Surrogate Model Based Control Considering Uncertainties for Composite Fuselage Assembly

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ABSTRACT

Shape control of composite parts is vital for large-scale production and integration of composite materials in the aerospace industry. The current industry practice of shape control uses passive manual metrology. This has three major limitations: (i) low efficiency: it requires multiple trials to achieve the desired shape during the assembly leading to longer assembly times; (ii) non-optimal: it makes it challenging to reach optimal deviation reduction; and (iii) experience-dependent: highly skilled engineers are required during the

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assembly process. This paper describes an automated shape control system that can adjust composite parts to an optimal configuration in a manner that is highly effective and efficient. The objective is accomplished by (i) building a finite element analysis platform, validated by experimental data; (ii) developing a surrogate model with consideration of actuator uncertainty, part uncertainty, modeling uncertainty, and unquantified uncertainty to achieve predictive performance and embedding the model into a feed-forward control algorithm; (iii) conducting multivariable optimization to determine the optimal actions of actuators. We show that the surrogate model considering uncertainties achieves satisfactory prediction performance and that the automated optimal shape control system can significantly reduce the assembly time with improved dimensional quality.

Keywords: composite assembly, shape control, assembly deviation prediction, surrogate model, finite element analysis, uncertainty

1. INTRODUCTION

Composite parts have been widely used in industry due to their unique characteristics, such as high strength-to-weight ratio, high stiffness-to-weight ratio, potentially long life and low life-cycle cost. Over the past decades, aircraft manufacturers have been gradually increasing the application of composite parts in the design of aircrafts. As an example, a Boeing 787 aircraft has major structural parts (e.g. fuselage, wings, floor beams et al.) made of composite materials and the composite parts represent more than 50% by weight [1]. In addition, the Boeing 787 was designed to be the first commercial airliner with the fuselage comprised of one-piece composite barrel section. Assembly of composite parts is a manufacturing process of joining two or more composite parts together using various joining techniques. Due to multiple suppliers and multiple manufacturing batches, dimensional variability of composite parts inevitably exists when
the composite parts are assembled. For example, significant dimensional issues were discovered when two fuselage sections were assembled [2]. Therefore, effective methodologies are needed for dimensional analysis, variation reduction, and optimal shape adjustment for composites parts assembly to achieve satisfactory dimensional accuracy.

In order to achieve dimensional (or shape) adjustment of a fuselage section, a number of adjustable actuators, as shown in Fig. 1 (a, b), may be used in practice to provide push or pull forces to reduce dimensional deviations and generate the required shape of the parts. In Fig. 1 (b), the center of the fuselage edge circle without the actuators is defined as the original point of the coordinate system. The horizontal intermediate cross plane is defined as the X-Y plane of the coordinate system. Z-axis is vertical upwards with the X-Y plane. The responses are the total dimensional deviations under the impact of actuators' forces, (Here we focus on multiple key points in fuselage edge plane near the actuators, as shown in Fig. 1 (c)). Currently, the adjustment of each actuator is conducted by a trial-and-error method based on in-situ measurements of the section until reaching a desired acceptable shape. The current practice has three limitations: (i) low efficiency: it may take long time and multiple trials to adjust actuators to achieve a desired shape during the assembly process; (ii) non-optimality: it may reach an acceptable dimensional quality rather than the optimal deviation reduction; (iii) highly skilled engineers requirement: the quality and efficiency of assembly depend on the skills of engineers, which increases the uncertainties of the time and quality of the fuselage assembly task. In order to reduce the cycle time, increase the productivity, as well as
decrease the dimensional variation of the composite assembly, an automatic optimal shape control (AOSC) system will be developed to realize composites modeling and control for optimal adjustment of all actuators simultaneously.

Fig. 1 Schematic diagram for shape adjustment

In the literature, there are numerous research topics have been conducted for modeling and analysis of dimensional variation reduction and control for the assembly of isotropic metal parts. Jin and Shi introduced a state-space modeling methodology for dimensional control of sheet metal assembly [3], and later extended to the Stream of Variation (SoV) methodology that has been developed and implemented in various multistage manufacturing processes [4]. Djurdjanovic and Ni proposed a linear state-space SoV modeling by deriving explicit expressions for the influence of the errors in fixtures, locating datum features and measurement datum features in the multistation machining process [5, 6]. A method of influence coefficient (MIC) was exploited to combine engineering structural mechanics with statistical methods to model the relationships between the incoming part deviations and the output assembly deviations in single-station [7] and multi-station assembly processes [8]. A detailed literature review
of modeling and variation analysis of compliant assembly of metal parts can be found in the papers [9, 10]. It is a more challenging task to model the dimensional variation of compliant composite parts with anisotropic characteristics. Zhang and Shi built a stream of variation (SoV) model for compliant composite part assembly in single-station [9] and multi-station processes [10]. In their model, part manufacturing errors, fixture position errors, and relocation induced errors were considered for the analysis of dimensional variation and its propagation in a multistage process. However, this SoV modeling method cannot be directly used in the AOSC system of composite fuselage assembly because that the SoV model in [9] and [10] is based on engineering physical mechanics, and material property parameters such as equivalent stiffness matrix and compliance matrix about the composite parts are required. It is difficult to obtain an accurate estimation of those parameters, especially for large composite parts with complex structures. Other than the SoV type of models [3-6, 9-10], other types of models have been used to improve the quality control of assembly process, such as robust pattern-matching technique for variation source identification [11], adaptive product, process and tooling design strategy for optimal dimensional quality [12], modeling of operator effects on process quality [13], and variation analysis using component geometric covariance [14] et al. However, these models are mainly focused on variation analysis and modeling instead of shape control, thus they cannot be used directly in the composite fuselage shape control problem.

In the literature about control strategies, Zhong et al. [15] proposed a feed-forward control strategy that explicitly took the uncertainties of model coefficients into account. However, the model with uncertainties was developed for rigid metal parts and
the parameters depended on the geometrical transformation of rigid parts, which was not suitable for composite parts with highly nonlinear anisotropic properties. Djurdjanovic and Ni [16] proposed a novel SoV model based stochastic control of dimensional quality to minimize the least square of difference between the dimensions at the end of the line and the nominal dimensions. In their model, the measurement and process noise, as well as the accuracy of actuation of flexible tooling elements are considered. As this control strategy is built upon a SoV model with the state space structure [5, 6], which is not suitable for the fuselage shape control where a different model structure is adopted. For other control strategies considering model uncertainties, adaptive control [17], H-Infinity optimal control [18], and Fuzzy control [19] were proposed and applied in the dynamic systems. However, these methods cannot be directly used in the shape control of composite fuselage assembly process because of the difference between dynamic model and static dimensional model for variation reduction [15]. Thus, an effective modeling and control methodology needs to be developed for the AOSC system of composite fuselage assembly.

Next, we review the literature about model uncertainty. Since the model uncertainty is an important and widely encountered problem, which is associated with control theory, automation, mechanical engineering, manufacturing, statistics and applied mathematics [20-24], it is a challenging task to illustrate all different techniques of treating model uncertainty in all those domains. For model uncertainty in control theory [20, 21], uncertainty is illustrated via uncertain parameters or disturbances within a typically compact set, and robust or adaptive control is developed to deal with
uncertainty explicitly. Decision support models evaluate the extent of uncertainties and realize the balance between decision benefits and risk management [22]. Statistical models consider structural uncertainty via model selection as well as model validation, and consider parametric uncertainty by specified stochastic terms with random distributions [23]. More literature on uncertainty modeling can be found in [24]. In this paper, we are mainly focused on the review of the model uncertainty corresponding to statistical models, which is closely related to the topic of this paper. In Section 2.1, the authors review the conventional methods including the multivariate Regression model, the Universal Kriging model, and the Stochastic Kriging model, where uncertainties are illustrated by different stochastic terms.

In the paper, we propose a methodology to develop a surrogate model considering various uncertainties and applying this model to achieve automatic optimal shape control. An overview of the proposed methodology is given in Fig. 2. According to the tooling parameters (e.g. number, positions, and maximum forces of actuators), material parameters, and dimensional parameters of the composite part, we build a finite element analysis (FEA) platform for the methodology development and validation. The FEA platform is validated by a set of real experiments with industrial set-ups. Based on the validated FEA platform, a scheme of computer experiment is generated by design of experiment and then conducted to collect input variables and output responses. A surrogate model considering part uncertainty, actuator uncertainty, and model uncertainty is developed to link the relationship between inputted actuators’ forces and outputted dimensional deviations. The surrogate model is validated by the testing
experiments. After that, the surrogate model considering uncertainties is embedded into an automatic optimal shape control algorithm. A feed-forward control law is obtained by solving the optimization problem that minimizes the weighted sum of square of dimensional deviations between the real dimensional positions and the designed positions of the part. The main contribution of this paper is to study the composite fuselage shape control problem. In conventional control papers, most of the models are differential or difference equations, e.g. dynamic equations, which cannot be adequately used to describe the dimensional control problem for composite fuselage. In this paper, we use a surrogate model with Gaussian process term and other random terms, which is able to model the relationships among the fuselage dimension and actuators’ forces, as well as address actuator uncertainty, part uncertainty, modeling uncertainty and unquantified uncertainty. With this new type of model, we propose a feedforward control strategy and apply it to a composite fuselage shape control problem.
The remainder of this paper is organized as follows. Section 2 illustrates the surrogate modeling process considering various uncertainties. A maximum likelihood estimation (MLE) algorithm is developed to realize the parameter estimation and the response prediction for incoming inputs. Section 3 describes a feed-forward automatic control algorithm for shape control of composite assembly process. Section 4 presents the process of building the finite element analysis platform and its validation by real experimental data. In addition, a case study is conducted to demonstrate the implementation procedures, including design of experiment, prediction results, control results, sensitivity analysis, and stress analysis. Finally, a brief summary is provided in Section 5.
2. SURROGATE MODEL CONSIDERING UNCERTAINTIES

In this section, we are interested in building a model to link the response variables (dimensional deviations) with control variables (actuators’ forces). The model should have sufficient prediction capability and precision with consideration of various uncertainties. We will first review the conventional multivariate regression model, the Universal Kriging Model (UKM), and the Stochastic Kriging Model (SKM). Then, a novel Surrogate Model considering Uncertainties (SMU) will be proposed. After that, a maximum likelihood estimation (MLE) algorithm is developed to estimate the parameters of the SMU.

2.1 Review of Conventional Models

2.1.1 Multivariate Regression Model

According to the mechanics of composite material and classical lamination theory, a linear relationship between dimensional deviations and actuators’ forces is expected within the elastic zone [25]. Assuming that there is a noise term to describe the unquantified errors, such as error from modeling nonlinear property by linear approximation, error from part uncertainty et al., we can use a linear regression model [26] to describe how the actuators’ forces impact on the part deviations,

\[ Y_i(F) = FS + \varepsilon_i \]  

where \( F_{1 \times q} \) is the actuator force vector with dimension of \( 1 \times q \); \( Y_i(F) \), with dimension \( 1 \times p \), is the output dimensional variable under the \( i^{th} \) replication at \( F \); \( S \) is the sensitivity matrix that quantifies the relationship between actuators’ forces and fuselage
deviations; $\varepsilon_i$ is assumed to follow a multivariate normal distribution $\varepsilon_i \sim \mathcal{N}(0, \Sigma_e)$, that represents the unquantified errors.

### 2.1.2 Universal Kriging Model

Now consider an ideal simulation experiment where the response could be observed without noise. The Universal Kriging Model (UKM) could be developed to show the deterministic relationship in a statistical framework [27]:

$$ Y_j(F) = FS_j + z_j(F) $$

where $Y_j(F)$ denotes the $j^{th}$ dimensional variable of fuselage at the actuators’ forces $F$; $S_j$ represents the sensitivity matrix corresponding to the $j^{th}$ response, $j = 1, 2, \ldots, p$; $z_j(F)$ is a stochastic process with mean 0. Usually, we consider $z_j(F)$ as a Gaussian process $z_j \sim \mathcal{GP}(0, \Sigma_{z_j})$, where $\Sigma_{z_j}$ is a covariance matrix of stochastic process. Similar to the notation in the reference [27], the covariance between vector $a$ and vector $b$ satisfies $\Sigma_{z_j}(\tau_j^2, \theta, a, b) = \tau_j^2 R_{z_j}(\theta, a - b)$, where $R_{z_j}(\theta, a - b)$ is one of the correlation functions. With spatial correlation, $z_j(F_m)$ and $z_j(F_n)$ tend to be similar (e.g. $R_{z_j,(m,n)} = R_{z_j}(\theta, F_m, F_n)$ tends to be large) if $F_m$ and $F_n$ are close. Note that, since the response of simulation could be observed without noise, there is no replication needed.

### 2.1.3 Stochastic Kriging Model

Considering both a noise term $\varepsilon_i$ and a stochastic process term $z_j(F)$ that are regarded as intrinsic uncertainty and extrinsic uncertainty respectively, the Stochastic Kriging model can be developed [28] as

$$ Y_{ij}(F) = FS_j + z_j(F) + \varepsilon_{ij}, $$
where \(Y_{ij}(F)\) denotes the \(j^{th}\) dimensional variable of the fuselage under the \(i^{th}\) replication at the actuators’ forces \(F\); \(S_j\) represents the sensitivity matrix corresponding to the \(j^{th}\) response, \(j = 1, 2, \ldots, p\); \(z_j(F)\) is a stochastic process with mean 0, which represents the extrinsic uncertainty from randomly sampling from functional mapping during surrogate modelling; The intrinsic noise \(\varepsilon_{ij}\) is assumed to be independent and identically distributed with a normal distribution \(\varepsilon_{ij} \sim \mathcal{N}(0, \sigma_{\varepsilon j})\).

For experiment setting, pairs \((F_t, n_t)\), \(t = 1, 2, \ldots, k\), where \(n_t\) is the number of simulation replications taken at the design setting \(F_t\), are implemented to collect the simulation data. The stochastic process \(z_j(F)\) is usually assumed to be a Gaussian process \(\mathcal{G}(0, \Sigma_{zj})\) with covariance matrix \(\Sigma_{zj}\) with dimension \(k \times k\). For any two vectors of the actuators’ forces, the covariance is \(\Sigma_{zj}(F_m, F_n) = \text{cov}[z_j(F_m), z_j(F_n)]\). Let \(\Sigma_{zj}(F_{0,t}) = (\text{cov}[z_j(F_0), z_j(F_1)], \ldots, \text{cov}[z_j(F_0), z_j(F_k)]\) as the covariance between \(z_j\)’s at design points and new actuators’ forces \(F_0\).

We know the sample mean at \(F_t\) as \(\bar{Y}_j(F_t) = \frac{\sum_{i=1}^{n_t} Y_{ij}(F_t)}{n_t}\). Let \(\bar{Y}_j = (\bar{Y}_j(F_1), \ldots, \bar{Y}_j(F_k))^T\), and the noise covariance matrix is \(\Sigma_{\varepsilon j}\), a \(k \times k\) covariance matrix with \((a, b)\) element is \(\text{cov}\left[\sum_{i=1}^{n_a} \varepsilon_{ij}(F_a)/n_a, \sum_{i=1}^{n_b} \varepsilon_{ij}(F_b)/n_b\right]\). Let \(F_{DOE} = (F_1; \ldots; F_k)\) denote all the design points of the actuators’ forces. Let \(R_j(F_{0,t}) = (\text{cov}[Y_j(F_0), \bar{Y}_j(F_1)]; \ldots; \text{cov}[Y_j(F_0), \bar{Y}_j(F_k)]\).

We can get the MSE-optimal linear predictor as

\[
\hat{Y}_j(F_0) = F_0 S_j + R_j(F_{0,t})^T [\Sigma_{zj} + \Sigma_{\varepsilon j}]^{-1} (\bar{Y}_j - F_{DOE} S_j).
\]

**2.2 Surrogate Model Considering Uncertainties**
The conventional models discussed in Section 2.1 cannot handle all the uncertainties in our application, such as uncertainties from actuators variability and part-to-part variability (uncertainties sources will be explained further in Section 2.4). We propose a novel surrogate model considering various uncertainties. Experiment design pairs \((F_t, n_t), t = 1, 2, \ldots, k\), are implemented to collect the response, which is the dimensional deviations \(Y_{ij}(F_t)\). Afterwards, a surrogate model considering uncertainties is proposed as

\[
Y_{ij}(F_t) = F_t S_j + F_t \bar{S}_j + \bar{F}_t S_j + z_j(F_t) + \epsilon_{ij}(F_t),
\]

where \(i = 1, 2, \ldots, n_t\); \(j = 1, 2, \ldots, p\); \(p\) is the number of output responses (key dimensional features). \(F_t\) is the target actuators’ forces vector (row vector) to be implemented; \(\bar{F}_t\) is an additional random deviations of actuators’ forces that results from the actuators’ uncertainty, whose distribution \(\mathcal{N}(0, \Sigma_F)\), can be obtained from the tolerance of actuators instructions; \(F_t + \bar{F}_t\) is a true actuators’ force vector at the \(i^{th}\) replication of \(F_t\). \(S_j\) is an ideal sensitivity vector (column vector) and \(\bar{S}_j\) represents the random sensitivity vector variability from the part uncertainty, which is assumed to follow \(\mathcal{N}(0, \Sigma_S)\). Both \(S_j\) and \(\Sigma_S\) are unknown. \(z_j(F_t)\) is a zero mean stochastic process, which is assumed to be a stationary Gaussian process \(z_j \sim \mathcal{GP}(0, \Sigma_z)\). \(\epsilon_{ij}(F_t)\) is assumed to follow an independent normal distribution \(\epsilon_{ij}(F_t) \sim \mathcal{N}(0, \sigma^2_{ij}(F_t))\), that represents the inherent simulation variability in a stochastic simulation. \(\epsilon_{ij}, \bar{F}_t, \bar{S}_j\), and \(z_j\) are assumed to be mutually independent, and their higher order interaction terms \(\bar{F}_t \cdot \bar{S}_j\) is assumed to be zero. The model can be interpreted as a decomposition of the response \(Y_{ij}(F_t)\) into
three parts: a regression term \( F_t S_j + F_t \tilde{S}_j + \bar{F}_t S_j \), a Gaussian process term \( z_j(F_t) \), and a noise term \( \epsilon_{ij}(F_t) \).

Let \( \bar{Y}_j(F_t) \) be the sample mean at \( F_t \), that denotes \( \bar{Y}_j(F_t) = \frac{\sum_{i=1}^{n_t} Y_{ij}(F_t)}{n_t} \). Let \( F_{DOE} = (F_1; ...; F_k) \) denote the design matrix of actuators’ forces. Let \( Y_j(F_0) \) be the true response at \( F_0 \). The purpose of building a surrogate model is to realize accurate prediction on \( Y_j(F_0) \). Let \( \Sigma_{\epsilon_j} \) represent the covariance matrix associated with the noise term with \( (a,b) \) element \( \text{Cov} \left[ \sum_{i=1}^{n_a} \epsilon_{ij}(F_a)/n_a, \sum_{i=1}^{n_b} \epsilon_{ij}(F_b)/n_b \right] \), and \( \Sigma_{\epsilon_j} \) is assumed to be diagonal, which indicates that the correlations among the noise terms under different design points are zeros.

In our method, we model the actuators’ uncertainty, part uncertainty, and unquantified uncertainty by using random vectors with Gaussian distributions and describe the modeling uncertainty with a Gaussian process term. The reason to use this technique is that the proposed surrogate model considering uncertainties can decompose the dimensional deviations into multiple components. These components are associated with different physical sources of uncertainties. In addition, the assumption of Gaussian distribution for actuators’ forces and sensitivity vector is reasonable because of engineering tolerance of specific variables. The Gaussian process can extract the complex material properties information from the training samples and realize good prediction performance.

Firstly, we derive the joint distribution shown in Proposition 1.

**Proposition 1.** The model (4) implies that \( Y_j(F_0) \) and a sample mean vector \( \bar{Y}_j = [(\bar{Y}_j(F_1), \bar{Y}_j(F_2), ..., \bar{Y}_j(F_k))]^T \) follows a multivariate normal distribution
\[
\begin{align*}
\left( \frac{Y_j(F_0)}{\bar{Y}_j} \right) & \sim N_{1+k} \left( \begin{bmatrix} F_0S_j \\ F_{DOE}S_j \end{bmatrix}, \begin{bmatrix} \tau_j^2 & R^T(F_0) \\ R(F_0) & R \end{bmatrix} \right), \\
\text{where} \quad R_j(F_0) & = (\text{Cov}[Y_j(F_0), \bar{Y}_j(F_1)]; \ldots; \text{Cov}[Y_j(F_0), \bar{Y}_j(F_k)]) = \left( F_0\Sigma_sF_1^T + \tau_j^2R_{xz}(\theta, F_0 - F_1); \ldots; F_0\Sigma_sF_k^T + \tau_j^2R_{xz}(\theta, F_0 - F_k) \right), \quad \text{and} \quad R_j = F_{DOE}\Sigma_sF_{DOE}^T + \Sigma_z + S_j^T\Sigma_FS_j \cdot I + \Sigma_{\epsilon_j}.
\end{align*}
\]

**Proof:**

According to the model (4) and its assumptions,

\[
\text{Cov}[Y_{mj}(F_a), Y_{hj}(F_b)] = \text{Cov}[\tilde{F}_aS_j + \tilde{F}_bS_j + z_j(F_a) + \epsilon_{mj}, \tilde{F}_bS_j + \tilde{F}_bS_j + z_j(F_b) + \epsilon_{hj}]
\]

\[
= F_a\Sigma_sF_b^T + S_j^T \text{Cov}[\tilde{F}_a, \tilde{F}_b]S_j + \tau_j^2R_{xz}(\theta, F_a - F_b) + \text{Cov}[\epsilon_{mj}, \epsilon_{hj}]
\]

\[
= \begin{cases} 
F_a\Sigma_sF_a^T + S_j^T\Sigma_FS_j + \tau_j^2 + \sigma_{\epsilon_j}^2(F_a) & a = b, m = h \\
F_a\Sigma_sF_a^T + S_j^T\Sigma_FS_j + \tau_j^2 & a = b, m \neq h,
\end{cases}
\]

Moreover, similarly, we can derive that

\[
\text{Cov}[Y_j(F_0), Y_{mj}(F_a)] = F_0\Sigma_sF_a^T + \tau_j^2R_{xz}(\theta, F_0 - F_a).
\]

The noise item \(\epsilon_{ij}\) is independent among simulation replications and actuator force vectors, thus obtaining the sample mean of replications at actuator force vector \(F_a\) only affects

\[
\text{Cov}[\bar{Y}_j(F_a), \bar{Y}_j(F_a)] = F_a\Sigma_sF_a^T + S_j^T\Sigma_FS_j + \tau_j^2 + \sigma_{\epsilon_j}^2(F_a)/n_a.
\]

Let \(R_j(F_0, \cdot)\) be the \(k \times 1\) vector \((\text{Cov}[Y_j(F_0), \bar{Y}_j(F_1)]; \ldots; \text{Cov}[Y_j(F_0), \bar{Y}_j(F_k)]) = (F_0\Sigma_sF_1^T + \tau_j^2R_{xz}(\theta, F_0 - F_1); \ldots; F_0\Sigma_sF_k^T + \tau_j^2R_{xz}(\theta, F_0 - F_k))\). Let \(R_j = \ldots\)
\( F_{DOE} \Sigma S F_{DOE}^T + \Sigma z_j + S_j^T \Sigma F S_j \cdot I + \Sigma_{\epsilon_j} \), where \( F_{DOE} = (F_1; \ldots; F_k) \), and \( I \) denotes identity matrix.

Therefore, we can get the multivariate normal distribution as

\[
\begin{pmatrix} Y_j(F_0) \\ \bar{Y}_j \end{pmatrix} \sim \mathcal{N}_{1+k} \left( \begin{pmatrix} F_0 S_j \\ F_{DOE} S_j \end{pmatrix}, \begin{pmatrix} \tau^2_j & R_j(F_0,\cdot) \\ R_j(F_0,\cdot) & R_j \end{pmatrix} \right).
\]

**Proposition 2.** Assume that \( S_j, \Sigma z_j, \Sigma F, \Sigma \epsilon, \) and \( \Sigma S \) are known, the best MSPE (mean square prediction error) linear unbiased predictor is

\[
\hat{Y}_j(F_0) = F_0 S_j + R_j(F_0,\cdot) R_j^{-1}(\bar{Y}_j - F_{DOE} S_j),
\]

where \( R_j = F_{DOE} \Sigma S F_{DOE}^T + \Sigma z_j + S_j^T \Sigma F S_j \cdot I + \Sigma_{\epsilon_j} \), and \( R_j(F_0,\cdot) = \left( F_0 \Sigma S F_1^T + \tau^2_j R_j(\theta, F_0 - F_1); \ldots; F_0 \Sigma S F_k^T + \tau^2_j R_j(\theta, F_0 - F_k) \right). \)

The mean square error (MSE) of the predictor \( \hat{Y}_j(F_0) \) is \( \text{MSE}^* = \tau^2_j - R_j(F_0,\cdot) R_j^{-1} R_j(F_0,\cdot). \) The best MSPE linear unbiased predictor is also called simply a best linear unbiased predictor (BLUP).

**Proof:**

From Proposition 1, we get that the \( (Y_j(F_0); \bar{Y}_j) \) follows a multivariate normal distribution. According to the standard conclusions for the multivariate normal distribution [29], we can get the distribution of \( Y_j(F_0) \) given \( \bar{Y}_j \) as

\[
Y_j(F_0) | \bar{Y}_j \sim \mathcal{N}(\bar{\mu}, \bar{\Sigma}),
\]

where \( \bar{\mu} = F_0 S_j + R_j(F_0,\cdot) R_j^{-1}(\bar{Y}_j - F S_j), \bar{\Sigma} = \tau^2_j - R_j(F_0,\cdot) R_j^{-1} R_j(F_0,\cdot). \)

According to the theorem 3.2.1 [27], the best MSPE predictor of \( Y_j(F_0) \) is

\[
\hat{Y}_j(F_0) = E[Y_j(F_0)|\bar{Y}_j] = F_0 S_j + R_j(F_0,\cdot) R_j^{-1}(\bar{Y}_j - F_{DOE} S_j).
\]
It is obvious that the MSE of the predictor \( \hat{Y}_j(F_0) \) is \( \sigma^2 = \mathbb{E}[ (\hat{Y}_j(F_0) - Y_j(F_0))^2 ] = \Sigma = \tau_j^2 - R_j^T (F_0, \cdot) R_j^{-1} R_j (F_0, \cdot) \).

Following similar procedures in [28], we can prove that \( \Sigma_{\epsilon_j} \) and \( S_j^T \Sigma_F S_j \) inflate the MSE. However, we cannot mathematically prove if \( \Sigma_S \) inflates or deflates the MSE.

### 2.3 Maximum Likelihood Estimation

In this section, we will derive the maximum likelihood estimation of \( (S_j, \tau_j^2, \theta_j, \Sigma_S) \) with the assumption that \( (\Sigma_F, \Sigma_\epsilon) \) is known. \( \Sigma_F \) can be obtained from the tolerance of the actuators’ forces, and \( \Sigma_\epsilon \) can be estimated from the samples.

Under the multivariate normal distribution, the log-likelihood function of \( (S_j, \tau_j^2, \theta_j, \Sigma_S) \) is

\[
\ell(S_j, \tau_j^2, \theta_j, \Sigma_S) = -\frac{1}{2} \ln[(2\pi)^k] - \frac{1}{2} \ln[\det(R_j)] - \frac{1}{2} (\bar{Y}_j - F_{DOE} S_j)^T R_j^{-1} (\bar{Y}_j - F_{DOE} S_j),
\]

(5)

where \( R_j = F_{DOE} \Sigma_S F_{DOE}^T + \Sigma_{\epsilon_j} + S_j^T \Sigma_F S_j \cdot I + \Sigma_{\epsilon_j} \).

To estimate the \( \Sigma_{\epsilon_j} \) from the samples, we can use the sample variance

\[
\hat{\Sigma}_{\epsilon_j} = \text{diag}\left[ \frac{1}{n_1-1} \sum_{i=1}^{n_1} (Y_{ij}(F_1) - \bar{Y}_j(F_1))^2, \ldots, \frac{1}{n_k-1} \sum_{i=1}^{n_k} (Y_{ij}(F_k) - \bar{Y}_j(F_k))^2 \right],
\]

(6)

which is strongly consistent for \( \Sigma_{\epsilon_j} \). Recalling Equation (5), \( R_j = F_{DOE} \Sigma_S F_{DOE}^T + \Sigma_{\epsilon_j} + S_j^T \Sigma_F S_j \cdot I + \Sigma_{\epsilon_j} \). When we consider the impact of estimating \( \Sigma_{\epsilon_j} \), we can regard \( F_{DOE} \Sigma_S F_{DOE}^T + \Sigma_{\epsilon_j} + S_j^T \Sigma_F S_j \cdot I \) in Equation (5) as one term. Then, it is similar to \( R_j = \Sigma_M + \Sigma_\epsilon \) in the Stochastic Kriging [28]. According to similar proof procedure of Theorem 1 in the Stochastic Kriging [28], we can show that estimating \( \Sigma_{\epsilon_j} \) in this way introduces no prediction bias.
The MLE-based estimation procedure can be summarized as Algorithm 1.

**Algorithm 1: MLE based algorithm for the surrogate model considering uncertainties**

While \( j = 1: p \)

Initialization:

(i) estimate the \( \Sigma_{\epsilon j} \) from equation (6);

(ii) set the \( \Sigma_F \) according to the tolerance of actuators’ forces;

End

Estimation:

(i) set the starting points \( S_{j0} = (F_{DOE}^T * F_{DOE})^{-1} F_{DOE} Y_j(F_{DOE}), \tau^2_{j0} = \text{var}(Y_j(F_{DOE}) - F_{DOE} S_{j0}), \theta_{j0} = 1/2, \Sigma_{S0} = 0; \)

(ii) Use \( \hat{\Sigma}_{\epsilon j} \) to replace \( \Sigma_{\epsilon j} \) and then maximize the log-likelihood function (5) over \( \hat{S}_j, \hat{\tau}_j, \hat{\theta}_j, \hat{\Sigma}_S. \)

End

Prediction:

(i) predict \( \hat{Y}_j(F_0) \) by the equation

\[
\hat{Y}_j(F_0) = F_0 \hat{S}_j + \hat{R}_j(F_0; \cdot) \hat{R}_j^{-1}(\hat{Y}_j - F_{DOE} \hat{S}_j)
\]

where \( R_j = F_{DOE} \hat{S}_j F_{DOE}^T + \hat{S}_{\epsilon j}^T \Sigma_F \hat{S}_j \cdot I + \hat{\Sigma}_{\epsilon j}. \)

(ii) calculate the MSE of the estimator by

\[
\text{MSE}_j = \hat{\tau}_j^2 - \hat{R}_j(F_0; \cdot) \hat{R}_j^{-1}(\hat{R}_j(F_0; \cdot))
\]

where \( \hat{R}_j(F_0; \cdot) = (F_0 \hat{S}_j F_1^T + \hat{\tau}_j^2 R_{\epsilon j}(\hat{\theta}, F_0 - F_1); ...; F_0 \hat{S}_j F_k^T + \hat{\tau}_j^2 R_{\epsilon j}(\hat{\theta}, F_0 - F_k)). \)

End

2.4 Uncertainty Analysis

In Sections 2.2 and 2.3, we have investigated the theoretical impact of different uncertainties. In this section, we analyze the uncertainty sources based on the engineering knowledge during the composite part assembly process. Four kinds of uncertainties are considered in the equation (4). Those uncertainties are actuator uncertainty (\( \bar{F} \)), part uncertainty (\( \bar{S}_j \)), modeling uncertainty (\( \bar{z}_j(F) \)), and unquantified uncertainty (\( \epsilon_{ij} \)), respectively. We analyze sources of each uncertainty as follows.
(i) $\vec{F}$: Actuator uncertainty. When a force is implemented by an actuator, it may not be exactly the ideal magnitude and direction. The magnitude and direction of the forces may vary naturally due to the fabrication device tolerances of the hydraulic or electromechanical system of actuators. Part of directional variability is due to the deviations of contact geometry of actuators and their installations.

(ii) $\vec{S}_j$: Part uncertainty. A large composite part is usually manufactured by multiple batches and steps. The part uncertainty is originated in variability of raw materials (e.g. thickness variability of carbon fiber fabrics and organic impurities in epoxy resin) and the fabrication process (e.g. uncertainty of carbon fiber orientation, existence of delamination, resin rich/starved areas, air bubbles or blisters et al.) [30]. The part uncertainty will impact the mechanical properties and quantitative values of the sensitivity matrix.

(iii) $z_j(F)$: Modeling uncertainty. The surrogate modeling can be regarded as a process to build the link between control variables and responses. $z_j(F)$ is a realization of a mean 0 stochastic process which indicates the values being randomly sampled from a space of functional mappings. This stochastic property contributes to the modeling uncertainty.

(iv) $\epsilon_{ij}$: Unquantified uncertainty. When we measure a part in practice, measurement errors inevitably exist in the sensor readings. Moreover, if we use finite element analysis (FEA) to mimic real experiments, there are errors between the FEA model and the real system. Computational accuracy, relevant to the meshing resolution, will also impact the precision of the FEA model. These errors, together with other
random noise or unquantified errors, will be classified into the unquantified uncertainty.

To emphasize the motivation of introducing this novel methodology, we want to compare the Stochastic Kriging with our method. In the Stochastic Kriging, the model covers all the uncertainties by extrinsic uncertainty and intrinsic uncertainty. It is a generic decomposition without considering physical sources and characteristics of different uncertainties. In the proposed surrogate model, we bring in two more stochastic terms to separate the actuators’ variability and part-to-part variability. There are two advantages: (i) We can make full use of corresponding engineering knowledge to enhance the prediction accuracy of the model. For example, we can get actuators’ force tolerance from engineering knowledge, and then we integrate this information into the model to achieve better prediction. (ii) A more detailed decomposition helps us understand the impacts of different uncertainties with physical interpretations. Potentially, it lays a foundation for further exploration of optimal uncertainty control to improve the quality of the composite assembly process.

3. FEED-FORWARD AUTOMATIC OPTIMAL SHAPE CONTROL

After a surrogate model considering uncertainties is built and validated, the surrogate model will be embedded into the AOSC system. A feedforward control strategy will be applied as [31]. Control actions will be implemented by ten actuators for the shape control.

We develop the following feed-forward automatic optimal control algorithm to realize the shape adjustment of a fuselage, as shown in Fig. 3. For an incoming fuselage,
its dimensional information $Y$ will be measured with an in-line 3D laser metrology system. Given any virtual tooling adjustment with actuators’ forces $F$, we can use the surrogate model to predict the fuselage dimensions $\hat{Y}(F)$ when force $F$ is applied to the fuselage. Next, the predicted shape deviations from the target can be derived by subtracting designed dimensions $Y^*$ from the predicted dimensions $\hat{Y}(F) + Y_c$. Then a weighted multivariable optimization criterion can be used to determine optimal control actions among iterative runs of virtual shape adjustment. After the optimal actuators’ forces $F^*$ is obtained, a true shape adjustment can be implemented. We will introduce details about the objective function for the optimization as follows.

![Fig. 3 Feed-Forward Automatic Optimal Shape Control Algorithm](image)

The objective of feed-forward automatic optimal shape control algorithm is to minimize the deviations between the current dimensions and the designed dimensions by iteratively virtual shape adjustment. Besides, the optimization should also take the magnitude constraints of actuators’ forces into consideration. Therefore, the objective function is
\[
\min_{\mathbf{F}} J = (\mathbf{Y}_c + \hat{\mathbf{Y}}(\mathbf{F}) - \mathbf{Y}^*)^T \mathbf{W}(\mathbf{Y}_c + \hat{\mathbf{Y}}(\mathbf{F}) - \mathbf{Y}^*)
\]

s.t. \( F_L \leq \mathbf{F} \leq F_U \)

where \( \mathbf{Y}_c \) is an in-line measured dimensional vector of current fuselage. \( \hat{\mathbf{Y}}(\mathbf{F}) \) is the predicted dimensional deviation vector, \( \hat{\mathbf{Y}}_j(\mathbf{F}) = \mathbf{F} \mathbf{S}_j + \mathbf{R}_j^T(\mathbf{F},\cdot) \mathbf{R}_j^{-1}(\mathbf{V}_j - \mathbf{F}_{DOE} \mathbf{S}_j); \mathbf{Y}^* \) is the designed target dimensional vector; \( \mathbf{W} \) is the weighting coefficients matrix, whose diagonal elements reflect the relative importance of corresponding dimensional variables; Non-diagonal elements of \( \mathbf{W} \) are assumed to be zeros; \( \leq \) denotes component-wise inequality; \( F_L \) and \( F_U \) are the lower bound and the upper bound of actuators force vector \( \mathbf{F} \). Thus, we can write the objective function as

\[
\min_{\mathbf{F}} J = \sum_{j=1}^{p} w_j \cdot \left[ \mathbf{F} \mathbf{S}_j + \mathbf{R}_j^T(\mathbf{F},\cdot) \mathbf{R}_j^{-1}(\mathbf{V}_j - \mathbf{F}_{DOE} \mathbf{S}_j) + \mathbf{Y}_c - \mathbf{Y}^*_j \right]^2
\]

s.t. \( F_L \leq \mathbf{F} \leq F_U \)

where \( w_j \) is a weighting coefficient, which is the \( j^{th} \) diagonal entry of \( \mathbf{W} \). \( \mathbf{R}_j = \mathbf{F}_{DOE} \mathbf{S}_j \mathbf{F}_{DOE}^T + \mathbf{S}_j \mathbf{S}_j \mathbf{I} + \Sigma_{\epsilon} \), \( \mathbf{R}_j(\mathbf{F},\cdot) = \left( \mathbf{F} \Sigma_{\epsilon} \mathbf{F}_1^T + \tau_j^2 \mathbf{R}_{2j}(\theta, \mathbf{F} - \mathbf{F}_k) \right) \). \( \mathbf{Y}_c \) and \( \mathbf{Y}^*_j \) are the \( j^{th} \) entry of \( \mathbf{Y}_c \) and \( \mathbf{Y}^* \) respectively.

The control method takes weighted summation of square of dimensional deviations as an objective function with a bounded constraint of actuators’ forces. Some general control methods in the literature usually calculate the optimum control actions via minimization of both the response deviations \( J \) and control efforts (e.g. a weighted \( L_2 \) norm loss \( \mathbf{W}_F \mathbf{F}^T \) with a weight matrix \( \mathbf{W}_F \) ). In this way, they can realize acceptable
control performance as well as reserve control energy. However, in the shape control problem of the composite fuselage, the objective is to achieve minimized dimensional deviations of the fuselage to the target dimensions. The adjusting actuators’ forces is very easy with no constraints in energy consumption. Thus, there is no need to include a weighted loss $FWF^T$ in the objective function, which will dilute the performance of dimensional quality control. Instead, we consider that the actuators’ forces are bounded by the actuators’ capability and engineering specifications of their maximum forces. Thus, the control function in our method considers minimizing dimensional deviations and limiting the bounds of actuators’ forces. After we get the control objective function, we can find that this is a quadratic programming problem with box constraints, which can be solved by the interior point method [32].

4. CASE STUDY

4.1 FEA modeling and validation

The finite element analysis (FEA) is an effective numerical technique for composite part analysis. During the development of the AOSC system, a set of testing experiments need to be implemented for providing training data and testing data. If we can build a finite element model that is consistent with a real composite fuselage, it will increase the flexibility and efficiency of system development.

ANSYS Composite PrepPost is an add-in module to the ANSYS Workbench and is integrated with standard analysis functions for composite parts [33]. By Composite PrepPost, two kind of materials, carbon fiber and epoxy resin, are used to generate multiple fabrics with different geometric parameters. Fabrics can be stacked up
depending on specific sequence and orientations. Moreover, then, stack-ups are used to generate sub-laminates and further integrated into a composite part. Furthermore, the boundary conditions and composite meshing are applied to the structure in the pre-processing stage. A post-processing is used to evaluate the design performance or material failure. In another word, the FEA model can be used to predict the finished product under simulated real-world fabrication conditions. Both stress and deviations in addition to a range of failure criteria can be analyzed by using the FEA model.

After a composite fuselage is built in the ANSYS Workbench, two support areas and one 4-inch width strap are used to fix the fuselage. Ten actuators are applied to adjust the shape of the composite fuselage. The dimension between actuators and edge of the fuselage is 12 inches. The FEA model, as shown in Fig. 4 (a), is used to mimic the fuselage and assembly fixture set-up during assembly.

Fig. 4 Comparison between FEA simulation model and real testing experiment set-up
In order to validate whether the FEA model is consistent with the real composite part, a physical testing experiment is set up to measure the deviations of the fuselage under different actuators’ forces. We ensure that the key parameters including length, width, thickness, weight, and maximum deviation under gravity are consistent between the FEA simulated part and the real fuselage. In the validation test, the actuators’ forces are changed from 100 pounds to 600 pounds. The differences of the dimensional deviations of outputs between the FEA simulated part and the real fuselage under different actuators’ forces are shown in Fig. 5. The horizontal axis is the circumferential distance from the center, and the vertical axis is the dimensional deviation under the actuator’ force. We can find that the dimensional outputs from the FEA model are relatively consistent with the dimensional measurements of the real fuselage testing. As the actuator force increases, the difference between FEA simulated part and the real fuselage has a slight increase. However, the maximum difference is about five mils (Note: 1 mil is a thousandth of an inch) even with 600 pounds of actuator forces.
We can find that there is a systematic error between the FEA simulation and the physical experiment. In order to calibrate the model, physical experimental observations are collected. However, there is a challenge to calibrate a bunch of system parameters with very limited physical experimental observations. We have investigated this particular problem and proposed an effective calibration method with sensible variables identification and adjustment [34]. Because this paper is focused on the shape control problem for a composite fuselage, we take the FEA model and model calibration as a given in this paper. In the rest of this paper, we use this FEA model to generate data, do the modeling and control, and validate the methodology we proposed. It is worth pointing out that the physical experimental data is only used in the validation of the FEA simulation.
platform. Then all the prediction results and control results shown in Section 4.3-4.6 are based on the simulated data by using the validated FEA simulation platform.

4.2 Design of Experiment Considering Uncertainties

In practice, the shapes of fuselages are not exactly same between different fuselages due to the part uncertainty discussed in Section 2.4. Thus, we may need several fuselages to collect training dataset by experiments. In this case study, two fuselages with a different thickness ratio of carbon fiber and epoxy resin are generated to mimic the part uncertainty. When generating those fuselages with different dimensions/shapes, we fix the thickness of carbon fiber fabrics and change the thickness ratio of carbon fiber and epoxy resin by revising the thickness of epoxy resin. Four groups of fuselages are built considering the different degree of part uncertainty, which is shown in Table 1.

Taking group 1 with uncertainty 1% as an example, epoxy resin thicknesses of fuselages 1 and 2 are 0.826×(101%) inch and 0.826×(99%) inch. The uncertainty degree for groups 1 to 4 is 1%, 2%, 4%, 7%, respectively. It needs to be pointed out that the tolerance of thickness of regular carbon fiber/epoxy resin fabrics is 2%. Thus, we have covered a larger range of part uncertainties than the tolerance of parts in the design. After we generate these four groups of fuselages, we also validate them by maximum deviation under gravity. Their maximum deviations under gravity are within the range of 7.573 to 9.444 inches, which are consistent with the range of real fuselages in the production floor.

For each fuselage, 30 training samples are generated by a Latin hypercube design [25]. Therefore, for each group of training dataset, there are 60 training samples from different fuselages will be used. These samples are generated from the FEA model. The
FEA model described early has been calibrated separately in Section 4.1. Thus the output of the FEA model is reflected as a calibrated one.

### Table. 1 Design of experiment considering different degree of part uncertainty

<table>
<thead>
<tr>
<th>Fuselage</th>
<th>Group 1 / inch</th>
<th>Group 2 / inch</th>
<th>Group 3 / inch</th>
<th>Group 4 / inch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>Epoxy resin thickness 0.826×(101%)</td>
<td>0.826×(102%)</td>
<td>0.826×(104%)</td>
<td>0.826×(107%)</td>
</tr>
<tr>
<td></td>
<td>Max deviation under gravity 8.295</td>
<td>8.167</td>
<td>7.921</td>
<td>7.573</td>
</tr>
<tr>
<td>Group 2</td>
<td>Epoxy resin thickness 0.826× (99%)</td>
<td>0.826× (98%)</td>
<td>0.826× (96%)</td>
<td>0.826× (93%)</td>
</tr>
<tr>
<td></td>
<td>Max deviation under gravity 8.561</td>
<td>8.698</td>
<td>8.985</td>
<td>9.444</td>
</tr>
</tbody>
</table>

After we obtained those fuselages, designed experiments are conducted to generate the datasets of actuators’ forces and their corresponding fuselage shape changes. In the design of experiment, the input variables are ten actuators’ forces with a range of ± 1000 pounds. The responses are dimensional deviations of the fuselage. The design of experiment will mimic the real experiments to collect training dataset and testing dataset for calibration of the AOSC system. The responses under different actuators’ forces are able to provide training dataset for determining the parameters in the model. And then testing dataset is used to mimic the real application of the AOSC system and test the performance. Examples of datasets generated with a designed experiment are shown in Fig. 6.
When we use the AOSC system, incoming new fuselages to be assembled are different from the fuselages used to calibrate the AOSC system. To mimic this real production situation, 20 fuselages with different dimensions are generated to mimic incoming new fuselages to be assembled, which provide the testing dataset. After the parameter estimation based on the training dataset, a surrogate model considering uncertainties is built and validated by the testing dataset. Hereafter, a multivariable optimization is conducted to obtain the optimal control for best dimensional quality.
Twenty testing fuselages with different dimensions are used to validate the effectiveness and efficiency of the AOSC system.

4.3 Surrogate Modeling and Prediction Results

After we implement the design of experiment in the FEA simulation platform, the training dataset and testing dataset are collected. This training dataset is used to develop the surrogate model discussed in Section 2. A parameter estimation is conducted by using the training dataset. The simulation is conducted in a computer with Intel Core i7-4500U CPU with 8.00GB memory. In order to evaluate the obtained surrogate model, a prediction performance is evaluated for both the training dataset and the testing dataset. The concept of the prediction evaluation is shown in Fig. 1 (c). Predicted dimensional deviations are compared with the FEA outputted deviations, and the prediction errors are calculated.

The comparison of prediction errors among four methods is shown in Fig. 7 and Fig. 8. Fig. 7 is the boxplot based on the training dataset and Fig. 8 is the boxplot based on the testing dataset. The X axis denotes the output index associated with different quality features on the fuselage. The Y axis represents dimensional prediction errors with a unit inch. Mean absolute deviation is calculated for each method in order to show the quantitative performance. Meanwhile, the run time per sample is collected. All these results are summarized in Table 2. We can see that the Universal Kriging model (UKM) has the least training prediction error, which is reasonable because of the properties of Kriging. While the prediction error of the UKM based on the testing dataset is the worst, which can be explained by overfitting. Quantitatively, the mean absolute deviations of
training samples for the regression model, the UKM, the SKM, and the proposed SMU are 0.0381, $2.59 \times 10^{-16}$, 0.0021 and 0.0018, respectively. The mean absolute deviations of the testing samples for the regression model, the UKM, the SKM, and the proposed SMU are 0.0030, 0.0097, 0.0029 and 0.0027, respectively. Therefore, the proposed SMU performs better than the regression model and the SKM in both the training samples and testing samples. Thus, the proposed SMU can realize the best prediction capability than the other three benchmark methods. Even though the run time per sample for the SMU is the largest, 0.3140 second is acceptable for the feedforward control of shape adjustment of the composite fuselage.

(Note: The UKM denotes the Universal Kriging Model, the SKM denotes the Stochastic Kriging Model, and the SMU denotes the proposed surrogate model considering uncertainties)

Fig. 7 Prediction errors of the four methods based on the training dataset
It is worth noting that the prediction errors of the four methods follow a specific pattern for different output indices. For example, the prediction errors for the output index 1-28 (corresponding to the lower half of the composite fuselage) tend to be smaller than the ones for the output index 29-57 (corresponding to the upper half of the composite fuselage). The reason is that the adjustable actuators are installed in the lower half of the fuselage. Thus, the sampling data can capture more information for the lower half and realize better prediction. The prediction errors for output index 13-17 are relatively larger than the neighboring key points because of the impact of supporting fixture constraints. In addition, this pattern can be changed if we adjust the positions of
the actuators. Where and how to distribute the actuators are future topics to be investigated.

4.4 Automatic Shape Control Results

A case study is conducted to illustrate the effectiveness and efficiency of the feed-forward control strategy developed in the paper. The case study is motivated by the real application of the AOSC system. We summarize the procedure of the proposed AOSC system as following steps:

(i) The initial incoming fuselage dimensions are measured by in-situ 3D metrology.

(ii) The obtained dimensional measurements will be fed into the control algorithm.

(iii) The control algorithm will do an iterative optimization to find the optimal forces to be used in the dimensional control. The convergence of virtual shape adjustment can be guaranteed because the optimization problem is convex. The algorithm can get a global optimal solution.

(iv) A feedforward control strategy is applied for the shape control by using the obtained optical forces. The shape adjustment is achieved by adjusting the physical actuators once in practice. There are no iterations in this step.

For verification, twenty new incoming testing fuselages with different dimensions are used to test the system. The control results are shown in Fig. 9 and Table 3. We can see that the AOSC system based on the surrogate model considering uncertainties can achieve the best control performance, which indicates the smallest deviations after
control. Quantitatively, the mean of the absolute deviations after control is $1.26 \times 10^{-4}$ inch, which is lower than the control requirement. The run time per sample for the control system based on the SMU is a little larger than other methods. However, 2.9738 seconds is acceptable for the shape adjustment of the composite fuselage according to engineering knowledge.

![Fig. 9 Deviations after control based on the four models](image)

**Table. 3 Design of experiment considering different degree of part uncertainty**

<table>
<thead>
<tr>
<th></th>
<th>Regression</th>
<th>UKM</th>
<th>SKM</th>
<th>SMU</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAD After Control / inch</td>
<td>3.34\times 10^{-4}</td>
<td>4.16\times 10^{-4}</td>
<td>1.48\times 10^{-4}</td>
<td>1.26\times 10^{-4}</td>
</tr>
<tr>
<td>Run Time Per Sample / second</td>
<td>0.1727</td>
<td>2.9516</td>
<td>2.0641</td>
<td>2.9738</td>
</tr>
</tbody>
</table>

We can find that the deviations after control also follow a specific pattern similar to the pattern of prediction errors in Section 4.3. This control performance is mainly determined by the prediction errors in Section 4.3. However, this control performance of each dimensional variable can be changed by adjusting the weight matrix $W$ in Equation
(7). Usually, the weight matrix $W$ is adjusted by the relative importance of corresponding dimensional variables according to engineering knowledge.

4.5 Sensitivity Analysis

We also conduct the sensitivity analysis of the control performance. For one thing, we explore how the magnitude of fuselage variability impacts the control performance; for another, we make a thorough inquiry about the maximum actuator force should be used in the AOSC system. The maximum actuator force determines the bounds $F_L, F_U$ in the control system.

![Figure 10 Sensitivity analysis for fuselage variability and maximum actuators’ forces in the AOSC system](image)

The results of sensitivity analysis are shown in Fig. 10. It shows that as fabrics thickness variability increases from 1% to 7%, the average of maximum absolute deviations also becomes larger. However, even for fabrics thickness variability with 7%, the average of maximum absolute deviations is still $1.2 \times 10^{-3}$, which is within the engineering specifications of the assembly requirement. For the sensitivity analysis of the maximum actuators’ forces, we assume that the fuselage variability is 2%. In this case,
when the maximum actuator force is lower than 600 pounds, the increase of maximum actuator force will improve the control performance dramatically. Also, no further improvements can be observed if the maximum actuator force is increased more than 600 pounds. Thus, 600 pounds is the upper limit that can provide sufficient control capability for the fuselage variability with 2%.

4.6 Stress Analysis and Failure Test

Engineers may worry about whether large actuators’ forces may damage the composite fuselage. In order to address this concern, we conducted the stress analysis and failure test. The result of stress analysis is shown in Fig. 11. We can see that the maximum/middle/minimum principal stress, the maximum shear stress and equivalent (von Mises) stress all become larger as the magnitude of actuators’ forces increases from 100 to 1000 pounds. Even for the 1000 pounds, the maximum stresses are within the limit of stress. We also implement failure test based on multiple popular criteria including Max strain/stress, Tsai-Wu, Tsai-Hill, Hoffman, Hashin criteria [25]. The result of failure test shows that Inverse Reserve Factor (IRF), which defines the inverse margin to failure, is 0.27 and is lower than the failure threshold 1.00.
5. SUMMARY

Composite parts have been widely used in the airplanes due to its numerous superior material properties. However, dimensional control and variation reduction of composite parts are not as well understood as comparing with the conventional metal parts such as aluminum and titanium.

Motivated by reducing dimensional errors when joining two composite parts, an automatic optimal shape control system has been developed in this paper. Firstly, a finite element analysis (FEA) platform is built to mimic the composite part fabrication process and simulate the dimensional changes of a composite part under various external forces. The FEA platform is validated with experimental data obtained from a real part set-up in a production floor. The validated FEA model is to generate the dimensional response data under different set of actuators’ forces designated by designed experiments. Based on
those datasets, a surrogate model considering four types of uncertainties (actuator uncertainty, part uncertainty, modeling uncertainty, and unquantified uncertainty) has been developed to achieve good prediction performance. An MLE estimation algorithm has been used for parameter estimation and response prediction. Afterward, the surrogated model considering uncertainties is embedded into a feedforward control algorithm, which is achieved by conducting multivariable optimization to minimize the weighted summation of dimensional deviations of the response to the target.

A case study reveals that the surrogate model considering uncertainties achieves better prediction performance than three other benchmark methods (e.g. the regression model, the Universal Kriging model, and the Stochastic Kriging model). The automatic optimal shape control system can significantly improve the dimensional product quality and reduce the cycle time in the assembly process of composite parts. Furthermore, a sensitivity analysis has been conducted to explore the control performance for different fuselage variability and different bounds in the AOSC system. Stress analysis and failure test show that the actuators force 1000 pounds will not damage the composite parts.

ACKNOWLEDGMENT

The work is funded by the Strategic University Partnership between the Boeing Company and the Georgia Institute of Technology.
NOMENCLATURE

UKM The Universal Kriging Model

SKM The Stochastic Kriging Model

SMU The Surrogate Model considering Uncertainties

\( F \) An actuators’ forces vector with dimension of \( 1 \times q \)

\( \bar{F}_t \) An additional random deviations of actuators’ forces that results from the actuator uncertainty

\( (F_t, n_t) \) Experiment design pairs, \( n_t \) is the number of simulation replication taken at the design setting \( F_t \)

\( F_0 \) A new actuators’ forces vector

\( F_{DOE} \) \( F_{DOE} = (F_1; \ldots; F_k) \) denote the design matrix of actuators’ forces, \( k \) is the number of design vector of actuators’ forces

\( S_j \) The sensitivity matrix corresponding to the \( j^{th} \) response

\( \bar{S}_j \) The random sensitivity vector variability from the part uncertainty

\( y_{ij}(F) \) The \( j^{th} \) dimensional variable of the fuselage under the \( i^{th} \) replication at the actuators' forces \( F \)

\( \bar{y}_j(F) \) The sample mean of the \( j^{th} \) dimensional variable at \( F \)

\( \bar{Y}_j \) The sample mean vector \( \bar{Y}_j = [\bar{y}_j(F_1), \bar{y}_j(F_2), \ldots, \bar{y}_j(F_k)]^T, F_1; \ldots; F_k \) denote the design vectors of actuators’ forces.

\( y_j(F_0) \) The \( j^{th} \) dimensional variable of the fuselage at the actuators’ forces \( F_0 \)
\( \bar{Y}_j(F_0) \) A predictor of \( Y_j(F_0) \)

\( z_j \) A stochastic process, which represents the extrinsic uncertainty. In the paper, it is considered as a Gaussian process \( GP(0, \Sigma_{zj}) \) with mean 0 and covariance matrix \( \Sigma_{zj} \)

\( \varepsilon_{ij} \) Noise in the \( j^{th} \) dimensional variable under the \( i^{th} \) replication

\( \Sigma_F \) The covariance matrix of additional random deviations of actuators’ forces

\( \Sigma_S \) The covariance matrix of part sensitivity vector

\( \Sigma_{zj} \) The covariance matrix of stochastic process.

\( \Sigma_{\varepsilon j} \) The noise covariance matrix
REFERENCES


[34] Wang, Y., Yue, X., Tuo, R., Hunt, J. H., Shi, J. “Effective Model Calibration via Sensible Variable Identification and Adjustment, with application to Composite Fuselage Simulation,” to be submitted, (currently under approval for release by the project sponsor company).
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Fig. 2 Overview of the proposed methodology

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Fig. 6 Examples of datasets generated with a designed experiment

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Fig. 8 Prediction errors of the four methods based on the testing dataset

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Fig. 5 The dimensional deviations under different actuator’ forces in the FEA simulation and the physical experiment.
Fig. 6 Examples of datasets generated with a designed experiment

![Graphs showing DOE 1, 2, 3, and 4 with different forces applied on ideal and deformed parts.](Image)
Fig. 7 Prediction errors of the four methods based on the training dataset
Fig. 8 Prediction errors of the four methods based on the testing dataset
Fig. 9 Deviations after control based on the four models
Fig. 10 Sensitivity analysis for fuselage variability and maximum actuators’ forces in the AOSC system
Fig. 11 Maximum stress under different magnitude of actuators’ forces
Table 1 Design of experiment considering different degree of part uncertainty

<table>
<thead>
<tr>
<th>Fuselage</th>
<th>Group 1 / inch</th>
<th>Group 2 / inch</th>
<th>Group 3 / inch</th>
<th>Group 4 / inch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuselage 1</td>
<td>Epoxy resin thickness</td>
<td>0.826×(101%)</td>
<td>0.826× (102%)</td>
<td>0.826× (104%)</td>
</tr>
<tr>
<td></td>
<td>Max deviation under gravity</td>
<td>8.295</td>
<td>8.167</td>
<td>7.921</td>
</tr>
<tr>
<td>Fuselage 2</td>
<td>Epoxy resin thickness</td>
<td>0.826× (99%)</td>
<td>0.826× (98%)</td>
<td>0.826× (96%)</td>
</tr>
<tr>
<td></td>
<td>Max deviation under gravity</td>
<td>8.561</td>
<td>8.698</td>
<td>8.985</td>
</tr>
</tbody>
</table>
Table. 2 Mean absolute deviations (MAD) and run time for the four methods

<table>
<thead>
<tr>
<th></th>
<th>Regression</th>
<th>UKM</th>
<th>SKM</th>
<th>Proposed SMU</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAD Training / inch</td>
<td>0.0381</td>
<td>2.59e-16</td>
<td>0.0021</td>
<td>0.0018</td>
</tr>
<tr>
<td>MAD Testing / inch</td>
<td>0.0030</td>
<td>0.0097</td>
<td>0.0029</td>
<td>0.0027</td>
</tr>
<tr>
<td>Run Time Per Sample / second</td>
<td>0.0483</td>
<td>0.0538</td>
<td>0.0706</td>
<td>0.3140</td>
</tr>
</tbody>
</table>
Table 3 Design of experiment considering different degree of part uncertainty

<table>
<thead>
<tr>
<th></th>
<th>Regression</th>
<th>UKM</th>
<th>SKM</th>
<th>SMU</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAD After Control / inch</td>
<td>3.34×e-04</td>
<td>4.16×e-04</td>
<td>1.48×e-04</td>
<td>1.26×e-04</td>
</tr>
<tr>
<td>Run Time Per Sample / second</td>
<td>0.1727</td>
<td>2.9516</td>
<td>2.0641</td>
<td>2.9738</td>
</tr>
</tbody>
</table>