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Robust Fixture Layout Design for a Product Family Assembled in a Multistage Reconfigurable Line

Reconfigurable assembly systems enable a family of products to be assembled in a single system by adjusting and reconfiguring fixtures according to each product. The sharing of fixtures among different products impacts their robustness to fixture variation due to trade offs in fixture design (to allow the accommodation of the family in the single system) and to frequent reconfigurations. This paper proposes a methodology to achieve robustness of the fixture layout design through an optimal distribution of the locators in a multistation assembly system for a product family. This objective is accomplished by (1) the use of a multistation assembly process model for the product family, and (2) minimizing the combined sensitivity of the products to fixture variation. The optimization considers the feasibility of the locator layout by taking into account the constraints imposed by the different products and the processes (assembly sequence, data scheme, and reconfigurable tools' workspace). A case study where three products are assembled in four stations is presented. The sensitivity of the optimal layout was benchmarked against the ones obtained using dedicated assembly lines for each product. This comparison demonstrates that the proposed approach does not significantly sacrifice robustness while allowing the assembly of all products in a single reconfigurable line. [DOI: 10.1115/1.3123320]

Keywords: *fixture layout, robust design, product family, multistation assembly*

1 Introduction

Traditionally, mass production of complex products has been accomplished using dedicated manufacturing systems. Such systems are characterized by high productivity and low flexibility, which work well for a relatively static market. However, today's market features rapid changes in demand and short product lifecycle. Those changes have obliged manufacturers to increase product variety and reduce lot size. Therefore, manufacturers are continuously developing new products and production systems. The development of product families has helped manufacturers to meet customer requirements in terms of variety. An example of a product family is presented in Fig. 1, where three car models of different sizes form the family. The use of reconfigurable manufacturing systems and controls has given manufacturers the possibility to cost effectively produce the family of products through systematic reconfigurations.

In the automotive industry, the body assembly process is less flexible than general assembly. Therefore, it has been receiving a lot of attention nowadays in pursuing flexibility. Autobodies are usually assembled in multistation sequential process (up to 70 stations), where at each station, fixtures are used to locate and clamp the parts for joining. These fixtures play a critical role in controlling the position of the parts and subassemblies on each station, and on the final product quality. Traditionally, fixtures are dedicated to one product type, thus limiting the possibility to reuse them for other products. Since fabricating assembly systems for each product type in the family can be very expensive, there is a necessity for fixture flexibility to allow the assembly of a product family in a single line.

Reconfigurable assembly systems using flexible fixtures allow the assembly of different products in a single assembly line by sharing process tools. An example of such flexible fixture, also called programmable tools (PTs), is the FANUC robot F-200iB [1], which can hold different part-types in automobile body assembly lines. As the product changes from one type to another, the PTs change their positions as needed by the new part geometry, thus allowing the assembly of different product types in the same production line. The disadvantages of such systems are that assembling multiple products in a single reconfigurable line imposes additional constraints on product design, and the frequent changeover between products is an additional source of process variation, which impacts the final product quality.

Product quality is usually characterized by the fulfillment of customer's specifications and product functionality. In the automotive industry, the parameters that determine product quality are known as the key product characteristics (KPCs). The KPCs are, in general, quantitative features of the product such as relative position of parts, flushes, and gaps. Fixtures have a key role in determining the position of the parts, and doing so, on the achievement of the KPC specifications. For this reason, the fixtures form part of the key control characteristics (KCCs) of the process [2]. Figure 2 represents a part (a rectangular sheet) mounted on a 3-2-1 fixture formed by three NC blocks. Two of the blocks have pins that restrict the in-plane motion of the part. The pins locate the part by fitting into a hole and a slot previously

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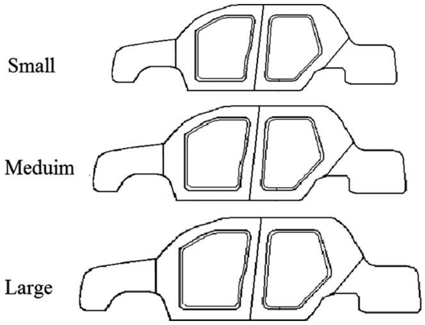

Fig. 1 A product family consisting of sedans of small, medium, and large sizes

pierced on the part. The three blocks also position and restrain the part in the direction normal to the plane. The 3-2-1 locating points are known as principal locating points (PLPs). PLPs' position and their interaction with the fixture play an important role on the quality of the product (e.g., the position of the KPC points M_1 and M_2 in Fig. 2). Dimensional variations caused by the pins and the NC blocks have different effects on the part. Variation in the pins generally causes a global rigid body motion of the part (e.g., displacement of the part in the *z*-*x* plane in Fig. 2); however, NC variation causes local out of plane deformation of parts (e.g., deformation in the *y*-*z* plane in Fig. 2). In this study, we are interested in the global deviations of the part; therefore, we simplified the problem by considering only the pin layout, but still refer the problem as PLP layout.

When dedicated fixtures are used for each product at each station, it is possible to optimize the location of the PLPs in terms of their robustness to fixture variation. However, when multiple products of the same family are assembled in a single line, the products have to share fixtures. Sharing fixtures may result in a distribution of the PLPs that is not optimal for each individual product. Therefore, it is important to determine a robust distribution of the PLPs for the product family considering fixture sharing.

This paper presents a methodology to design robust PLP layouts for a product family assembled in a single line using reconfigurable fixtures. The requirements to solve such a problem are as follows.

- To obtain an expression that relates the PLP layout (design variables) to the final product variation, applicable to all products in the family.
- To define the search space for the location of the PLPs and the constraints for their location. In the case of the product family, the constraints for the solution not only include product-part geometry, but also consider multiproduct fixture sharing and the reconfigurable fixtures' workspace.
- To minimize the effect that fixture variation has on product variation without violating the constraints.

The remainder of the paper is organized as follows: Section 2 reviews the state of the art in multistation assembly variation propagation models, fixture design, and reconfigurable fixturing systems. Section 3 addresses the design problem of determining the optimal distribution of the PLPs for a product family. A case study is presented in Sec. 4, with the conclusions given in Sec. 5.

2 Literature Review

The literature review covers the following three areas related to the proposed research: multistation assembly models, fixture design, and reconfigurable fixturing systems.

2.1 Multistation Manufacturing Processes. To establish relations between part and process variation and the final product quality in a multistation assembly process, it is necessary to have a model of the process. Such a model was first developed for autobody assembly at the station level [3]. The modeling of a multistation assembly process was first attempted by Shiu et al. [4], where a kinematics-based model of the process was developed. One of the main findings in Ref. [4] was the identification of the "reorientation" effect that occurs in multistation assembly processes. This effect occurred when subassemblies are located again in downstream stations where the PLPs may not be the same as in prior stations. Figure 3 illustrates the effects of fixture deviation and the reorientation, where Fig. 3(*a*) presents the effect that a displacement of the two-way pin (P_2) has on the part, and especially in the location of points M_1 and M_2. Figure 3(*b*) shows the reorientation effect on a subassembly as it moves from station *k*-1 to station *k*. Then, variation in station *k*-1 is transmitted to station *k* due to the reorientation, which is the major difference between the single-station and multistation variation modeling.

A formal representation of the multistation assembly process was developed by Jin and Shi [6] where a state space representation of the assembly process was used to determine the final product variation given the variation of the incoming parts and fixtures

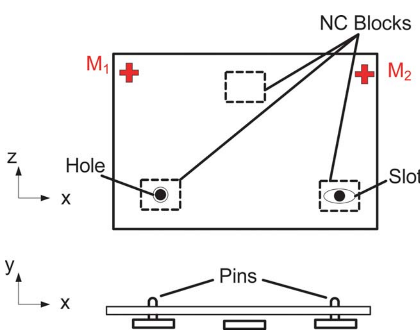

Fig. 2 Top and side views of the 3-2-1 fixture layout

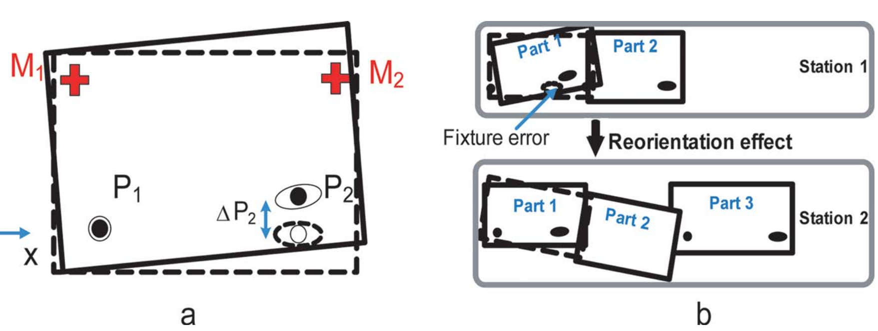

Fig. 3 Effect of the fixture deviation and reorientation (adapted from Ref. [5])

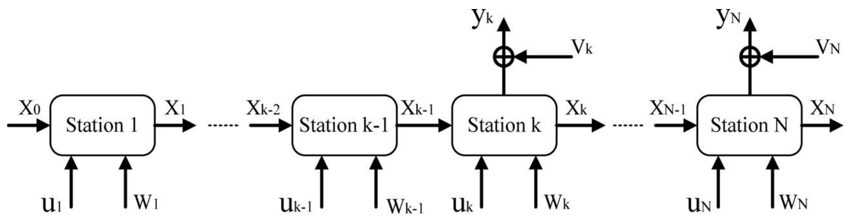

Fig. 4 Diagram of the multistation assembly process with *n* stations (adapted from Ref. [2])

or KPCs and KCCs' relations. This model assumes that (i) parts are rigid, (ii) the types of part-to-part joint used are all lap joints, and (iii) variation only occurs in the plane and not out of plane (e.g., 2D parts where variation only occurs in the *z*-*x* plane in Fig. 3(*a*)). Another multistation modeling method was proposed by Mantripagrada and Whitney [7]. The authors used the state transition model to predict the variation propagation of 2D rigid parts in the plane, and to perform assembly corrections. Since the variation propagation model is fundamental to establishing the relation between KCCs and KPCs' deviations, the state space model is described next.

A schematic of a multistation assembly process is presented in Fig. 4. Observing this figure, it is possible to understand how the subassemblies are transferred from one station to another, accumulating variation along the process. The variation accumulated up to station k (translations and rotations of the parts) is represented by the variable $\mathbf{x}_k \in \Re^n$ in Eq. (1). This variable depends on the deviation accumulated up to station k-1 plus fixture's deviations, $\mathbf{u}_k \in \Re^p$, and other unmodeled deviation characterized by $\mathbf{w}_k \in \Re^n$. The reorientation effect of the subassembly coming from station k-1 in station k is represented by matrix $\mathbf{A}_{k-1} \in \Re^{n\times n}$. This matrix relates the fixture layout of two adjacent stations and determines the repositioning necessary for the subassembly entering station k (see Fig. 3(*b*)). The impact of fixture deviations in station k is determined by matrix $\mathbf{B}_k \in \Re^{n\times p}$. On the other hand, the measurements or outputs $\mathbf{y}_k \in \Re^m$, if they exist at station k, depend on the position of the selected measurement points for the assembly, which normally correspond to the product KPCs. The relation between the variation of the part and the measurement points is given by matrix $\mathbf{C}_k \in \Re^{m\times n}$. Usually the measurements are not perfect and there exists noise represented by $\mathbf{v}_k \in \Re^m$. All the aforementioned matrices are obtained based on kinematic relationships, which are detailed in Refs. [5,6].

The complete state space representation of the dimensional relationships is given below

$$\begin{aligned} \mathbf{x}_k &= \mathbf{A}_{k-1} \cdot \mathbf{x}_{k-1} + \mathbf{B}_k \cdot \mathbf{u}_k + \mathbf{w}_k \\ \mathbf{y}_k &= \mathbf{C}_k \cdot \mathbf{x}_k + \mathbf{v}_k \end{aligned} \tag{1}$$

Based on the linear properties of the model, it is possible to represent the deviation of the measurement points in the last station N as

$$\mathbf{y}_N = \mathbf{C}_k \cdot \mathbf{\Phi}_{N,0} \cdot \mathbf{x}_0 + \sum_{k=1}^{N} \mathbf{C}_N \cdot \mathbf{\Phi}_{N,k} \cdot \mathbf{B}_k \cdot \mathbf{u}_k + \sum_{k=1}^{N} \mathbf{C}_N \cdot \mathbf{\Phi}_{N,k} \cdot \mathbf{w}_k + \mathbf{v}_N \tag{2}$$

where $\mathbf{\Phi}$ is the state transition matrix [5] and can be calculated as

$$\begin{aligned} \mathbf{\Phi}_{k,i} &\equiv \mathbf{A}_{k-1} \cdot \mathbf{A}_{k-2} \cdot \mathbf{A}_{k-3} \cdot \ \ldots \ \cdot \mathbf{A}_i \\ \mathbf{\Phi}_{i,i} &\equiv \mathbf{I} \end{aligned} \tag{3}$$

Equation (2) can be simplified to

$$\mathbf{y}_N = \mathbf{\Psi}_0 \cdot \mathbf{x}_0 + \sum_{k=1}^{N} \mathbf{\Gamma}_k \cdot \mathbf{u}_k + \sum_{k=1}^{N} \mathbf{\Psi}_k \cdot \mathbf{w}_k + \mathbf{v}_N \tag{4}$$

where

$$\mathbf{\Gamma}_k = \mathbf{C}_N \cdot \mathbf{\Phi}_{N,k} \cdot \mathbf{B}_k \quad \text{and} \quad \mathbf{\Psi}_k = \mathbf{C}_N \cdot \mathbf{\Phi}_{N,k} \tag{5}$$

Since the type of process analyzed in Fig. 4 involves a serial assembly line with only one assembly station per stage, the terms station and stage are used interchangeably in the remainder of the paper.

2.2 Fixture Design. Early research in fixture design did not consider the existence of external variation sources [8,9]. Later, researchers considered the existence of errors in fixtures and/or parts. In this area, the research is divided into two categories based on whether the workpiece is considered rigid or compliant. In both categories, the common approach is to determine the position of the locators and clamps that ensures a correct location of the workpiece and minimizes the effect of external variation sources.

In the case of rigid parts, the research has been focused on the robust layout design of fixtures and clamps. Cai et al. [10] proposed a variational method for robust fixture configuration design of 3D rigid parts. Wang and Pelinescu [11] developed an algorithm for fixture synthesis for 3D workpieces by selecting the positions of the clamps from a collection of discrete candidate locations called point set.

In the design of fixtures for complaint parts, finite element method was used to model and analyze workpiece behavior including the effect of friction forces [12]. Menassa and DeVries [13] used optimization to assist in the evaluation and selection of the 3-2-1 fixtures and clamps for prismatic parts aiming to minimize workpiece deflection. Cai and Hu [14] studied the use of more complex fixture scheme, the "N-2-1" fixture, to provide additional support for the part, and used optimization to distribute the fixtures in order to reduce the part's deformation. Camelio et al. [15] determined the optimal fixture location to hold sheet metal parts considering out of plane variation of fixtures and welding guns' position, and the springback effect on the subassembly after it is removed from the fixture.

All the previous works are based on single-station synthesis of fixture layout. The problem of distributing the fixtures or its simplification to PLPs' distribution in a multistation process is more challenging due to the interstation relation of variation propagation caused by subassembly's reorientation. Kim and Ding [16] were the first to address this problem for the case of 2D rigid parts considering lap joints. They determined the distribution of PLPs for rigid parts that is robust to fixture variation for a single product assembled in a multistation process. To this end, they developed a sensitivity index that relates PLP layout to final product variation (KPCs) and used several optimization methods to determine the distribution. Kim and Ding [16] centered their effort on reducing the impact of fixture variation on the final product quality. Following this approach, Eq. (4) can be simplified as

$$\mathbf{y}_N \equiv \sum_{k=1}^{N} \mathbf{\Gamma}_k \cdot \mathbf{u}_k = \mathbf{D} \cdot \mathbf{u} \tag{6}$$

where $\mathbf{u}$ is the stack up vector of all the fixture deviation, and matrix $\mathbf{D}$ is calculated as

$$\mathbf{D} = [\mathbf{\Gamma}_1 \quad \mathbf{\Gamma}_2 \quad \cdots \quad \mathbf{\Gamma}_N] \tag{7}$$

Kim and Ding [16] ignored the last term $\mathbf{\Gamma}_N$ in matrix $\mathbf{D}$ because it is the final measurement station; hence the fixtures should have tighter tolerances and a better maintenance policy. Using that simplification, they proposed the calculation of a sensitivity index that relates the deviation sum squares of the output measurements $\mathbf{y}^T \cdot \mathbf{y}$ as presented in Eq. (8), where the subindex N was dropped for simplification, resulting in

$$\mathbf{y}^T \cdot \mathbf{y} = \mathbf{u}^T \cdot \mathbf{D}^T \cdot \mathbf{D} \cdot \mathbf{u} \tag{8}$$

Then, the input/output sensitivity S can be calculated as the ratio of the sum of the KPCs' squares to the square of the input's variation

$$S = \frac{\mathbf{y}^T \cdot \mathbf{y}}{\mathbf{u}^T \cdot \mathbf{u}} = \frac{\mathbf{u}^T \cdot \mathbf{D}^T \cdot \mathbf{D} \cdot \mathbf{u}}{\mathbf{u}^T \cdot \mathbf{u}} \tag{9}$$

When analyzing the sensitivity index, it is possible to observe that if the product $\mathbf{D}^T \cdot \mathbf{D}$ is "small," then the effect of the fixture variation is minimized. This is precisely the objective of a robust locator layout: minimizing the impact that the fixture variation has on the KPC. To achieve this goal, one of the several criteria can be used. These criteria, most of which have an origin in the optimal design of experiments, are as follows: A-optimality, which is to minimize the trace of $\mathbf{D}^T \cdot \mathbf{D}$; D-optimality, which is to minimize the determinant of $\mathbf{D}^T \cdot \mathbf{D}$; and E-optimality, which is to minimize the extreme (maximum or minimum) eigenvalue of $\mathbf{D}^T \cdot \mathbf{D}$.

The A-optimality criterion is equivalent to minimizing the sum of all the eigenvalues and can be understood as minimizing the sum of all the sensitivities of the process. The D-optimality criterion corresponds to minimizing the multiplication of the eigenvalues. This criterion has been widely use in design of experiments due to its clear interpretation, which is the minimization of the uncertainty on the parameters estimated using least-squares. However, this criterion cannot be used in fixture design because matrix $\mathbf{D}$ is singular due to the singularity of the A's matrices used to form it [17]. The E-optimality criterion is equivalent to minimizing the square root of the 2-norm of $\mathbf{D}$. In practice, this is equivalent to minimizing the worst possible deviation in the process, which is associated with the maximum eigenvalue of $\mathbf{D}$. Using the E-optimality criterion, the optimization problem can be stated as determining the location of the locators described by vector φ, which minimizes the upper bound of the sensitivity and does not violate the product/process constraints $\mathbf{g}(\varphi)$, that is,

$$\min_{\varphi} S_{\max} \equiv \lambda_{\max}(\mathbf{D}^T \cdot \mathbf{D}) \quad \text{s.t.} \quad \mathbf{g}(\varphi) \leq \mathbf{0} \tag{10}$$

where $\lambda_{\max}(\cdot)$ stands for the maximum eigenvalue of a matrix, and the geometric constraints $\mathbf{g}(\varphi)$ consider that the locators have to be located in the feasible region inside the parts.

To solve the optimization problem in Eq. (10), Kim and Ding [16] used several different methods, such as sequential quadratic programming, simplex, basic exchange, modified Fedorov, and revised exchange. Since the problem in Eq. (10) is nonlinear and may have several local minima, the global optimality of the solution cannot be guaranteed.

2.3 Reconfigurable Fixturing Systems. There have been many attempts to use reconfigurable fixturing systems in manufacturing aiming to reduce cycle time, fixture costs, and process variation [18]. The first automatically reconfigurable assembly fixture was developed by Asada and By [19]. They studied reconfigurable or adaptive fixture systems using kinematical and mechanical approaches. Since then, research has been done in the area such as assembly flexibility [20], and on quality by error compensation [21].

In machining, Walczyk and Longtin [22] studied the use of reconfigurable fixtures for compliant parts. They analyzed the performance of a reconfigurable system formed by a matrix of extendable pins, used to locate a workpiece, in terms of the forces applied and the system accuracy. More recently, Shen et al. [23] developed a reconfigurable fixturing system that can be relocated in the pallet as different parts enter the machining station.

The aforementioned efforts were mainly focused on the design of reconfigurable fixture devices. However, they do not consider the layout of the fixture (e.g., distribution and selection of reconfigurable devices). The single-station layout design for a family of products was first studied by Lee et al. [18]. They investigated the use of reconfigurable equipment to fixture a family of sheet metal parts using the N-2-1 scheme. The problem addressed was to determine the feasible position of the PTs in the station to ensure that all the parts can be processed. They also determined the minimal size of the required working spaces in order to use small PTs.

Table 1 Comparison of modeling and fixture design methodologies

		Single product	Multiple products
Single-station level	Modeling and fixture design	[3,8–15]	[18–23]
Multistation level	Modeling	[4–7]	Each product can be treated independently
	Fixture design	[16]	To be developed in this paper

Table 1 summarizes the methodologies presented in this review section. Previous work in robust design and reconfigurable fixtures has been based on a single machine (station) level for a single or multiple products. On the other hand, the multistation approach has only been considered for a single product. Therefore, there is a need to develop a methodology to design a robust reconfigurable fixture layout for a product family assembly in a single line.

3 Optimal Locators Layout for a Product Family

This section presents a methodology to solve the problem of distributing the locators for a product family in which the products share fixtures. This problem can be formulated as a constrained optimization problem, including the determination of the objective function, the definition of the constraints and the optimization method to search the solution.

3.1 Objective Function. Minimizing the sum of squares of the final product deviations ($\mathbf{y}^T \cdot \mathbf{y}$) is equivalent to optimize dimensional quality. Therefore, we propose that the objective function $f(\cdot)$, used to determine the optimal PLPs' location, is a function of the upper sensitivity of all the products. Then, for a product family, consisting of r products or models sharing the same assembly line, the problem can be formulated as

$$\min_{\varphi^1, \varphi^2, \ldots, \varphi^r} f(S_{\max-1}, S_{\max-2}, \ldots, S_{\max-r}) \quad \text{s.t.} \quad \mathbf{g}(\varphi^i) = \mathbf{0} \qquad \forall i = 1 \cdots r \tag{11}$$

In particular, we consider the case where the function $f(\cdot)$ is the weighted sum of the sensitivities' upper bound for the whole family, as presented in Eq. (12). The reasons for selecting this function are the following: (i) it directly incorporates all products into the objective function; (ii) it allows the use of weights to accounts for difference in importance between the different products in the family; and, finally, (iii) in case that a cost-quality-sensitivity model were available, the use of the proposed objective function, using the sensitivities summation, will allow designers to quantify the maximum potential cost or the cost upper bound incurred (due to the increment of variation) by using a single reconfigurable line (the development of a cost model is a topic of future research).

$$f(S_{\max-1},S_{\max-2},\ldots,S_{\max-r}) = \sum_{i=1}^{r} w_i \cdot S_{\max-i}\sum_{i=1}^{r} w_i \cdot \lambda_{\max}(\mathbf{D}_i^T \cdot \mathbf{D}_i) \tag{12}$$

After replacing Eq. (12) with Eq. (11) we have the formulation of the fixture layout problem as

$$\min_{\varphi^1,\varphi^2,\ldots,\varphi^r} \sum_{i=1}^{r} w_i \cdot \lambda_{\max}(\mathbf{D}_i^T \cdot \mathbf{D}_i) \quad \text{s.t.} \;\; \mathbf{g}(\varphi^i) = \mathbf{0}, \qquad \forall\, i = 1 \cdots r \tag{13}$$

The use of weights $w_i(w_i > 0)$ allows designers to incorporate in the formulation the relative importance that each product has on the family. A possible criterion to select the weights is to consider the expected demand for each product (the product with a higher expected demand can have higher weights, i.e., defining the weights as the product mix-ratio). Another possibility is to use the normalized expected profit of each product as weights. The normalized expected profit can be obtained by dividing the profit of each model by the total expected profit of the product family. Exploring possible selections of the weights is not the scope of this work; hence, we assume all the weights equal to 1.

The constraints $\mathbf{g}(\varphi^i)$ correspond to the set of geometric constraints that limit the PLPs' location φ^i of the ith product. They contain information about the feasible region where the locators can be placed, PT dimensions, and the relations between the parts and the different products of the family (their derivation is presented in Sec. 3.2). In this case, the design vector φ^i contains the position of the $2m$ PLPs required to locate the m parts or components of product i in the X-Z plane. The PLPs' locations are directly related with the position of the two NC blocks containing pins (see Fig. 2). The location of the third NC block is not considered in this analysis because it does not impact the in-plane variation of the part. The PLPs' locations for the ith product are denoted by $\varphi^i=[\mathbf{p}_1^i\mathbf{p}_2^i\cdots\mathbf{p}_{2m}^i]$, where $\mathbf{p}_k^i(\forall k=1,2,\ldots,2m)$ has two coordinates (one in X and one in Z); consequently, the design variables can be rewritten for the ith product as $\varphi^i=[(x_1^i,z_1^i)\times(x_2^i,z_2^i)\cdots(x_{2m}^i,z_{2m}^i)]$.

Since the objective is to minimize the maximum eigenvalue of $\mathbf{D}_i^T\cdot\mathbf{D}_i$, it is important to analyze the sensitivity of the eigenvalue calculation to modeling and computational errors. Model errors are caused by errors in the generation of the system matrices A's, B's, and C's, and computational errors are inherent in calculations with floating point arithmetic [24]. In this research, both errors can be seen as perturbations of the true matrix product $\mathbf{D}_i^T\cdot\mathbf{D}_i$. A special property of symmetric matrices, such as $\mathbf{D}_i^T\cdot\mathbf{D}_i$ (the multiplication of a nonsymmetric matrix by its transpose results in a symmetric matrix), is that they have the lowest possible eigenvalue conditioning or sensitivity of the eigenvalue calculation to perturbations [24,25]. Therefore, the selected criterion, based on minimizing the maximum eigenvalue (Eqs. (10) and (13)), is robust to modeling and computation errors.

3.2 Constraints' Definition. The constraints define the feasible space where the PLPs can be located as well as the necessary conditions to ensure that the assembly is feasible. Thus, they define the viability of the assembly. Before describing the constraints for the product family design problem, it is necessary to present some process conditions or considerations that make the problem addressed in this research closer to the reality; those are the following.

- Each part has only one set of PLPs. This implies that in later stations each subassembly must be held using some of the previously used locating points available on the parts. The use of only one set of locators per part is a common practice in industry because it helps to minimize the cost of the parts.
- Each PT carries the set of fixture elements (blocks and pins) necessary to hold a part or subassembly. This condition avoids the use of multiple PTs to carry each part or subassembly, to save cost and space.
- To avoid increasing the mechanical complexity and cost of the PT, it is considered that the distance between the pins installed on the PT is constant. This distance is a design variable, which has to be the same for all the products, and cannot vary from product to product.

Considering the aforementioned conditions, it is possible to define the constraints for the product family as follows (the mathematical description of the constraints can be found in Appendix A).

(a) All the PLPs must be positioned within the feasible area of an individual part. This area includes all the part and excludes the internal holes on the part. A safety margin of 30 mm is defined along all the part contours (internal and external) to ensure that the locators are not too close to the edges. The verification of the belonging or not of a point to the feasible region of a part was done using an image-matrix of the geometric shape of every part. Then, a value of 0 was assigned to the "in" or feasible region and 1 to the outer or infeasible region (including cavities on the parts). Doing so, the verification of the in/out location of a point was done by checking if the coordinates of the point correspond to 0 or 1 in the appropriate image (part). The advantages of this method are that it is simple to check, and the image has to be calculated only once, then stored and used every time it is required. The generation of the image requires information of the position of both external and internal vertices that defines the part, and an algorithm to check if a point belongs or not to a certain region. There are many algorithms to perform this type of verification, one of those is the point inclusion test widely use in the CAD-CAM and the computational geometry fields [26].

(b) The distance between the locators on each part-type (d) and subassembly-type (s) should be the same for all models (see Fig. 5). This means that the distance between the two locators used to hold the same type of part or subassembly is fixed. However, the position of the pins in the station can be adjusted using the PT to accommodate the different products. If the distances between the locators used to hold a given part-type or a subassembly-type are not the same for all the models, then one or more assemblies are not feasible because the parts or subassemblies do not fit into the fixtures. Figure 5(a) presents graphically the constraint for the part-type (products A, B, and C), and Fig. 5(b) presents the constraints for the subassembly-type (only products A and B are shown).

(c) The PT has to be able to locate the fixture elements in the appropriate position; therefore, at least one point in between both pins has to belong to the workspace of the PT (e.g., the middle point between the pins). Graphically this can be presented in Fig. 6, where the locator's middle points, represented by triangles, are inside the PT workspace. For the case where the workspace is circular, the radius of the minimum circle that contains all the middle points must be smaller than the workspace radius. The problem of determining the circle with minimum radius that contains a set of points is known as the minimum circle enclosing problem [18].

(d) Another constraint that can be included is that the PLPs on each part have to be aligned along one of the principal axes of the part. This prevents the coupling of the errors

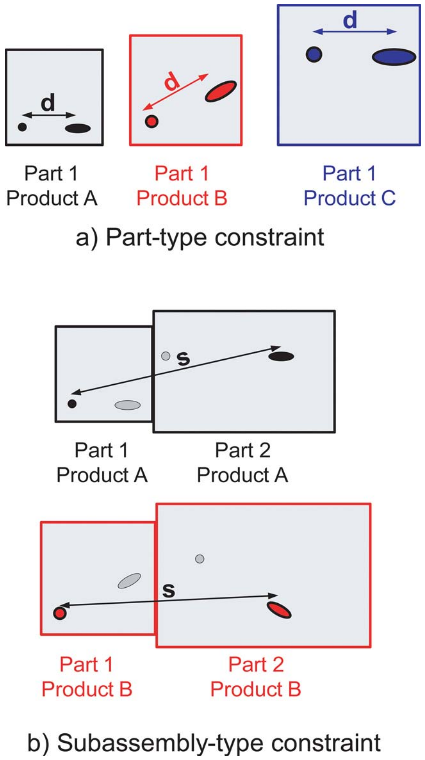

Fig. 5 Distance constraint in parts and subassemblies for different products

in the three axes. Therefore, having the PLPs aligned with the principal axis of the part is a recommended practice. Mathematically, the constraints can be represented as the product of the differences in location of the hole and the slots in the X and Z directions, which has to be equal to zero to ensure the correct alignment.

The aforementioned constraints characterize the multiproduct fixture design problem and do not consider, for simplicity, other fixture functional requirements such as part static stability, clamping stability, and total restraint [9]. These unconsidered requirements can be incorporated as additional constraints into the proposed formulation.

3.3 Optimization and Optimality. Due to the nonlinear nature of the problem and the constraints, sequential quadratic programming was chosen to perform the optimization. This optimization method is frequently used for fixture design [14,16,27]. One of the properties of the gradient-based method is that it tends to converge rapidly. A disadvantage of this method is that it can be easily entrapped in a local optimum. Therefore, different initial conditions can be used to perform the search for a good locator layout.

Due to the complexity of the objective function and the constraints, solving the problem as proposed in Eq. (11) is difficult. On top of this, obtaining a feasible initial condition that satisfies all the constraints is also challenging. Therefore, the problem was solved first using the relaxed formulation (Lagrange relaxation), which is, in general, easier to solve compared with the original one [28]. Equation (14) presents the relaxed formulation, where the objective function directly includes the squares of the constraints multiplied by a constant factor or Lagrange multiplier $\beta(\beta > 0)$.

$$J = \min_{\varphi^1,\varphi^2,\ldots,\varphi^r} \sum_{i=1}^{r} w_i \cdot \lambda_{\max}(\mathbf{D}_i^T \cdot \mathbf{D}_i) + \beta \cdot [\mathbf{g}(\varphi^i)^T \cdot \mathbf{g}(\varphi^i)] \quad (14)$$

The relaxed form of the problem has the advantage of allowing a slight violation of the constraints. Therefore, it can be used as a starting point for the solution of the constrained problem (13). The selection of the multiplier β is done to ensure a reasonable solution (low value of β) that tolerates some constraints' violation, and then, it is increased to look for a solution that is closer to the one of the real problem. Finally the true problem (13) can be solved starting from the result of the one with the highest factor β.

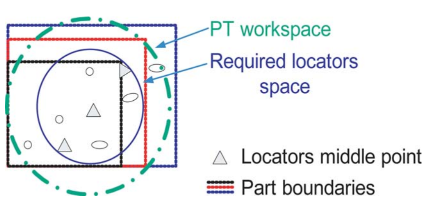

Fig. 6 Workspace verification

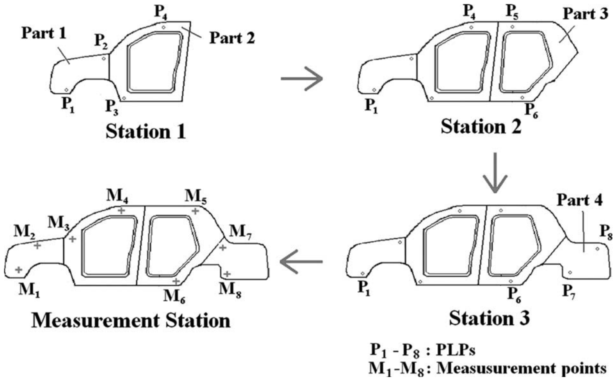

Fig. 7 Assembly sequence of a sedan side frame

4 Case Study

The case study selected is the assembly of the side frame of a family of sedans, as presented in Fig. 1. The side frames are composed of four parts each and are assembled in the process pictured in Fig. 7. The process consists of three assembly stations and a final measurement station, where the location of the KPCs defined for this process are measured (points M).

The data scheme defined for this process is the following: In station 1 the locators used are $\{(P_1,P_2),(P_3,P_4)\}$; this means that the first part is held using locators P_1 and P_2, and the second part using locators P_3 and P_4. In station 2 the locators used are $\{(P_1,P_4),(P_5,P_6)\}$, in station 3 $\{(P_1,P_6),(P_7,P_8)\}$, and in the measurements' station $\{(P_1,P_8)\}$.

The feasible region where the PLPs must be placed can be determined based on the location of the internal and external vertices. The locations of all the vertices and KPCs are presented in Appendixes B and C for the small sedan. To obtain these sets for the medium and large sedans, it is necessary to multiply each coordinate by 1.06 and 1.12, respectively.

The PTs used in the assembly were assumed to be robots with three degrees of freedom in the plane as presented in Fig. 8(a), which corresponds to a revolute-revolute-revolute type robot. Due to the robot characteristics, they have a circular working space as shown in Fig. 8(b). The radius e of the workspace was selected to be 500 mm.

The results of the PLP layout for a product family are presented next. The results are benchmarked with the optimal solutions obtained for each product as it were assembled in a dedicated assembly line (dedicated line for each product). This comparison

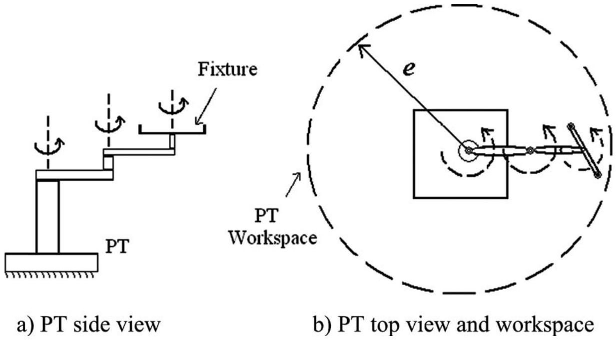

Fig. 8 Views of a programmable tool and its workspace

provides information of the performance compromised, in terms of robustness to fixture variation, by using a single reconfigurable line.

Due to the existence of several local minima, in accordance with the results obtained by Kim and Ding [16], 100 random initial conditions were used to search for a good layout of the PLPs for both the product family and the single products (case of dedicated lines). In the product family case the multiplier β was first set to 5. Later, the layout with lower J was optimized after β increased to 50, and then to 750.

4.1.1 Fixture Layout for a Dedicated Line. In the optimization for each single model, considering dedicated lines, two cases were analyzed. Case 1 has no constraint in the alignment of the locators and case 2 impose constraints on the alignments of the locators. In both cases the optimization was performed 100 times starting from random initial conditions of the locators for each model independently. Figures 9 and 10 present the location of the fixtures for each model for the cases with and without locator's alignment. Table 2 presents the values of λ_{max} for each configuration. The table also includes a row showing the sum of the λ_{max} for later comparison with the product family solution.

For the case of aligned pins, the solution obtained in height (z) is close to the "center of gravity" of the sensor points in the same direction. Therefore, for this case where the pins have to be aligned, their locations tend to be equally distant to the "upper" and "lower" sets of measurement points. In that way, the effect of fixture variation will be minimized in average.

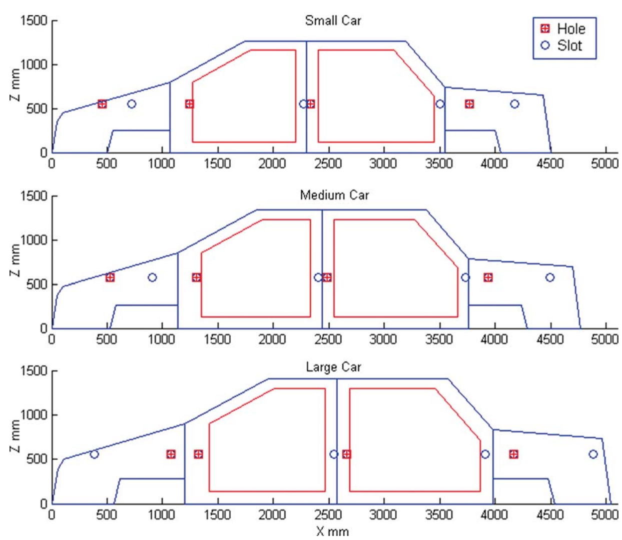

Fig. 9 Location of the PLPs for dedicated lines with the alignment constraint

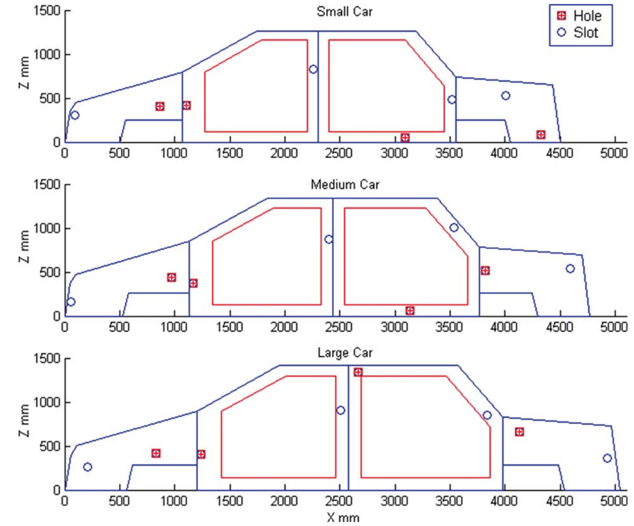

Fig. 10 Location of the PLPs for dedicated lines without the alignment constraint

4.1.2 Fixture Layout for a Reconfigurable Line. Figure 11 presents the final location of the PLPs for the family, where the distances of the hole and the slot are the same across the three products. The values of the upper bound of the sensitivity (λ_{max}), obtained for each model and for the product family (sum of the λ_{max}), are presented in Table 2.

No results are presented for the case of the product family with aligned pins since there is no feasible solution to that problem for

Table 2 Results of the optimization for each single model (λ_{max})

	Dedicated lines	Dedicated lines (aligned pins)	Reconfigurable line
Small car	18.04	20.28	23.93
Medium car	18.01	18.56	25.07
Large car	18.02	19.98	21.83
Sum λ_{max}	54.07	58.70	70.93

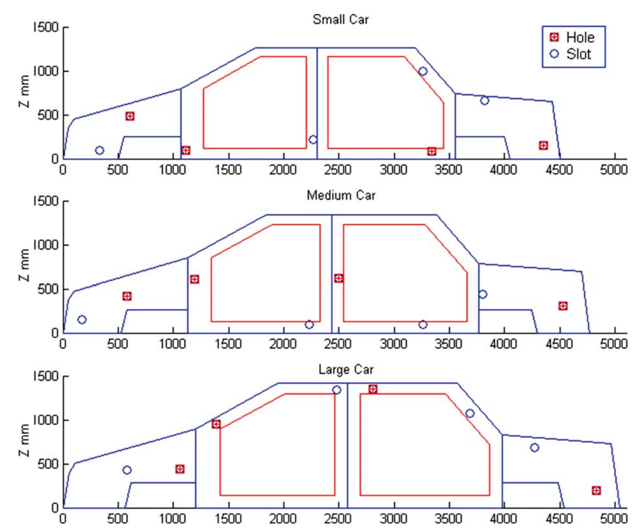

Fig. 11 Location of the PLPs for reconfigurable line (note that the distance between the hole and the slot remains the same for each part-type across the three models)

the products considered here.

The difference between the sum of λ_{max} for the reconfigurable line and the dedicated lines (nonaligned and aligned cases) are 16.76 and 12.13, respectively, which corresponds to increments of 31% and 21% for each case. Those increases can be judged as reasonable considering the complexity of the geometries, the amount of constraints that the reconfigurable line imposes in the assembly, and the differences in sizes of the cars. It is important to note that the value obtained with λ_{max} corresponds to the upper bound on the sensitivity. Therefore, it corresponds to the worst case scenario. The increase in the sensitivity of the product family can be compensated through an appropriate distribution of the tolerances in the parts and tools and a good maintenance strategy that keeps the variation low.

5 Conclusions

This paper proposes a methodology for fixture configuration design for a family of products assembled in a single reconfigurable line. The problem is formulated as a constrained optimization by considering part geometry, PTs' workspace, and pins' alignment. Sequential quadratic programming and Lagrange relaxation were used to search for a robust design. The resulting layout of the PLPs using a reconfigurable line is compared with the case of single product dedicated lines in term of the quality of the solution. Two different scenarios were analyzed: no alignment restriction on the PLPs, and the PLPs have to be aligned (in *X* or *Z* directions). The result obtained for the product family is feasible; however, the sensitivity is 31% higher than the one for dedicated lines considering the worst case. This increment does not imply that the product family assembly is in general worse than the single lines. Obviously, there is a tradeoff between the achievement of production flexibility by using a reconfigurable line, and the robustness of the system to fixture variation for the product family. An enterprise level evaluation of the pros and cons of both approaches (reconfigurable versus dedicated) seems to be an appropriate method to decide which production scheme is better considering expected demands, product and process costs, flexibility, and quality among other factors. It is the aim of this research to assist that type of decision through the development of tools that help to perform such evaluation, and also assist designers on the development of this type of assembly process.

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Appendix A: Constraints

First, the nomenclature used to formulate the constraints is presented; the locators $\mathbf{p}^i_{p\text{-}j\text{-hole}}$ and $\mathbf{p}^i_{p\text{-}j\text{-slot}}$ are vectors containing the position (in the *x*-*z* directions) of the hole and slot, respectively, for the *j*th part of the *i*th. The terms $\mathbf{p}^i_{s\text{-}j\text{-hole}}$ and $\mathbf{p}^i_{s\text{-}j\text{-slot}}$ stand for the vectors containing the position (in the *x*-*z* directions) of the hole and slot, respectively, for the *j*th subassembly of the *i*th model. The number of parts on each model is *m*, the number of subassemblies is *q*, and the number of models is *r*.

Constraint a. The locators should be inside the feasible region of the parts

$$\mathbf{p}^i_{p\text{-}j\text{-hole}} \text{ and } \mathbf{p}^i_{p\text{-}j\text{-slot}} \in \text{feasible region of part } j \text{ of product } i \qquad \forall j = 1 \cdots m, \quad \forall i = 1 \cdots r \tag{A1}$$

Constraint b. The distance $d^i_j = \text{dist}(\mathbf{p}^i_{p\text{-}j\text{-hole}}, \mathbf{p}^i_{p\text{-}j\text{-slot}})$ between the locators (hole and slot) used in the *i*th part of the *j*th model has to be the same across models, where dist$(\mathbf{a}, \mathbf{b})$ stands for the Euclidian distance between vectors **a** and **b**.

$$d^1_j = d^2_j = \cdots = d^r_j, \quad \forall j = 1 \cdots m \tag{A2}$$

Also for each subassembly, the distance $s^i_j = \text{dist}(\mathbf{p}^i_{s\text{-}j\text{-hole}}, \mathbf{p}^i_{s\text{-}j\text{-slot}})$ between the locators used in the *i*th subassembly of the *j*th model has to be the same across models

$$s^1_j = s^2_j = \cdots = s^r_j, \quad \forall j = 1 \cdots q \tag{A3}$$

Constraint c. The position of the fixture containing the pins used to hold each part or subassembly must be inside the workspace of the PT that carries it. Then, the position of the point where the PT holds the fixture, which lies between the two locators, must be inside the workspace of the PT. This point can be described, for the case of a part, as

$$\mathbf{f}^i_{p\text{-}j} = \mathbf{p}^i_{p\text{-}j\text{-hole}} + \alpha_j(\mathbf{p}^i_{p\text{-}j\text{-hole}} - \mathbf{p}^i_{p\text{-}j\text{-slot}}) \tag{A4}$$

and for a subassembly it can be described as

$$\mathbf{f}^i_{s\text{-}j} = \mathbf{p}^i_{s\text{-}j\text{-hole}} + \alpha_j(\mathbf{p}^i_{s\text{-}j\text{-hole}} - \mathbf{p}^i_{s\text{-}j\text{-slot}}) \tag{A5}$$

where α_j is a constant $(0 \leq \alpha_j \leq 1)$; without lost of generality we can assume $\alpha_j = 0.5$. Then, the condition for the point to belong to the workspace of the corresponding PT can be written for a part and a subassembly as

$$\mathbf{f}^i_{p\text{-}j} \in \text{workspace}_j, \quad \forall j = 1 \cdots m, \quad \forall i = 1 \cdots r \tag{A6}$$

$$\mathbf{f}^i_{s\text{-}j} \in \text{workspace}_j, \quad \forall j = 1 \cdots q, \quad \forall i = 1 \cdots r \tag{A7}$$

The workspace of each PT is defined by its own characteristics (e.g., dimensions, number of DOFs, type of joints, etc.), and it represents all the points that a PT can reach holding the fixture in the appropriate direction.

Constraint d. The locators on each part and subassembly have to be aligned along one of the principle axes of the part or subassembly; then, the product of the differences between the location of the hole and the slot along each axis, for each part/subassembly, must satisfy the following condition:

$$(\mathbf{p}^i_{p\text{-}j\text{-hole}}(x) - \mathbf{p}^i_{p\text{-}j\text{-slot}}(x)) \cdot (\mathbf{p}^i_{p\text{-}j\text{-hole}}(z) - \mathbf{p}^i_{p\text{-}j\text{-slot}}(z)) = 0 \qquad \forall j = 1 \cdots m, \quad i = 1 \cdots r \tag{A8}$$

$$(\mathbf{p}^i_{s\text{-}j\text{-hole}}(x) - \mathbf{p}^i_{s\text{-}j\text{-slot}}(x)) \cdot (\mathbf{p}^i_{s\text{-}j\text{-hole}}(z) - \mathbf{p}^i_{s\text{-}j\text{-slot}}(z)) = 0 \qquad \forall j = 1 \cdots q, \quad i = 1 \cdots r \tag{A9}$$

Appendix B: Geometry of the Parts (Small Car Vertices)

The following table shows the geometry of the geometry of the parts of small car vertices.

Part No.		Vertex location (*x*/*z*) (mm)
1	External vertices	(0/0); (500/0); (550/250); (1070/50); (1070/800); (100/450); (50/350)
2	External vertices	(1070/0); (2300/0); (2300/1260); (1740/1260); (1070/800)
	Internal vertices	(1270/120); (2200/120); (2200/1160); (1790/1160); (1270/800)
3	External vertices	(2300/0); (3550/0); (3550/740); (3190/1260); (2300/1260)
	Internal vertices	(2400/120); (3450/120); (3450/640); (3090/1160); (2400/1160)
4	External vertices	(3550/250); (4000/250); (4050/0); (4500/0); (4430/650); (3550/740)

Appendix C: Location of the Measurement Points (Small Car)

The following table shows the locations of the measurement points of small car vertices

Part No.	Position of measurement points (*x*/*z*) (mm)
1	(100/450); (1070/800)
	(1100/300); (1100/600); (1360/940); (2000/1200); (2300/1200); (2200/1000);
2	(2200/400); (2000/100); (1500/100); (2150/1260)
3	(3550/700); (3350/250); (3300/100); (2500/100)
4	(3800/685.8); (4430/650); (4050/0)

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