S}_m$$

and

$$\hat{\sigma}_{w,m}^2 = \frac{\left|\left[1 - \boldsymbol{X}\left(\boldsymbol{X}'\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\right] \delta \boldsymbol{S}_m\right|^2}{k - m - 1},$$

where $\delta \boldsymbol{S}_m = (s_m(t_1) - s_m(t_0), \ldots, s_m(t_k) - s_m(t_{k-1}))'$ and $\boldsymbol{X} = (1, \boldsymbol{y}'(t_0); 1, \boldsymbol{y}'(t_1); \ldots; 1, \boldsymbol{y}'(t_k))$. The r^2 value of each regression represents how well the linear assumption in (5) holds in the data. A high r^2 value will indicate a sufficient linearity. If the linearity does not hold, a higher order model may be necessary.

Usually, multiple runs of MMP operations are required to have a sufficient number of estimated values. Then, we can use multiple $\hat{R}_m$ to estimate the prior mean and variance of R_m, which are represented by $\mu_{R,m}$ and $\sigma_{R,m}^2$. We can also take the average of all $\hat{\boldsymbol{c}}_m'$ and $\hat{\sigma}_{w,m}^2$ to get a more reliability estimates.

In this numerical study, we generate a training data set that consists of simulated tool wear $\boldsymbol{S}(t)$ and quality degradation $\boldsymbol{Y}(t)$ from 50 runs of simulation according to the specifications listed in Table II. The average r^2 value from regression applied to the training set when m = 1,2,3,4,5,6 is equal to 0.98, 0.99, 0.91, 0.95, 0.42, 0.61, respectively. The fifth and sixth r^2 values that correspond to the outer and inner dies on the third stage are lower than those of dies on the other two stages. This is because according to the simulation configuration, the slope of regression represented by $\boldsymbol{c}_m'$ is smaller when m = 5,6 compared to when m = 1,2,3,4, which reduces the sum-of-square regression. From the overall r^2 values, we can see that the linear model in (5) can sufficiently represent the simulated data. We list the estimated values of $\widehat{\boldsymbol{C}}$, $\widehat{\boldsymbol{\Sigma}}_{\boldsymbol{W}}$, $\widehat{\boldsymbol{\mu}}_{\boldsymbol{R}}$ and $\widehat{\boldsymbol{\Sigma}}_{\boldsymbol{R}}$ based on the training data set in Table II. It is demonstrated in Table II that our estimated values are very close to their true values. These estimated parameter values are subsequently used to predict the system residual life in the following sections.

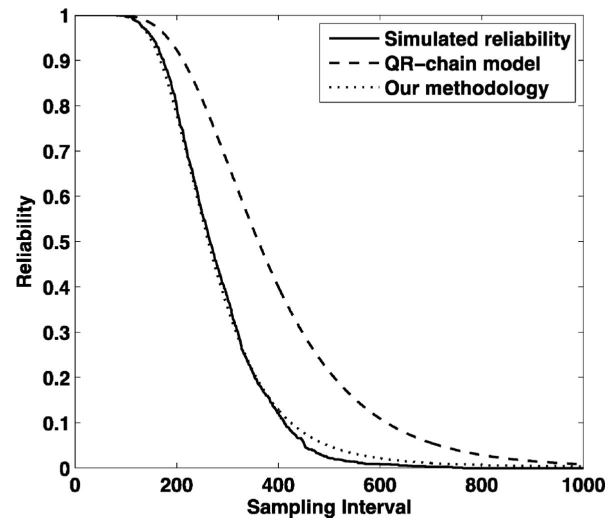

Fig. 3. System reliability when $\boldsymbol{C}$ is equal to the value in Table II.

C. Performance Comparison

1) When the Impact of Product Quality on Tool Wear Exists (i.e., $\boldsymbol{C}$ is Equal to the Value in Table II): First, we compare the performance of our methodology with the QR-chain model when no real-time quality measurements are available. Under this condition, according to the derivation in Section IV, our work will rely on historical information to derive a lifetime distribution/reliability of the system. Similarly, the QR-chain model also computes offline system reliability. To investigate the accuracy of reliability estimation using both methodologies, we simulate the actual system reliability by generating 1000 runs of simulation and record their actual lifetimes.

We present in Fig. 3 the estimated system reliability without incorporating real-time quality measurements based on our proposed model, the QR-chain model, and the actually simulated data. We observe that the reliability estimate based on our model is very close to the simulated reliability. However, the reliability estimate based on the QR-chain model demonstrates a bias caused by overestimate. We believe that this bias is mainly caused by the fact that the QR-chain model does not account for the effect of quality degradation on the instantaneous rate of tool wear, which tends to accelerate quality degradation and the system failure.

Next, we compare the performance of our methodology with the QR-chain model when real-time quality measurements are available. According to Section IV, our methodology can incorporates real-time quality measurements to online update the RLD of the system, which provides up-to-date information of the predicted lifetime. Since the QR-chain model does not incorporate real-time monitoring information to refine/update its model, the predicted lifetime is calculated solely based on the following logic: At any observation epoch, if an MMP has not failed yet, we compute the conditional lifetime distribution given the survival at current epoch using the reliability curve presented in Fig. 3. Then, we choose the median of the conditional lifetime distribution as the updated lifetime prediction.

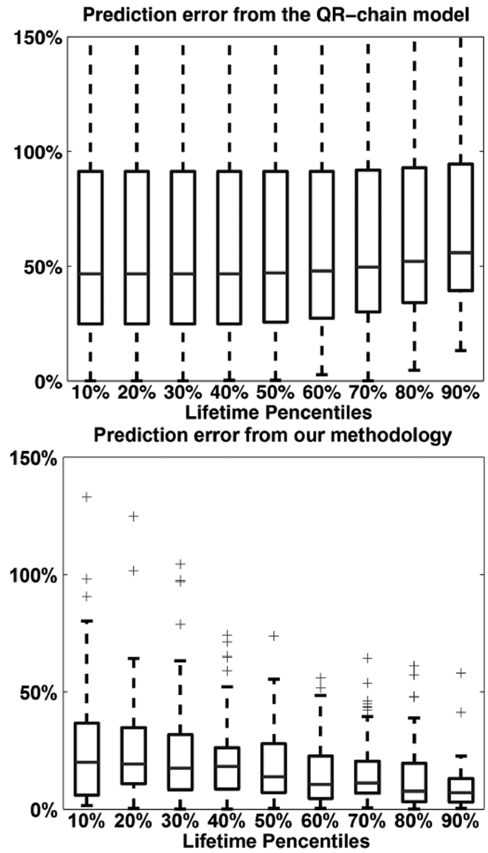

Fig. 4. Comparison of prediction error associated with our methodology and the QR-chain model when $\boldsymbol{C}$ is equal to the value in Table II.

The quantity of performance measurement is the lifetime prediction error defined below in (15)

$$\text{prediction error} \\ = \left| 1 - \frac{\text{Current observation epoch} + \text{Predicted residual life}}{\text{True lifetime}} \right|. \tag{15}$$

Here, current observation epoch + predicted residual life is equal to the predicted lifetime, and the true lifetime is obtained via the simulated signals.

Particularly, for each run of simulation, we predict the residual life of the MMP at the $10^{\text{th}}, 20^{\text{th}}, \ldots, 90^{\text{th}}$ lifetime percentiles. In other words, we estimate its residual life when $10\%, 20\%, \ldots, 90\%$ of the system lifetime is revealed. Then, we calculate the prediction error according to (15).

We generate a testing data set that consists of simulated data from 50 runs of simulation. The prediction errors of the testing data set using both our methodology and the QR-chain model are presented in Fig. 4. We observe that the prediction error from our methodology exhibits progressive reduction as we incorporate more real-time observations, while the prediction error from the QR-chain model remains high at all lifetime percentiles. We believe that this is due to two reasons: First, the QR-chain model does not account for the effect of quality degradation on tool wear, which tends to accelerate quality degradation and the system failure, and thus leads to a consistent bias in the prediction. Second, the estimated reliability according to the QR-chain model is a static estimate based on the reliability data, whereas our approach incorporates the real-time quality measurements that capture the latest status of the system.

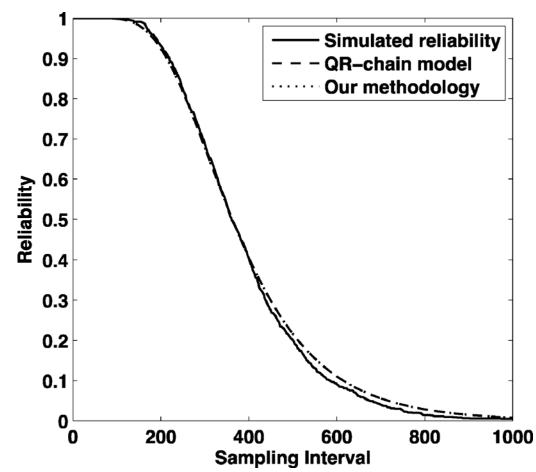

Fig. 5. System reliability when $\boldsymbol{C} = \mathbf{0}$.

2) When the Impact of Product Quality on Tool Wear Does not Exist (i.e., $\boldsymbol{C} = \mathbf{0}$): In this section, we focus on the situation when the impact of product quality on the rate of tool wear does not exist, which is the situation that the QR-chain model is capable of modeling. Same as Section V-C1, the performance comparison consists of two parts.

First, we compare the performance of our methodology with the QR-chain model when no real-time quality measurements are available. Actual system reliability is computed by generating 1000 runs of simulation and recording their lifetimes. Fig. 5 illustrates the estimated system reliability based on our methodology, the QR-chain model, and the simulated reliability. We observe that reliability estimates based on the two methodologies are both very close to the simulated reliability. This is because when the impact of product quality on the instantaneous rate of tool wear does not exist and no real-time quality measurements are available, both methodologies utilize the same model and the same historical data to capture system reliability.

Second, we compare the performance of our methodology with the QR-chain model when real-time quality measurements are available. Same as in Section V-C1, we generate data from 50 runs of simulation and show the lifetime prediction errors from both methodologies at the $10^{\text{th}}, 20^{\text{th}}, \ldots, 90^{\text{th}}$ lifetime percentiles in Fig. 6. We notice that the prediction error from the QR-chain model is significantly lower than what is presented in Fig. 4. This is mainly due to the fact that when the impact of quality degradation on the instantaneous rate of tool wear (represented by $\boldsymbol{C}$) does not exist, the lifetime distribution estimated from the QR-chain model is not biased. We also observe that the prediction error from our methodology exhibits similar progressive reduction as shown in Fig. 4. On the contrary, prediction error from QR-chain model remains at a high level at all lifetime percentiles. This difference is mainly due to the fact that the prediction using the QR-chain model solely based on the survival of MMP, whereas our approach incorporates real-time quality

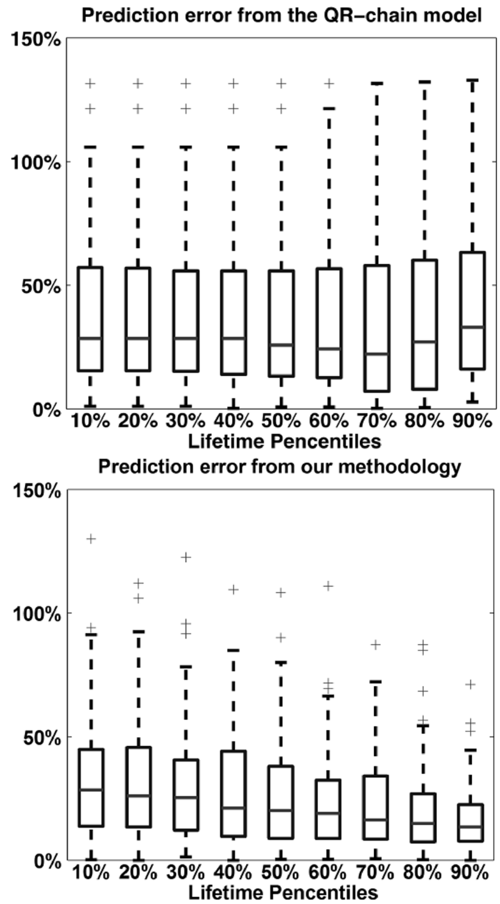

Fig. 6. Comparison of prediction error associated with our methodology and the QR-chain model when $\boldsymbol{C} = \mathbf{0}$.

measurements that capture the latest degradation status of the system to reason about the future degradation of the system.

D. Sensitivity Analysis

In this section, we investigate how the performance of our model changes at the values of key model parameters change. Specifically, we conduct three numerical experiments with the goal of studying how the accuracy of the lifetime prediction changes under: 1) different amplitudes of process noise; 2) different magnitudes of impact from quality degradation on the instantaneous rate of tool wear; and 3) different numbers of stages. To facilitate the simulation process, we begin with a baseline setup with parameter values chosen according to Table II. We then define two factors: n_1 and n_2, where n_1 is used to define different noise levels, and n_2 is used to scale the matrix $\boldsymbol{C}$. The detailed procedure of the sensitivity analysis is summarized below:

1) **Experiment I**: Evaluate the effect of the amplitude of process noise by multiplying the value of $\boldsymbol{\Sigma_Z}$ in Table II with the scale parameter n_1, where $n_1 = 0.1,0.5,1,2,5$.
2) **Experiment II:** Evaluate the effect of the magnitude of impact from quality degradation on the instantaneous rate of tool wear by multiplying the value of $\boldsymbol{C}$ in Table II with n_2, where $n_2 = 0,0.5,1,2,4$.
3) **Experiment III:** Evaluate the effect of the number of stages by choosing $M = 3,6,9,12,15$. The values of the other parameters related to this experiment are listed in the Appendix.

For each of these three experiments, we generated 50 runs of simulation and present the average prediction errors from both methodologies at the 30th, 60th, and 90th lifetime percentiles in Figs. 7–9, respectively. We observe that:

1) The prediction error from both models increase with the amplitude of process noise (scaled by n_1). This is because a higher process noise will lower the prediction accuracy.
2) As the magnitude of the impact from quality degradation on tool wear (scaled by n_2) increases, the prediction error from the QR-chain model increases while the prediction error from our methodology decreases. This is because a higher amplitude of matrix $\boldsymbol{C}$ accelerates the system failure, and thus the QR-chain model yields a higher bias in predicting the lifetimes. On the other hand, $\boldsymbol{C}$ is compounded in both the instantaneous rate of tool wear and the process noise. Under current parameter setting, increasing the amplitude of $\boldsymbol{C}$ increases the ratio of the instantaneous rate of tool wear versus the process noise, which improves the prediction accuracy of our methodology.
3) Prediction errors from both methodologies slightly decrease as the number of stages increase. This is because as the number of stages increases, the impact of tool wear and process noise from preceding stages on the quality degradation of subsequent stages increases (reflected in matrices $\boldsymbol{A}$ and $\boldsymbol{B}$), which tend to accelerate the system failure. This is equivalent to increasing the ratio of the instantaneous rate of tool wear versus the process noise. Thus, both methodologies show improved accuracy.
4) The prediction error from our methodology improves as the lifetime percentile increases. This is because our methodology leverages the real-time quality measurements to on-line update the RLD of the systems.

VI. Details for Industrial Implementation

In this section, we describe the procedure of applying our methodology in real industrial applications, which will serve as a guideline for practitioners.

First, several model parameters in our methodology need to be determined using the configuration of the MMPs: The constant model parameters $\boldsymbol{A}$ and $\boldsymbol{B}$ that represent the impact of tool wear and process noise on quality degradation should be derived based on the configuration of the MMP, which includes the layout of the manufacturing stages, the locations of quality measurement points, the characteristic of process noise, etc. Take the auto-body assembly process presented by Chen *et al.* [10] as an example. Matrix $\boldsymbol{A}$ can be calculated using the geometrical relationship between the locating pin (tool) and the quality measurement points on the raw parts. Matrix $\boldsymbol{B}$ can be calculated considering the randomness of allocating raw parts onto the locating pin. The variance of process noise $\boldsymbol{\Sigma}_z$ represents the significance of the noise, which may include the sensitivity of the quality monitoring sensors, the precision of allocating raw parts onto each stage, etc.

Second, model parameters regarding to tool wear, including $\boldsymbol{\mu_R}$, $\boldsymbol{\Sigma_R}$, $\boldsymbol{\Sigma_W}$, and $\boldsymbol{C}$, need to be estimated from historical data. Requirements of historical data are outlined next:

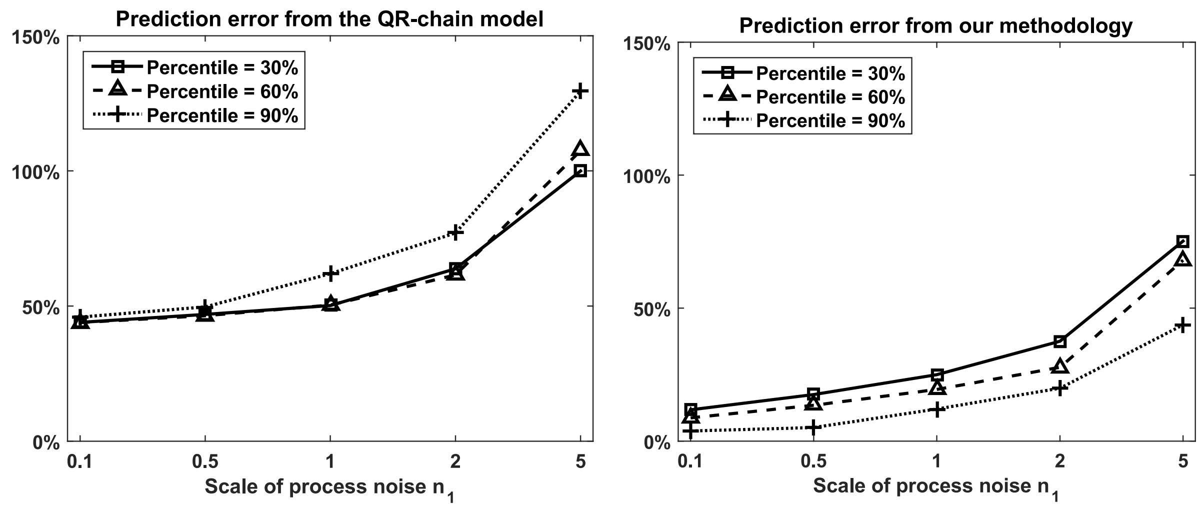

Fig. 7. Average prediction errors at different scales of process noise.

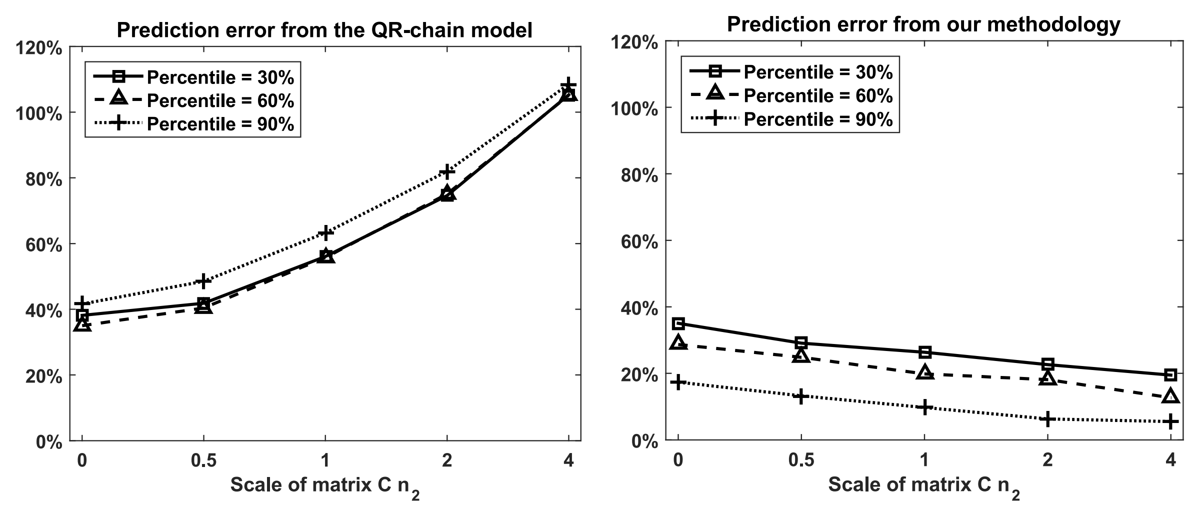

Fig. 8. Average prediction errors at different scales of matrix $\boldsymbol{C}$.

1) Several MMPs with an identical configuration need to be monitored from the beginning of operation until failure.
2) Both tool wear and product quality degradation in each MMP need to be observed periodically, i.e., using online quality and tool wear monitoring techniques.
3) Without loss of generality, the observation (sampling) interval can be set as a constant. The value of the interval may vary in different applications.

With the historical data available, parameters can be estimated by following the same linear regression procedure discussed in Section V-B.

Finally, with the above model parameters estimated off line, we can incorporate real-time quality measurements to on-line update the RLD of an MMP functioning in the field. This requires real-time quality measurements only. No tool wear monitoring is needed. The observation interval can be set to be equal to that of the historical data. The procedure of updating the RLD has been discussed in Section IV. In summary, with quality degradation being observed up to any epoch t_k, we first update the posterior distribution of the natural rate of tool wear $\boldsymbol{R}$ according to Proposition 1. Based on the posterior distribution, the RLD of the MMP is updated according to (12). Finally, we utilize the median of the RLD as the residual life prediction.

VII. Conclusion

In this paper, we focus on modeling the interactive relationship between tool wear and product quality degradation in MMPs. On one hand, tool wear in a stage may affect the quality degradation at current and subsequent stages. On the other hand, the quality degradation of incoming products may influence the tool wear process in subsequent stages. To capture this complex relationship, we propose an interaction model which incorporates a linear model to capture the effect of tool wear and process noise on quality degradation and a multidimensional linear SDE model to characterize the effect of quality degradation on the process of tool wear. In addition, we assume that the natural rate of tool wear in each stage is stochastic, which can be characterized by a prior distribution. Given the interactive model and the prior distribution, we propose to incorporate real-time quality measurements and a Bayesian updating scheme to online predict the RLDs of MMPs.

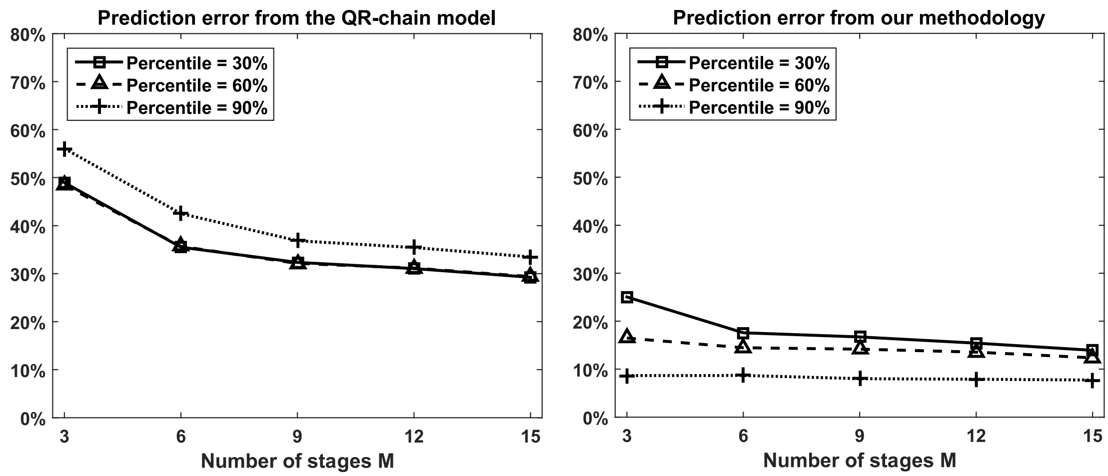

Fig. 9. Average prediction errors at different numbers of stages.

In order to validate our methodology, we conduct a series of numerical studies by simulation. The performance of our methodology is evaluated in terms of the lifetime prediction error. We also compare the performance of our methodology with the QR-chain model. From the simulation study, we conclude that the performance of both our methodology and the QR-chain model is sensitive to the process noise, the impact of quality degradation on tool wear, as well as the number of stages. Yet, our methodology consistently outperforms the QR-chain model due to considering the impact of quality degradation on the instantaneous rate of tool wear as well as incorporating real-time quality measurements.

Future work may focus on the implementation of our methodology into real manufacturing industries. This will help to validate and improve our model using real data. In addition, we will consider more complex models such as nonlinear models to capture the sophisticated relationship between tool wear and product quality degradation. Furthermore, we will consider incorporating catastrophic failure into our model.

Appendix

Choosing the parameter values for different numbers of stages M:

1) The elements of matrices $\boldsymbol{A}$, $\boldsymbol{B}$, are determined according to (14).
2) The elements of matrix $\boldsymbol{C}$ are determined as follows:

$$c_{i,j} = \begin{cases} 0.004, & \text{if } j = 2i - 1 \text{ or } 2i \\ 0.02, & \text{if } j = 2i - 3 \\ 0.012, & \text{if } j = 2i - 2 \\ 0, & \text{Otherwise} \end{cases}.$$

3) At every stage, the prior mean of the natural rate of tool wear and the variance of the Brownian motion noise in the tool wear model are equal to the values provided by Chen and Jin [40] multiply with 1000 (operations per sampling interval).
4) The prior variance of the natural rate of tool wear is determined as follows:

$$\boldsymbol{\Sigma_R} = \text{diag}\left(\sigma^2_{\boldsymbol{R},1}, \ldots, \sigma^2_{\boldsymbol{R},M}\right), \text{ where } \sigma_{\boldsymbol{R},i} = \frac{\mu_{\boldsymbol{R},i}}{3}.$$

5) The failure threshold for each stage $m = 1, \ldots, M$ is equal to 0.5.

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