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QUALITY AND RELIABILITY INFORMATION INTEGRATION FOR DESIGN EVALUATION OF FIXTURE SYSTEM RELIABILITY

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SUMMARY

Quality and reliability are two important factors in manufacturing-system design. However, the analysis and optimization of manufacturing-system reliability and product quality are normally conducted separately in practice. There is no general framework to integrate these two important factors, quantitatively analyze the interactions between them, and further study their integrated effects on the manufacturing-system performance. In this paper, the QR-Co-Effect of product/part quality and manufacturing-system component reliability is investigated in an assembly fixture system. The concept, model and analysis of QR-Co-Effect are addressed in this paper. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS: quality and reliability (QR) co-effect; manufacturing system reliability; product quality

1. INTRODUCTION

Manufacturing system reliability is a vital factor in ensuring product quality and productivity in a production process. Various efforts have been made to analyze and optimize manufacturing-system reliability in the component and system-design stage. In general, system-reliability analysis is based on component failures and the interdependency among those components. Various methodologies or techniques, such as fault-tree models, failure modes and effect analysis (FMEA), block reliability diagram, Petri nets, and the Markov model have been developed for this purpose [1].

However, the downtime of a manufacturing system is not only caused by manufacturing-system component failures, but also by nonconforming products produced by a degraded system. In the literature, system reliability is generally defined as the probability that a system will perform its intended function under operating conditions for a specified period of time [2]. Therefore, system-reliability analysis for a manufacturing system should consider not only its tooling and machine uptime, but also the produced-product quality.

In general, product quality has a complex relationship with the conditions, layout and interaction of manufacturing system components. Under given quality specifications, the determination of a threshold of a degraded component is very difficult due to the lack of a system model to integrate the product quality with the system-component reliability. Most of past research has focused on system degradation as a result of degraded system components. Various decision rules have been provided to determine the system failure by specifying a threshold as the maximum acceptable degradation for each component [3,4]. In those analyses, thresholds are mainly determined according to engineering experience or rule of thumb. There is no systematic analysis methodology to describe the interdependency between product quality and system-component degradation. As a result, either a conservative design of system reliability is used with excessively reliable components, or an over-optimistic reliable system is obtained leading to lots of unexpected downtime during production. There is therefore a pressing need to develop an integrated-system model by integrating product quality, manufacturing-system component degradation, and process-design information to enable manufacturing-system design evaluation and optimization. In order to develop such an integrated model, some terminology is defined.

- Manufacturing system—a system consisting of several tools or items of equipment performing the designed operations in a manufacturing process to produce products. As an example of

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an assembly process discussed in this paper, a fixture system consisting of locating pins and locating blocks, used to hold subassembly parts for welding operations to obtain an assembled product is considered as a manufacturing system. Detailed explanation of the functionality of locating pins and locating blocks in a fixture system will be given in Section 2.1.

- Manufacturing system component—a physical part of a manufacturing system, such as tools, equipment or machines. In this paper, locating pins and locating blocks are considered as manufacturing system components in a fixture system.
- Product—the final output or the end item of a manufacturing system. For example, a vehicle door frame is an output of a fixture system in an autobody assembly process.
- Incoming part (or simply called part)—a physical part of a product with its unique function in the product, such as the stamped parts of inner/outer apertures of a vehicle door frame, taken as the input of a manufacturing system. Other common examples are resistors inside TV products, etc.
- Quality—product assessment by using the statistics of the deviation of the quality characteristic measurements on the incoming parts or final products. In the paper, the 'product quality' of a fixture system is evaluated by the dimensional variance of the key product characteristic (KPC) measurements on the assembled products. 'Incoming part quality' is evaluated by the diameter variance of part-holes of incoming parts.
- Manufacturing-system reliability—the probability for a manufacturing system to work functionally for a specified period of time, that is, there are no catastrophic system failures, and the produced product quality is satisfied during the specified period of time. So, 'fixture system reliability' used in this paper is evaluated by the probability of producing satisfactory quality of products with no catastrophic failures of locating pins and locating blocks for a specified period of time.
- Manufacturing-system component reliability—generally described by two aspects: catastrophic failure and performance degradation. In the paper, catastrophic failure refers to a broken or loosened pin in a fixture system, and performance degradation means pin-diameter wear resulting in an increasing locating error.

The reliability analysis of a manufacturing system should consider both tool/machine component reliability and its output and input product/part quality. It can be seen that there is an interaction between manufacturing-system component reliability and the product/part quality, which is represented by the newly-defined concept 'QR-Co-Effect', illustrated in Figure 1. The functions of QR-Co-Effect have two aspects. One aspect is the degradation (δx) of a manufacturing-system component, with an impact on product quality (variance σ^2 and mean deviation μ). A product with unsatisfactory quality may be produced due to manufacturing-system component degradation before a catastrophic component failure is observed. This leads to manufacturing-system downtime due to defective products in production. The other aspect is the incoming-part quality that also has an impact on manufacturing system-component degradation δx and its probability of catastrophic failure. A larger variation of incoming parts may introduce more interference between subassembly parts and manufacturing-system components during operation. As a result, an accelerated degradation and/or more catastrophic failures may be observed when incoming parts have a larger variation or mean deviation, causing manufacturing-system components to fail sooner than those under consistently high quality of incoming parts. Without considering the QR-Co-Effect, the results of a manufacturing-system reliability analysis could be biased. However, there has been no systematic methodology available that incorporates the QR-Co-Effect into manufacturing-system reliability evaluations. The introduction of the QR-Co-Effect concept will address this issue.

Unlike conventional reliability analysis methods, the QR-Co-Effect modeling closely depends on both the given system functions and the user-defined product quality. In order to model such a QR-Co-Effect functionality, we need to address the following issues.

(1) Modeling the impact of manufacturing-system component degradation on product quality.
(2) Modeling manufacturing-system component degradation.
(3) Studying the impact of incoming-part quality on manufacturing-system component reliability.

Depending on product and manufacturing-system design, the quantitative relationship between incoming-part quality and manufacturing-system component reliability varies; the quantitatively modeling of the QR-Co-Effect in manufacturing-system reliability analysis is a very challenging problem. In this paper, the concept and the analysis framework of QR-Co-Effect are illustrated by using

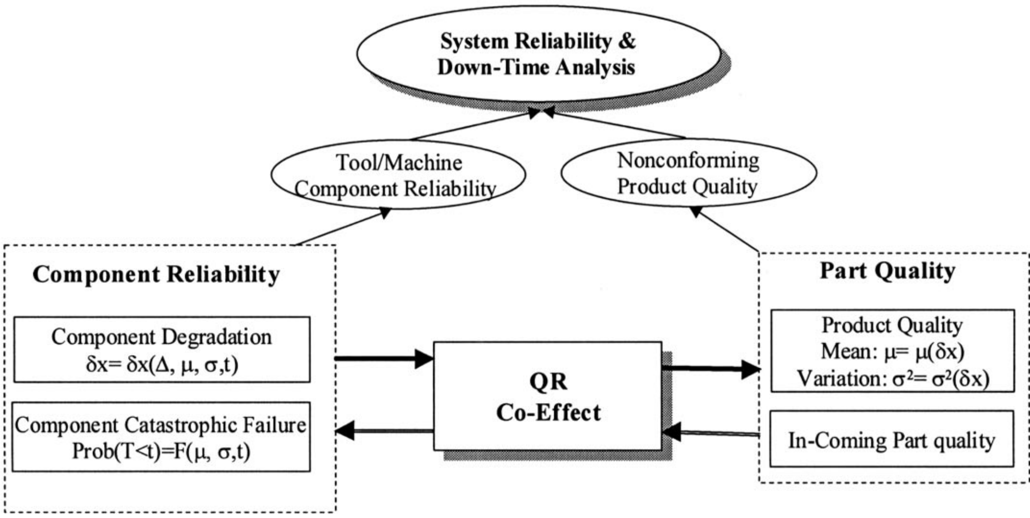

Figure 1. QR-Co-Effect function between quality and reliability

an assembly fixture system. A comparative study is conducted under different tooling layouts to explore the importance of integration of part quality and manufacturing-system component reliability information for system-reliability analysis. The potential use of the methodology for selection of manufacturing-system components, design of fixture layouts and improvement of manufacturing-system reliability is also discussed.

The paper is organized as follows: after a brief introduction, the effect of the fixture functionality on product quality is discussed in Section 2. In Section 3, the mechanism of manufacturing-system component wear is explained. Both qualitative and quantitative models are provided in this section. Then, quality and reliability information integration is addressed in Section 4. Section 5 provides an evaluation for the proposed methodology using the Monte Carlo simulation method. A comparison study is also given in this section to elaborate the importance of the proposed integrated reliability-analysis methodology. Afterward, more simulation analyses are provided in Section 6 to illustrate the potential usage and the effectiveness of the proposed methodology. Finally, the paper concludes in Section 7.

2. THE EFFECT OF FIXTURE FUNCTIONALITY ON PRODUCT QUALITY

The automotive body without doors, hood, fenders and trunk lid is called ‘body in white’ (BIW). In a BIW assembly line, depending on the complexity of products, there are typically 80 to 130 assembly stations which assemble 150 to 250 sheet-metal parts. Based on their functions, the components of a BIW are usually divided into structural and non-structural parts. Structural parts, such as rails, plenum and door-hinge reinforcements are much more rigid than non-structural parts, such as the door outer panel, cowl-side, roof, etc. Past research indicates that a structural part usually has a much greater impact on the automotive body dimensional accuracy [5,6]. Therefore, only structural parts will be considered in the modeling procedure. In this paper, the assumption of a rigid part and the sufficiency of using a 3-2-1 fixture (which will be discussed in Section 2.1) to locate a rigid panel are made in the analysis. These assumptions cover 68% of the total parts in a typical autobody [7].

In the BIW assembly, assembly fixtures play a critical role in ensuring both product quality and process throughput. If fixture components malfunction (e.g. if locating pins or locating blocks are loose or broken), parts will not be located in the correct position, leading to manufacturing-system downtime. Alternatively, fixture system components also gradually become worn during production; the wear of these components will lead to deterioration of accuracy of location, affecting product quality. Previous research indicates that 72% of the root causes of dimensional errors in BIW are due to fixture-system components [8].

Generally, system reliability due to system-component failure and degradation can be analyzed in the design stage using conventional methodologies, if the reliability information about system compo-

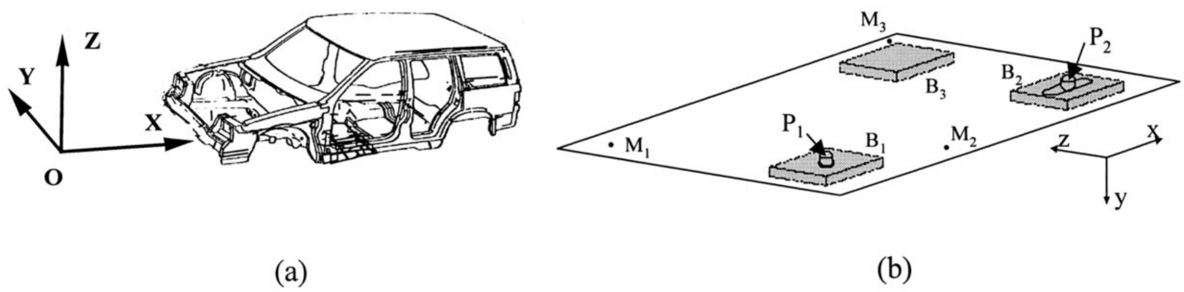

Figure 2. Automotive body and its assembly fixture: (a) body coordinate system; (b) 3-2-1 fixturing principle for rigid part

nents and their interactions is known [9–12]. However, these analysis methods cannot be applied directly to manufacturing-system reliability analysis because product-quality information and the interdependency between part quality and manufacturing-system component reliability are not considered. Consequently, a new methodology should be developed to integrate quality and reliability information for fixture design evaluation and optimization.

2.1. *Fixture layout and major components*

In this paper, a body coordinate system shown in Figure 2(a) is used. The origin of the body-coordinate system is defined in the front center of a vehicle and below its underbody. The X-Y-Z axes are shown in the figure. This definition of the body-coordinate system has been widely used in automotive industry in the product and process design.

In sheet-metal assembly, the position and orientation of the subassembly parts must be accurately located and remain fixed in the body-coordinate system during assembly operations. Generally, a 3-2-1 fixturing principle is widely used in assembly processes for locating a rigid part, using the minimum geometric features without creating interferences among locators. As an example, a typical 3-2-1 fixture system, as shown in Figure 2(b), is widely used in autobody assembly processes. In this fixture system, (1) a four-way locating pin (P_1) is used to locate the part datum feature, which is a hole in the part, to determine precisely the part-hole location in the X and Z directions; (2) a two-way locating pin (P_2) is used to locate another part datum feature, which is a slot in the part, to precisely determine the part-slot position in the Z direction. Therefore, these two locating pins (P_1 and P_2) constrain the part rotation and translation in the X–Z plane; and (3) three locating blocks (B_1, B_2, B_3) are used as locators working with clamps to constrain the part movement in the Y direction. Therefore, the combined functions of the fixture components (locating pins and locating blocks) constrain all six degrees of freedom of a rigid sheet-metal part in three-dimensional space.

In order to simplify the illustration of this new QR-Co-Effect concept, the fixture-system reliability analysis presented in this paper will focus on locating pins only. The analysis of the effect of locating blocks can be conducted following the similar procedure. Thus, all later analysis in this paper will be only discussed in the X–Z plane because locating pins control part position in the X–Z plane.

2.2. *The effect of fixture system component wear on product quality*

2.2.1. Product quality assessment. Product quality is generally defined by the dimensional accuracy of KPC points on a part, such as M_1, M_2 and M_3 shown in Figure 2(b). In the X–Z plane, the X–Z coordinates of these three measurement points are denoted as $M_i = (M_i(x), M_i(z))$ $(i = 1, 2, 3)$. Let $\mathbf{y} \equiv [M_1(x)M_1(z)M_2(x)M_2(z)M_3(x)M_3(z)]^{\mathrm{T}}$ be the 6-dimensional measurement vector for the KPC points. If the deviation (from nominal) of $\mathbf{y}$ is denoted as $\delta\mathbf{y}$, product quality can be assessed by both mean and variance of $\delta\mathbf{y}$, that is, if the mean or variance of any displacement δy_i $(i = 1, \ldots, 6)$ is larger than the quality specification of maximum mean shift γ_i or maximum variance η_i respectively, product quality is determined as nonconforming. In this case, the current fixture system is taken as failure. Thus, the decision rule used for determining manufacturing system status can be stated as: if $\bigcap_{i=1}^{6}\{(\mathrm{Var}(\delta y_i \mid \delta\mathbf{X}) < \eta_i) \cap (\mathrm{E}(\delta y_i \mid \delta\mathbf{X}) < \gamma_i)\}$ is not satisfied, then the current status of the manufacturing process is determined as failed, where $\delta\mathbf{X}$ denotes the current tool wear stage, and $\mathrm{Var}(\cdot)$ and $E(\cdot)$ represent the operator for calculating the variance and mean of a random variable. In order to use this decision rule, the effect of tool conditions on the product dimensional deviation $\delta\mathbf{y}$ should be

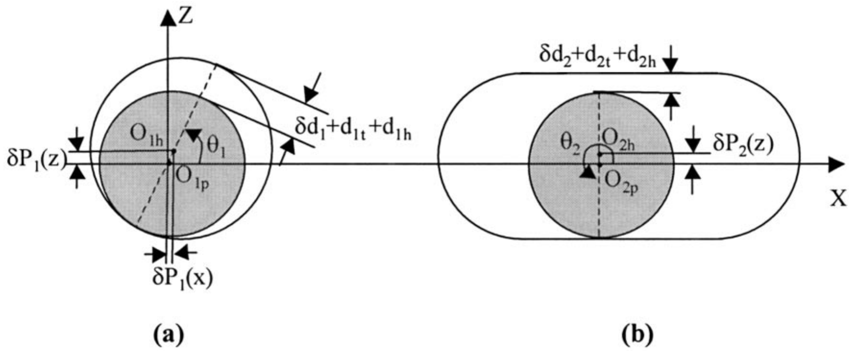

Figure 3. Locating error due to fixture system component wear: (a) four-way locating pin P_1; (b) two-way locating pin P_2

studied first, which will be discussed in the following subsections.

2.2.2. Part locating error due to fixture system component wear. In the X–Z plane, the effect of component wear on fixture system reliability is only associated with the wear of the locating pins. The wear of the locating pins mainly reflects on the reduction of pin diameters, which causes an increasing clearance between a pin and a part-hole. It is reasonable to assume that the hole on a part always contacts with the pin on one side when the part is located by a pin. Figure 3 shows different contacting orientations between pins and part-holes for Pin P_1 and Pin P_2.

In order to develop a model to describe the relationship between pin wear and a hole-center displacement of a part, some notation is defined as follows: $\delta P_i(x)$ and $\delta P_i(z)$ ($i = 1, 2$) are denoted as the displacements of the hole-center O_{ih} from the pin-center O_{ip} in the X and Z directions respectively, which are called *part locating errors* in this paper; δd_i represents the diameter reduction of Pin P_i due to pin wear; d_{it} is the design tolerance between Pin P_i and the corresponding part-hole; and d_{ih} represents the deviation of a hole diameter from its nominal due to the incoming-part dimensional error. In this paper, the quality of incoming parts is represented by the variance σ_{ih}^2 of the part-hole diameter deviations. In Figure 3(a), θ_1 is assumed to be a random variable following uniform distribution within $[0, 2\pi]$ denoted as $\theta_1 \sim U(0, 2\pi)$. Based on Figure 3(a), the relationship between the part locating errors ($\delta P_1(x)$ and $\delta P_1(z)$) and the diameter reduction of Pin P_1 can be obtained by

$$\delta P_1(x) = 0.5(\delta d_1 + d_{1t} + d_{1h}) \cos\theta_1 \quad (1)$$

$$\delta P_1(z) = 0.5(\delta d_1 + d_{1t} + d_{1h}) \sin\theta_1 \quad (2)$$

For Pin P_2, the pin wear effect on the part locating error in the X direction can be ignored. The relationship between the part locating error $\delta P_2(z)$ and the wear of Pin P_2 can be obtained by

$$\delta P_2(z) = 0.5(\delta d_2 + d_{2t} + d_{2h}) \sin\theta_2 \quad (3)$$

where θ_2 represents the orientation of the contacting surface. Because the part-hole makes contact with the Pin P_2 either on the upper side or the lower side in the Z direction, θ_2 is a random variable having two values of $-\pi/2$ and $\pi/2$ with the same probability of 0.5. In this paper, it is denoted as $\theta_2 \sim (-\pi/2, \pi/2)$.

2.2.3. Modeling the dependency of product quality on part locating errors. The relationship between the measurement deviation $\delta\mathbf{y}$ and the part locating error $\mathbf{v}$ in the X–Z plane can be described by a linear model [13]:

$$\delta\mathbf{y} = \mathbf{C}\mathbf{v}, \quad (4)$$

where $\mathbf{v} = [v_1 v_2 v_3]^{\mathrm{T}} = [\delta P_2(z) \delta P_1(x) \delta P_1(z)]^{\mathrm{T}}$ and $\mathbf{C}$ is:

$$\mathbf{C} = \frac{1}{P_2(x)} \begin{bmatrix} -M_1(z) & P_2(x) & M_1(z) \\ M_1(x) & 0 & P_2(x) - M_1(x) \\ -M_2(z) & P_2(x) & M_2(z) \\ M_2(x) & 0 & P_2(x) - M_2(x) \\ -M_3(z) & P_2(x) & M_3(z) \\ M_3(x) & 0 & P_2(x) - M_3(x) \end{bmatrix} \quad (5)$$

In this case, the coordinate origin point is simply defined as the position of P_1, and the X direction is defined as the connection line from P_1 to P_2. From equation (4), it can be seen that

$$\delta y_i = c_{i1}\delta P_2(z) + c_{i2}\delta P_1(x) + c_{i3}\delta P_1(z) \quad (6)$$

where c_{ij} is the (i, j)th entry of 6×3 matrix $\mathbf{C}$. Substitute equations (1)–(3) into equation (6), equation (6) can be rewritten as

$$\begin{aligned} \delta y_i = {} & 0.5c_{i1}(\delta d_2 + d_{2t} + d_{2h}) \sin\theta_2 \\ & + 0.5c_{i2}(\delta d_1 + d_{1t} + d_{1h}) \cos\theta_1 \\ & + 0.5c_{i3}(\delta d_1 + d_{1t} + d_{1h}) \sin\theta_1 \end{aligned} \quad (7)$$

For the given degradations of δd_1 and δd_2, the pin-wear effect on fixture-system reliability is assessed by the mean and variance of dimensional deviation δy_i on the product due to the randomness of θ_1, θ_2, d_{1h} and d_{2h}. The mean and variance of dimensional deviation δy_i can be calculated based on equation (7). Since $E(\sin\theta_1) = E(\cos\theta_1) = E(\sin\theta_2) = 0$, there can be obtained

$$E(\delta y_i(k) \mid \delta d_j, j = 1, 2) = 0 \tag{8}$$

It is reasonable to assume that θ_1 and θ_2 are independent. Thus, $\mathrm{Cov}[\sin\theta_2, \sin\theta_1]$ and $\mathrm{Cov}[\sin\theta_2, \cos\theta_1]$ are equal to zero. It can also be obtained that $\mathrm{Var}(\sin\theta_2) = 1$, $\mathrm{Var}(\sin\theta_1) = \mathrm{Var}(\cos\theta_1) = 0.5$ and $\mathrm{Cov}[\sin\theta_1, \cos\theta_1] = 0$. Thus, the part dimensional variance at the KPC points can be calculated by

$$\begin{aligned}\mathrm{Var}(\delta y_i \mid \delta d_j, j = 1, 2) &= 0.25c_{i1}^2(\delta d_2 + d_{2t})^2 + 0.25c_{i1}^2\sigma_{2h}^2 \\ &\quad + 0.125(c_{i2}^2 + c_{i3}^2)(\delta d_1 + d_{1t})^2 \\ &\quad + 0.125(c_{i2}^2 + c_{i3}^2)\sigma_{1h}^2\end{aligned} \tag{9}$$

From equation (8) it can be seen that the mean shift of the product dimensional deviation due to pin wear is equal to zero. Therefore, the assessment of product quality can be simply evaluated by the variance of the KPC points. Based on this, the decision rule used to judge a fixture system failure due to pin wear can be simplified to: if the condition $\bigcap_{i=1}^{6}(\mathrm{Var}(\delta y_i \mid \delta\mathbf{X}) < \eta_i)$ is not satisfied, then the current fixture system status is concluded as failed.

Remark. The assumption of uniform distribution for θ_1 and θ_2 is used to simplify the complexity of the contact position between the part-hole and the pin. It is a good approximation if the assembly process is under a stressless condition. However, the assumption of the uniform distribution is not a constraint for fixture system reliability analysis. The necessary condition here is to have an available way of finding the mean and variance of $\sin\theta_i$ and $\cos\theta_i$.

3. MODELING OF MANUFACTURING SYSTEM COMPONENT WEAR

3.1. Mechanism of fixture wear

In a fixture system, pin wear is the result of friction from the sliding movement between the pin and the hole of a part. Therefore, the pin wear is aggregated by all wear occurred during each operation. Archard [14] proposed a wear model based on the physical principle of the contacting and rubbing wear,

$$V = \frac{KFL}{3p} \tag{10}$$

where V represents the worn volume; F corresponds to the load force; L is the sliding distance; p is the penetration hardness of the softer material; and K is a random wear factor. Based on the sliding wear theory, the wear factor K closely depends on the contacting surface conditions [15,16]. This is because that the wear takes place at the contact points between aspirates on the sliding surface. Wallbridge and Dawson [15] proved that K generally follows a lognormal distribution, that is, $K \sim \mathrm{lognor}(\mu_K, \sigma_K^2)$, or equivalently, $\log(K) \sim N(\mu_K, \sigma_K^2)$. The density function $f(K)$ is

$$f(K) = \frac{1}{\sigma_K\sqrt{2\pi}}\,\mathrm{e}^{-[\ln(K)-\mu_K]^2/2\sigma_K^2} \tag{11}$$

where μ_K corresponds to the geometric mean of the distribution, and $K > 0$, to consider only the positive wear. Because of the random behavior of the wear factor K, the component wear V given in equation (10) is also considered as an independently lognormal distributed random variable with $V \sim \mathrm{lognor}(\mu_V, \sigma_V^2)$. The next subsection will discuss how to use a stochastic process model to describe the increasingly aggregated component wear δX.

3.2. Stochastic process modeling of fixture system component wear

The fixture-system component aggregated wear is increased with the number of operations, which can be described by a stochastic process model with an independently lognormal distributed increment:

$$\delta X(k) = \delta X(k-1) + \Delta(k) \tag{12}$$

where $\delta X(k)$ is the aggregated component wear after conducting k operations, and $\Delta(k)$ is the wear rate, i.e. a wear increment due to operation k. In the case of pin wear, $\delta X(k)$ is the pin diameter reduction (δd_1 or δd_2). Based on the result in Section 3.1, $\Delta(k)$, $k = 1, 2, \ldots$, are independently lognormal distributed random variables with $\Delta(k) \sim \mathrm{lognor}(\mu_\Delta(k), \sigma_\Delta^2(k))$. The mean and variance of the wear-rate equal to $E[\Delta(k)] = \exp(\mu_\Delta(k) + 0.5\sigma_\Delta^2(k))$ and $\mathrm{Var}[\Delta(k)] = \exp[2\mu_\Delta(k) + \sigma_\Delta^2(k)][\exp(\sigma_\Delta^2(k)) - 1]$. The aggregated wear after conducting k operations is

$$\delta X(k) = \sum_{j=1}^{k} \Delta(j) \tag{13}$$

When k is very large, based on the central limit theorem, $\delta X(k)$ approximately follows a normal distribution as

$$\delta X(k) \sim N\left[\sum_{j=1}^{k} E[\Delta(j)], \sum_{j=1}^{k} \mathrm{Var}[\Delta(j)]\right] \quad (14)$$

3.3. *Two stages of pin wear rate*

For a mechanical component, it is known that the initial wear rate is generally much higher than that afterwards [4]. Similarly, a higher wear rate $\Delta(k)$ can be observed on pins during the initial assembly operations because the clearance between a pin and a part-hole is very small at that time, which results in more surface contacting or interference forces. With an increasing number of operations, the clearance between the pin and the part-hole is increased as an effect of the aggregated pin wear due to the reduction of pin diameter. Thus, the initial higher wear rate will quickly reduce and tend to become stable under normal production conditions. In order to represent this early fast wear, and the random nature of pin wear rate, it is assumed that the mean of wear rate $E[\Delta(k)]$ is changed with operation k by the function

$$E[\Delta(k)] = \mu_0 + \mu_1 \mathrm{e}^{-\beta k} \quad (15)$$

where β describes how fast the mean of wear rate decreases at the early stage, $\mu_0 + \mu_1$ is the initial mean of the wear rate, and μ_0 is the stabilized mean of the wear rate.

4. QUALITY AND RELIABILITY INFORMATION INTEGRATION

The reliability of a fixture system closely depends on the condition of its components. In the fixture-design stage, a fixture-system failure is usually considered as its catastrophic component malfunctions, such as a broken or loosening pin due to a connection-bolt loosening etc. In this paper, in addition to considering this type of catastrophic component failure, a new type of fixture-system failure is first defined where product quality is used as a criterion to determine whether a fixture system is failing or not. Thus, excessive pin wear may also be regarded as a fixture-system failure if it affects product quality severely. Based on the logic relationship shown in Figure 1, a fault-tree model, as shown in Figure 4, is developed to describe the effect of these two types of component failures on fixture-system reliability.

In the first layer, an OR gate is used to describe the occurrence of a fixture-system failure due to either the catastrophic fixture-system component failure or the nonconforming product quality caused by the locating error of worn pins. Thus, the fixture system reliability R at time k can be calculated by

$$\begin{aligned} R(k) &= \text{Prob}\ (T > k) \\ &= \text{Prob (conforming product quality AND} \\ &\qquad \text{no catastrophic component failure} \\ &\qquad \text{during } [0, k]) \\ &= \text{Prob (conforming product quality during} \\ &\qquad [0, k]) \\ &\quad \times \text{Prob (no catastrophic component failure} \\ &\qquad \text{during } [0, k] \mid \text{conforming product} \\ &\qquad \text{quality)} \end{aligned} \quad (16)$$

where T as a random variable represents the fixture-system lifetime, and k is the fixture-system running time or the number of operations. It is reasonable to assume that pin catastrophic failure is independent of pin wear because the reduction of pin diameter has little impact on pin-failure rate. Moreover, the conformity of product quality is affected by the tool wear. Based on this assumption, equation (16) can be simplified as:

$$\begin{aligned} R &= \text{Prob (conforming product quality during } [0, k]) \\ &\quad \times \text{Prob(no catastrophic fixture failure during} \\ &\qquad [0, k]) \\ &= R_q(k) \times R_f(k) \end{aligned} \quad (17)$$

where $R_q(k)$ and $R_f(k)$ are the reliability in terms of product quality and catastrophic pin failure respectively. The detailed discussion for these two reliability calculations will be given in the following two subsections.

4.1. *Reliability modeling of component catastrophic failures*

In the second layer of the fault tree in Figure 4, the dependency of system catastrophic failure on each pin malfunction is described by an OR gate. Based on the OR gate function, the reliability R_f can be described by

$$R_f(k) = \prod_{i=1}^{n} R_i^f(k) \quad (18)$$

where R_i^f represents the reliability of Pin P_i in terms of whether it experiences a catastrophic failure. It is assumed that the probabilities of a pin broken or a pin loosened are independent of each other. n is the

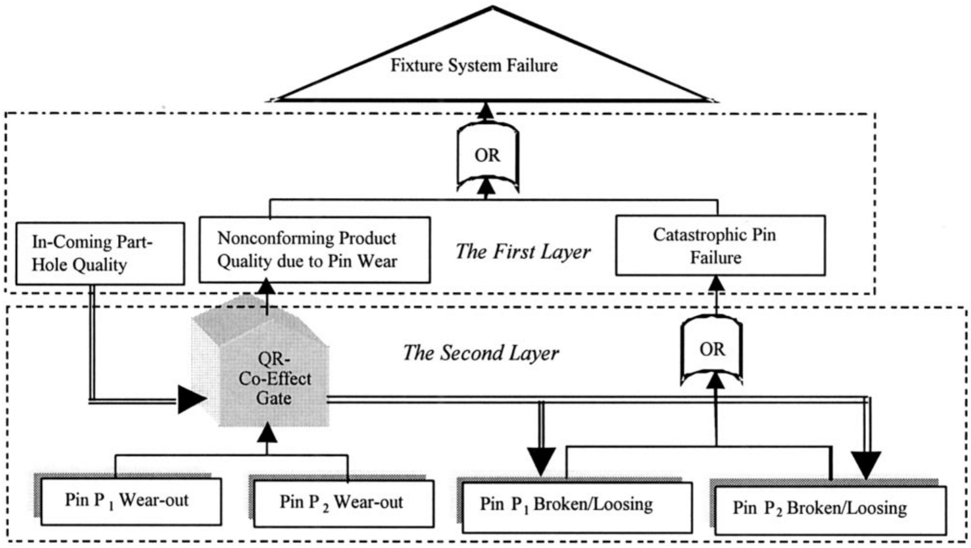

Figure 4. Fault tree analysis of a fixture system

number of pin components used in a fixture system; $n = 2$ in this paper.

For each pin component, its lifetime T_i in term of catastrophic failure is a random variable, which is assumed to follow an exponential distribution. Thus, fixture-system component reliability is the probability of a pin surviving operation k, that is,

$$R_i^f(t) = \text{Prob}(T_i > k) = e^{-\lambda_i k} \tag{19}$$

where λ_i represents the hazard rate of component i, and $1/\lambda_i$ equals the mean time to failure.

Remark. A brief justification of the exponential distribution of the pin component catastrophic failure time is as follows. We assume that the operational condition and the incoming part quality do not change significantly over production time. Moreover, the pin degradation will not change the pin diameter much (often less than 1%). Thus, pin degradation has little impact on the strength of pins. Under these assumptions, the pin catastrophic failure will have a constant hazard rate over production time, and its failure time is hence exponentially distributed.

In fixture systems, the value λ_i of each pin is affected by not only the pin quality itself (pin material and its coating, etc.), but also the hole quality of an incoming part. The reason for this is that if the hole sizes of incoming parts vary significantly, the magnitude and the direction of the contacting force between a pin and a hole will change more frequently. Thus, broken or loosened pins are more likely to occur due to component fatigue and bolt loosening. In this paper, the effect of incoming-hole diameter variations (σ_{ih}^2) on the component hazard rate (λ_i) is approximately described by the exponential function

$$\lambda_i = \lambda_{0i}\, e^{\alpha_i \sigma_{ih}} \tag{20}$$

where λ_{0i} is the original component hazard rate without considering the effect of incoming part-hole quality. α_i is a calibration factor, and σ_{ih} represents the standard deviation of hole diameters of incoming parts located by Pin P_i.

This new concept of how incoming part quality affects fixture-system reliability is shown by the double line in Figure 4. By borrowing the terminology of the functional gates (e.g. OR, AND) used in the traditional fault-tree model, a newly defined QR-Co-Effect gate is used to represent the function of this QR-Co-Effect for integrated reliability analysis, i.e. the interdependency between part quality and fixture-component reliability. The next subsection will further discuss the second function of the QR-Co-Effect gate.

4.2. *Reliability modeling of product quality*

4.2.1. $R_q(k)$ model. In this subsection, the fixture-system failure is considered to be indicated by a poor quality of products produced by a degraded fixture system. For this type of fixture-system failure, the dependency of fixture-system failure on component degradation is more complex

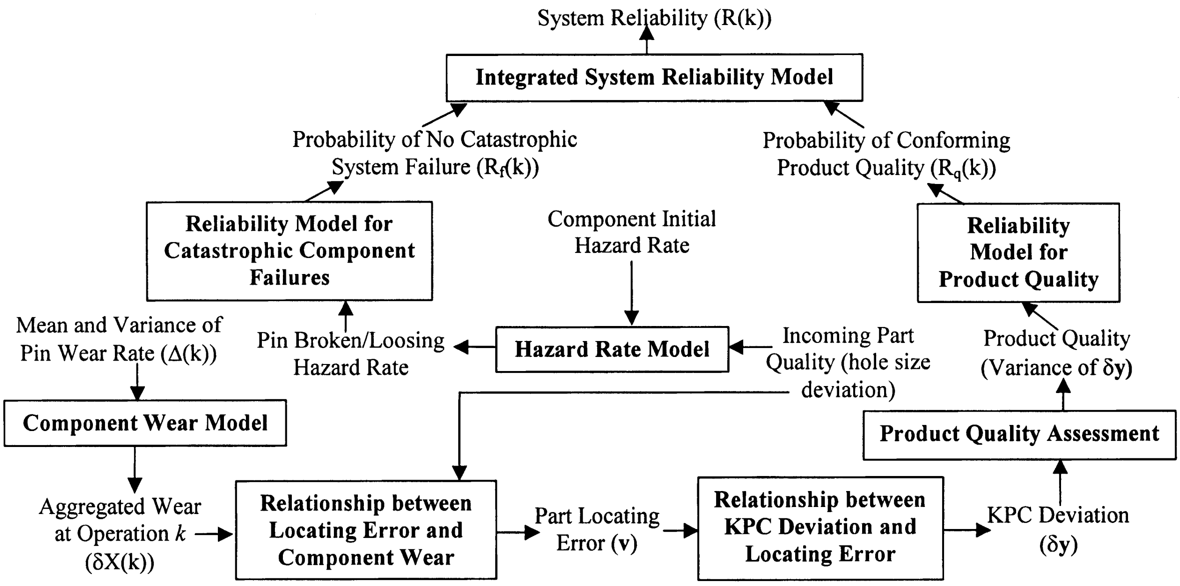

Figure 5. Summary of the QR-Co-Effect model for a fixture system

than that on catastrophic component failures. The reason is that the effect of fixture-system component wear on the product quality will depend on not only the amount of pin wear itself, but also on the design layout of fixtures and KPC points. In this situation, the traditionally-defined functional gates in the fault tree model, such as OR, AND, k-out-of-n voting gates etc. cannot be used directly. Therefore, the function of the QR-Co-Effect gate is to integrate the pin-component wear with product quality for fixture-system reliability analysis. As shown in Figure 4, the interaction of pin wear is connected with the QR-Co-Effect gate, and the output of QR-Co-Effect gate represents product quality. Thus, the second function of the QR-Co-Effect gate describes the effect of pin wear on product quality. Based on the quality assessment criteria, the reliability of $R_q(k)$ in the consideration of the effect of pin wear on fixture-system reliability can be calculated by

$$R_q(k) = \text{Prob}\left(\bigcap_{i=1}^{m} \text{Var}(\delta y_i(k) \mid \delta\mathbf{X}(k)) \le \eta_i\right) \quad (21)$$

where m is the number of KPC measurements. In this paper, $m = 6$.

4.2.2. Analytical solution for $R_q(k)$. Due to the geometrical relationship among KPC points, $\text{Prob}(\text{Var}(\delta y_i(k) \mid \delta\mathbf{X}(k)) \le \eta_i)$ is not independent of $\text{Prob}(\text{Var}(\delta y_j(k) \mid \delta\mathbf{X}(k)) \le \eta_j)$. Therefore, the quality constraints at all KPC measurements should be considered simultaneously. Thus, equation (21) is rewritten as

$$\begin{aligned} R_q(k) &= \text{Prob}\left\{\bigcap_{i=1}^{m} \text{Var}(\delta y_i(k) \mid \delta\mathbf{X}(k)) \le \eta_i\right\} \\ &= \int_{\bigcap_{i=1}^{m} \text{Var}(\delta y_i(k)|\delta\mathbf{X}(k))\le\eta_i} \mathrm{d}F^{\delta\mathbf{X}(k)}(\delta\mathbf{x}(k)) \end{aligned} \quad (22)$$

where $\delta\mathbf{X}(k) = [\delta X_1(k)\delta X_2(k)\ldots\delta X_n(k)]^{\mathrm{T}}$ representing the random vector of pin wear. In this paper, $\delta\mathbf{X}(k) = [\delta d_1 \delta d_2]^{\mathrm{T}}$. $F^{\delta\mathbf{X}(k)}(\delta\mathbf{x}(k))$ is the joint cdf of $\delta\mathbf{X}(k)$. It is reasonable to assume that all pin wears are independent of each other. Based on the stochastic process model of pin wear given in equation (14), equation (22) can be further expressed by

$$\begin{aligned} R_q(k) = \int_{\delta\mathbf{x}\in Q} \prod_{i=1}^{n} &\left(2\pi \sum_{j=1}^{k} \text{Var}[\Delta_i(j)]\right)^{-1/2} \\ &\times \exp\left[\left(-\left(\delta x_i - \sum_{j=1}^{k} E[\Delta_i(j)]\right)^2\right)\right. \\ &\times \left.\left(2\sum_{j=1}^{k} \text{Var}[\Delta_j(j)]\right)^{-1}\right] \mathrm{d}\delta x_i \end{aligned} \quad (23)$$

where $\delta\mathbf{x} \in \mathbf{Q} \Leftrightarrow \bigcap_{i=1}^{m} \text{Var}(\delta y_i(k) \mid \delta\mathbf{X}(k) = \delta\mathbf{x}(k)) \le \eta_i$.

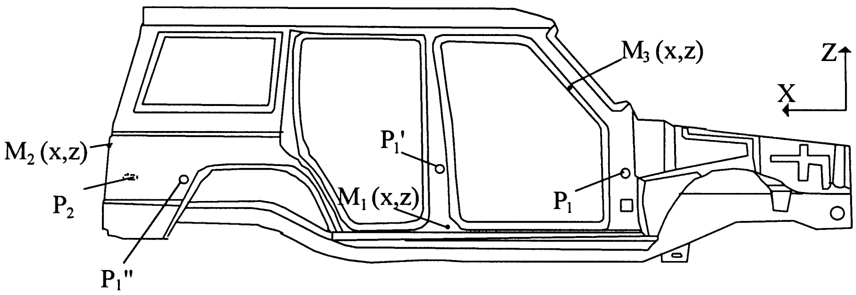

Figure 6. Layout of the fixtures and KPC points

Based on equation (9), it can be seen that the quality constraint of each KPC measurement represents an ellipse region in terms of δd_1 and δd_2. Therefore, all KPC quality constraints ($\delta \mathbf{x} \in \mathbf{Q}$) are just the intersection of m ellipses with the same ellipse center. Thus, the analytical solution of $R_q(k)$ can be obtained by the numerical solution of equation (23). Further evaluation of this analytical model will be given in Section 5.2 by comparing it with the Monte Carlo simulation result.

Table 1. Nominal x–z coordinates for pins and KPC points

	Nominal coordinates (mm)	
Point	x	z
P_1	2184	1489
P_1'	3166	1489
P_1''	4350	1489
P_2	4680	1489
M_1	3134	1200
M_2	4895	1608
M_3	2600	1921

4.3. An integrated reliability model for fixture system reliability analysis

Based on equations (17)–(20) and (23), the integrated system reliability model can be expressed by

$$R(k) = \left[\prod_{i=1}^{n} \exp\left(-\lambda_{0i} k\, e^{\alpha_i \sigma_{ih}}\right)\right] \times \int_{\delta\mathbf{x}\in Q} \prod_{i=1}^{n} \left(2\pi \sum_{j=1}^{k} \mathrm{Var}[\Delta_i(j)]\right)^{-1/2} \times \exp\left[\left(-\left(\delta x_i - \sum_{j=1}^{k} E[\Delta_i(j)]\right)^2\right) \times \left(2\sum_{j=1}^{k} \mathrm{Var}[\Delta_j(j)]\right)^{-1}\right] \mathrm{d}\delta x_i \quad (24)$$

Figure 5 provides a summary of the complex relationship between the manufacturing-system component reliability information and the product-quality information in a fixture system, where the rectangles represent all developed models, and the incoming and outgoing arrows indicate the inputs and outputs of the models, respectively.

5. EVALUATION OF THE INTEGRATED RELIABILITY MODEL OF A FIXTURE SYSTEM

5.1. An example of a fixture system in assembly processes

A real-world example of a side-frame assembly and its fixture systems, as shown in Figure 6, is used to illustrate the developed methodology. Table 1 gives all the dimensional layouts of the fixtures and the KPC points. In the simulation, product quality is indicated by three KPC points in the X–Z plane. The initial pin-hole clearance of Pins P_1 (or the alternative Pin P_1', Pin P_1'') and P_2 are $d_{1t} = d_{2t} = 0.1$ mm, and the design requirements of standard deviations of all KPC points (M_1, M_2 and M_3) in both X and Z directions are 0.1 mm, i.e. $\eta_i = 0.01$ mm^2 ($i = 1, \ldots, 6$). The reliability-related information and the parameters regarding pin component failure and wear models are summarized in Table 2.

5.2. Evaluation of the integrated reliability model using Monte Carlo simulation

Monte Carlo simulation is used to simulate the pin wear degradation process and its effect on the

Table 2. Summary of component failure models and the wear-out model

Failure definition	Model description	Reliability model & analysis	Parameters
Pin Broken or Loosing	Reliability model of pin failures	$R_i^f(k) = e^{-\lambda_i k}$; $\lambda_i = \lambda_{0i} e^{\alpha_i \sigma_{ih}}$	$\lambda_0 = 10^{-6}$; $\alpha_i = 50\ \text{mm}^{-1}$ $\sigma_{ih} = 10^{-2}$ mm; $i = 1, 2$.
Pin WearOut	Stochastic process model of pin wears	$\delta d(k) = \delta d(k-1) + \Delta(k)$ $\Delta(k) \sim \text{lognor}(\mu_\Delta(k), \sigma_\Delta^2(k))$ $E[\Delta(k)] = \mu_0 + \mu_1 e^{-\beta k}$	$\text{Var}(\Delta(k)) = 1.6 \times 10^{-9}\ \text{mm}^2$ $\beta = 10^{-3}$; $\mu_0 = 3 \times 10^{-6}$ mm; $\mu_1 = 8 \times 10^{-6}$ mm
	Pin wear effect on part locating errors	$\delta P_1(z) = 0.5(\delta d_1 + d_{1t} + d_{1h}) \sin\theta_1$ $\delta P_1(x) = 0.5(\delta d_1 + d_{1t} + d_{1h}) \cos\theta_1$ $\delta P_2(z) = 0.5(\delta d_2 + d_{2t} + d_{2h}) \sin\theta_2$	$d_{1t} = d_{2t} = 0.1$ mm $\theta_1 \sim U(0, 2\pi)$ $\theta_2 \sim (-\pi/2, \pi/2)$
	KPC deviation	$\delta \mathbf{y} = \mathbf{C}\mathbf{v}$	$\delta \mathbf{y} \equiv [\delta M_1(x)\delta M_1(z)\delta M_2(x)$ $\delta M_2(z)\delta M_3(x)\delta M_3(z)]^{\mathrm{T}}$ $\mathbf{v} = [\delta P_2(z)\delta P_1(x)\delta P_1(z)]^{\mathrm{T}}$
System Failures	System reliability	$R(k) = R_q(k) \cdot R_f(k)$	
	Component failures reliability	$R_f(k) = \prod_{i=1}^{2} R_i^f(k) = \prod_{i=1}^{2} \exp(-\lambda_{0i} k\, e^{\alpha_i \sigma_{ih}})$	
	Reliability in terms of product quality	Analytical reliability model: $R_q(k) = \int_{\delta \mathbf{x} \in Q} \prod_{i=1}^{n} \left(2\pi \sum_{j=1}^{k} \text{Var}[\Delta_i(j)]\right)^{-1/2} \times \exp\left[\left(-\left(\delta x_i - \sum_{j=1}^{k} E[\Delta_i(j)]\right)^2\right) \times \left(2 \sum_{j=1}^{k} \text{Var}[\Delta_j(j)]\right)^{-1}\right] \mathrm{d}\delta x_i$	$\eta_i = 0.01\ \text{mm}^2$ $i = 1, \ldots, 6$
		Monte Carlo simulation: $\hat{R}_q(k) = 1 - \dfrac{\left(\sum_{j=1}^{k} N_{\text{bad}}(j)\right)}{N_s}$	$N_s = 500$ (in simulation)

product-quality induced fixture-system failures. The simulation flowchart is shown in Figure 7. In the simulation, each fixture system is tested until it either fails or achieves a maximum number of operations ($K_{\max}$). In this paper, the maximum run of the system testing is set as $K_{\max} = 30\,000$ and 500 test systems are used, i.e. $N_s = 500$. The reliability $\hat{R}_q(k)$ is estimated from the simulation result as follows:

$$\hat{R}_q(k) = 1 - \frac{\sum_{j=1}^{k} N_{\text{bad}}(j)}{N_s} \tag{25}$$

where $N_{\text{bad}}(j)$ is the number of failed fixture systems at operation j, which are induced by nonconforming product quality due to excessive component wear. N_s is the total number of repetitions/systems used in the simulation.

The comparison of the Monte Carlo simulation with the integrated manufacturing-system reliability model of equation (24) is shown in Figure 8, where the horizontal axis indicates the number of system operations and the vertical axis is the fixture system reliabilities. From Figure 8, it can be seen that the numerical solution of the integrated fixture-system reliability model given in equation (24) is very consistent with that of the Monte Carlo simulation, which validates the effectiveness of the analytical model given in equation (24). In the discussion below, the integrated manufacturing system reliability model,

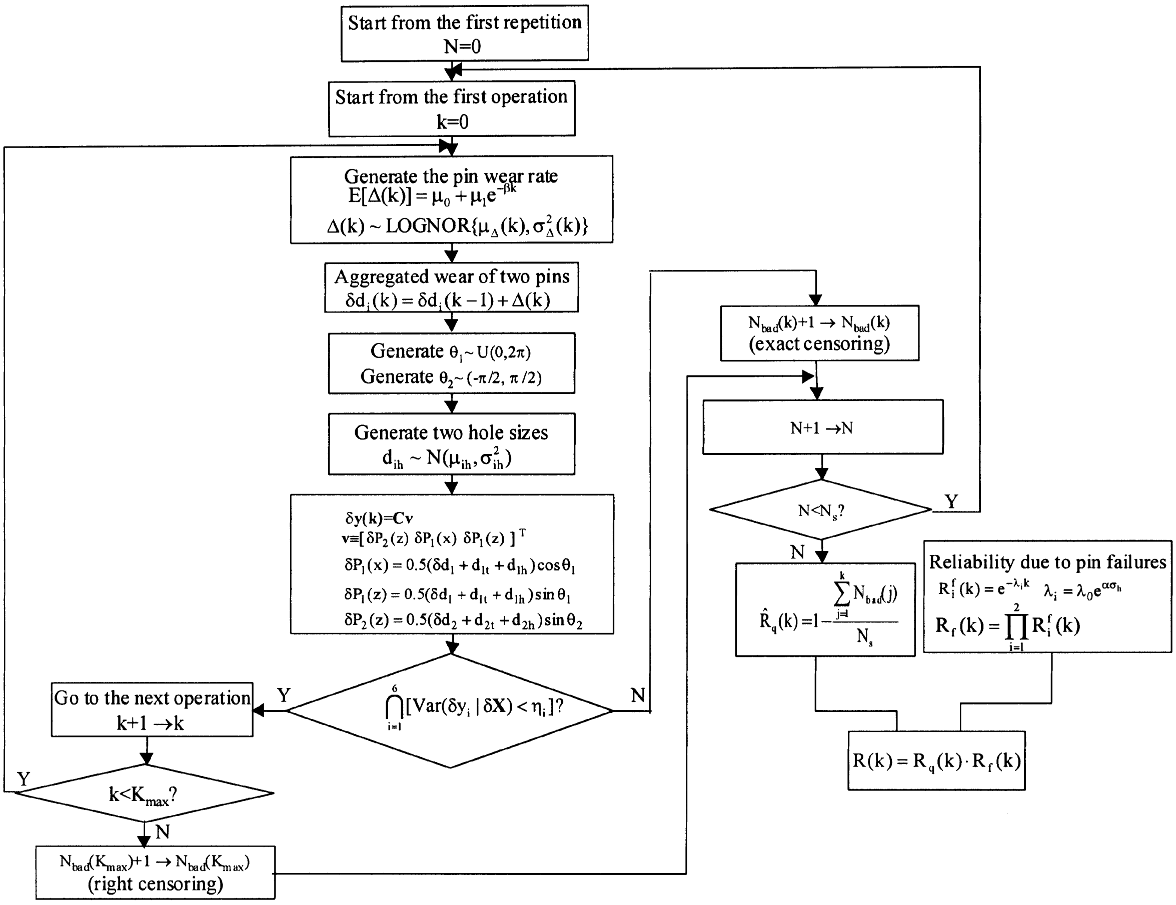

Figure 7. Monte Carlo simulation flowchart

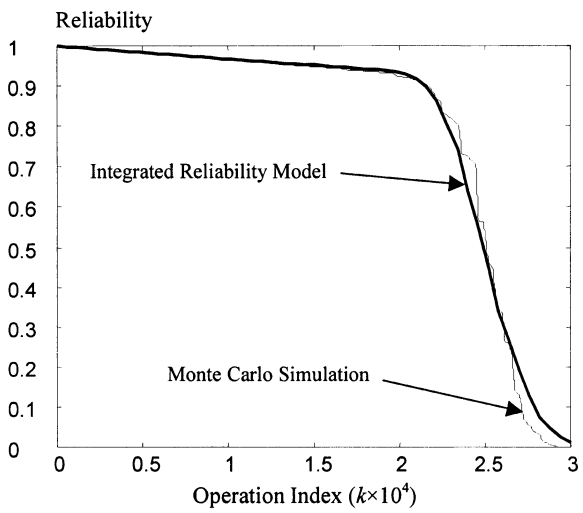

Figure 8. Comparison of the integrated reliability model with the simulation result

which has much less computation than the Monte Carlo simulation method, will be further used for fixture-system reliability evaluation.

5.3. *Comparison with other conventional system-reliability analysis methods*

5.3.1. Fixture system-reliability analysis with and without considering the QR-Co-Effect. The comparison of the proposed integrated manufacturing-system reliability model with other conventional system-reliability analysis methods is conducted based on the following three different definitions of system failures:

(a) only considering the catastrophic component failure (R_f with $\sigma_h = 0$);
(b) considering both the catastrophic component failure and product quality deterioration due to component wear ($R_f \times R_q$ with $\sigma_h = 0$), but

without considering the impact of incoming part quality; and

(c) considering the catastrophic component failure and the QR-Co-effect ($R_f \times R_q$ with $\sigma_h \neq 0$), i.e. product-quality deterioration due to the component wear and the impact of the incoming part quality on component failure.

Figure 9 gives the comparison of the different analysis results, where the horizontal axis is the number of system operations and the vertical axis is the system reliability. From this comparison, it can be found that the system reliabilities under definitions (a) and (b) are always larger than the system reliability under definition (c). As a result, the fixture-system reliability is reduced after considering the QR-Co-Effect. If a scheduled maintenance policy is planned based on the fixture-system reliabiliity using definitions (a) or (b), the fixture system may have lots of unscheduled down time since the interdependency between part quality and reliability is not considered.

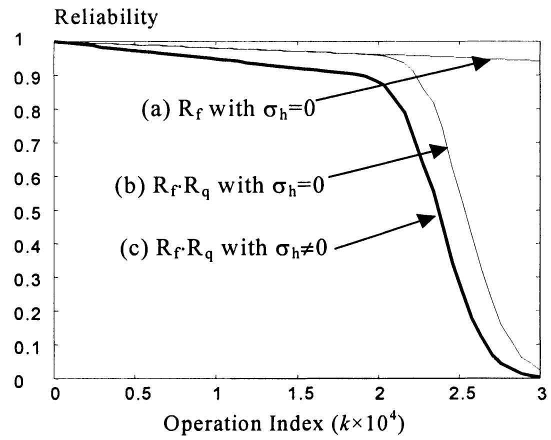

Figure 9. Fixture-system reliability analysis with and without considering the QR-Co-effect

5.3.2. Comparison of the KPC thresholds and the tool-wear thresholds. In conventional system-reliability analysis, thresholds are assigned to each pin as the maximum acceptable tool wear. A simple, commonly used way is to assign the same threshold to all pins, that is, $th_1 = th_2 = \ldots = th_n = th$, where th_i denotes the threshold for the ith pin-diameter reduction due to pin wear. Based on this threshold rule, the integral region of $R_q(k)$ in equation (22) is simplified, and the analytical solution of the fixture-system reliability can be calculated by multiplying n single-variable integrals, that is,

$$R_q(k) = \prod_{i=1}^{n} \int_0^{th} \left(2\pi \sum_{j=1}^{k} \mathrm{Var}[\Delta_i(j)]\right)^{-1/2} \times \exp\left[-\left(\delta x_i(k) - \sum_{j=1}^{k} E[\Delta_i(j)]\right)^2 \times \left(2\sum_{j=1}^{k} \mathrm{Var}[\Delta_j(j)]\right)^{-1}\right] \mathrm{d}\delta x_i(k) \quad (26)$$

The comparison of fixture-system reliability analysis using the thresholds assigned to tool wear and the thresholds assigned to each KPC measurement can be seen from the following two points of view.

(1) Impact of quality constraints on the fixture-system reliability. Comparing equation (22) with equation (26), it can be seen that different thresholds lead to different integral regions in the system-reliability calculation. In the integrated system-reliability model given in equation (22), the thresholds are assigned for each KPC measurement. Based on equation (9), it can be seen that the quality constraint at each KPC measurement is an ellipse function in terms of the pin wear δd_1 and δd_2. Moreover, all ellipses of the quality constraints at all KPC measurements share the same ellipse center. To simplify the illustration, two KPC points are used to represent the product quality. In this case, the integral region using these two quality constraints is the intersection region of two ellipses as shown in the shadowed region in Figure 10(a).

If the tool-wear thresholds are used for fixture-system reliability analysis, a squared integral region will be used in equation (26) for the equalized thresholds. When the tool-wear thresholds are selected within the quality constraints, as shown in Figure 10(b), the squared integral region is smaller than the intersection area of these two ellipses, leading to underestimated fixture-system reliability. In this case, either excessively reliable fixture-system components are required or an earlier preventive maintenance schedule has to be planned to achieve the desired fixture-system reliability. Similarly, if the tool-wear thresholds are selected beyond the quality constraints, as shown in Figure 10(c), the squared integral region is larger than the intersection area of two ellipses, leading to overestimated fixture-system reliability. As a result, this optimistic fixture-system reliability estimation will lead to the selection of insufficiently reliable components. In such a case, an excess of unscheduled downtime will occur during production due to nonconforming product quality.

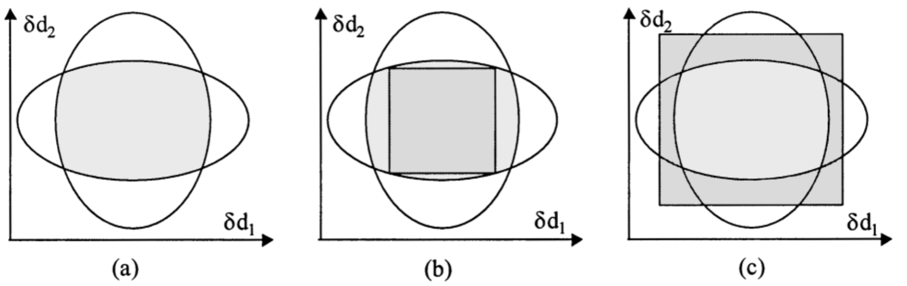

Figure 10. Comparison of KPC constraints and tool wear contraints: (a) quality constraint on KPC; (b) over-constraint on pin wear; (c) under-constraint on pin wear

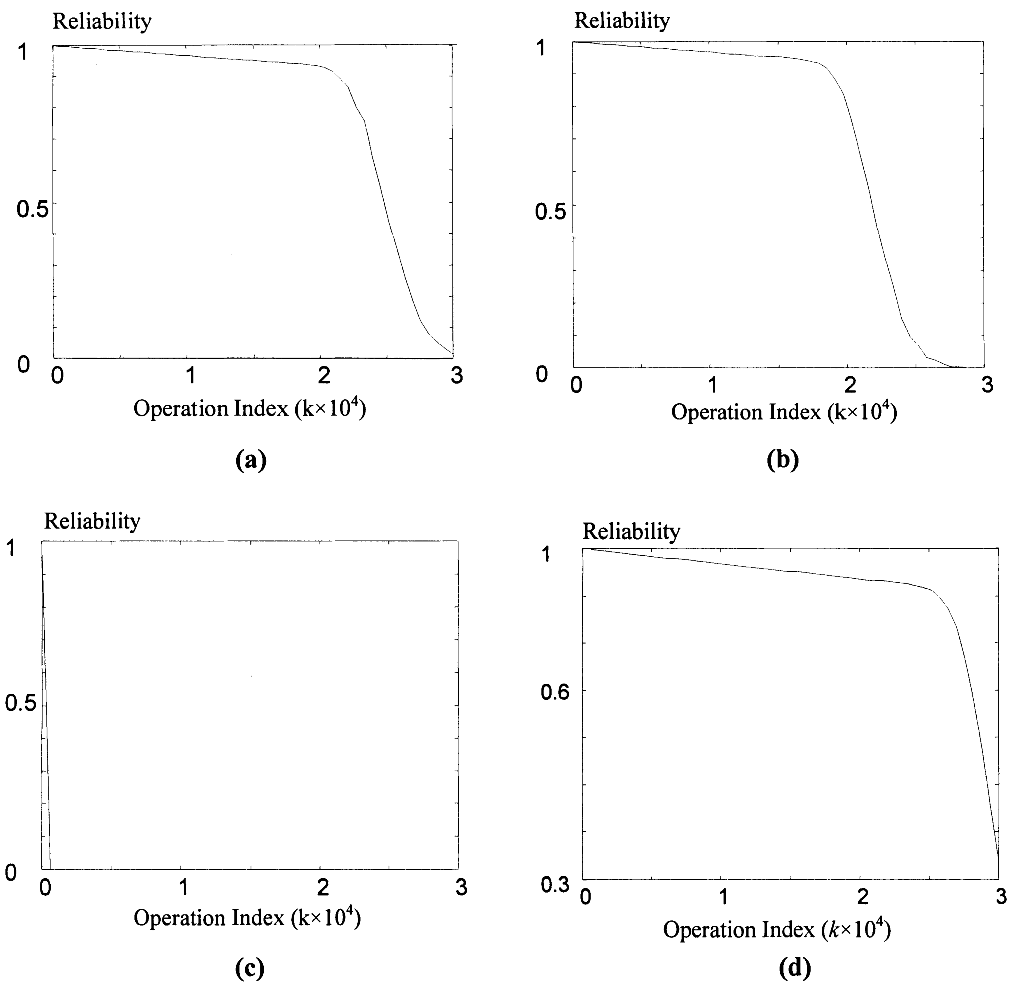

Figure 11. Impact of fixture layouts on fixture-system reliability using different threshold methods: (a) method 1 using P_1; (b) method 1 using P_1'; (c) method 1 using P_1''; (d) method 2 using P_1, P_1' and P_1''

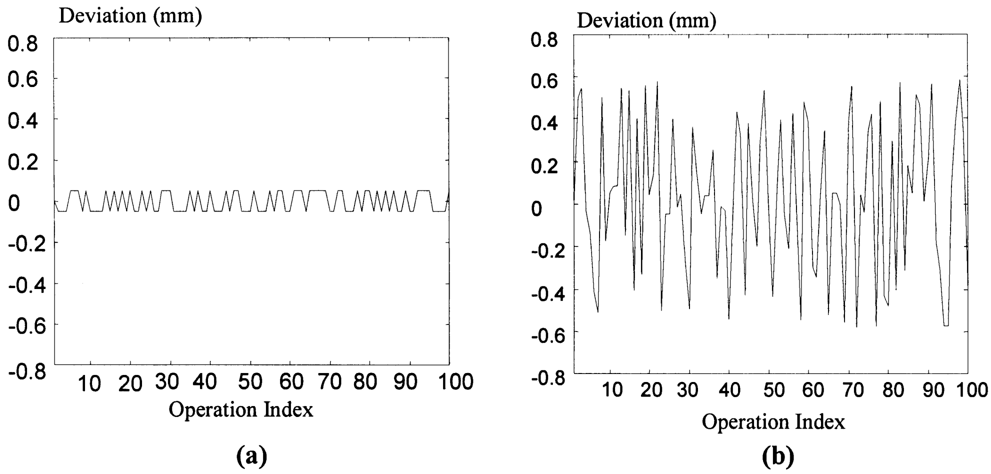

Figure 12. Comparison of part-locating error with KPC deviation: (a) early stage deviation at pin P_1''; (b) early stage deviation at M_3

Therefore, fixture-system reliability analysis using the thresholds assigned to KPC points is more meaningful than that using the thresholds assigned to tool wear.

(2) Impact of the tooling layout. When the KPC threshold is used for integrated fixture-system reliability analysis (method 1), the same magnitudes of tool wear with different tooling layouts will have different impacts on product quality according to equation (9). For example, if three different positions of the four-way pin P_1, P_1', and P_1'' are used, as shown in Figure 6, different fixture-system reliability results will be obtained, which are shown in Figures 11(a)–(c) for P_1, P_1', and P_1'' respectively. From these analysis results, it can be seen that Pin P_1 gives the highest fixture-system reliability (Figure 11(a)) while Pin P_1'' leads to the worst fixture-system reliability (Figure 11(c)). The reliability difference between Pin P_1 and Pin P_1'' is due to the sensitivity of the part quality to the locating error: case P_1'' is much more sensitive than case P_1. Thus, a small part-locating error due to the Pin P_1'' wear (Figure 12(a)) will lead to a significant error on the part quality (Figure 12(b)). As a result, the reliability for the case P_1'' quickly drops to zero (Figure 11(c)), even at an early stage of production, due to the nonconforming products produced in the fixture system.

The analysis results shown in Figures 11(a)–(c) illustrate that Pin P_1 is the best design in terms of fixture-system reliability, which is consistent with engineering knowledge of fixture systems. Therefore, the proposed integrated reliability-analysis methodology using KPC thresholds can be used as an effective method of tool-layout evaluation in the fixture-system design stage.

If the tool wear thresholds are used in the fixture-system reliability analysis (method 2), the design information of tooling layouts is not considered. Therefore, in contrast with method 1, the same analysis result of the fixture-system reliability is obtained no matter which tool layout is used. Figure 11(d) shows the analysis result for all three four-way pin positions, where $th_i = 0.1$ mm is assigned as the threshold of each pin-diameter reduction in equation (26). In this case, the worst performance of Pin P_1'' could not be revealed based on method 2.

6. APPLICATIONS OF THE INTEGRATED MANUFACTURING SYSTEM RELIABILITY MODEL

As discussed in Section 5, the proposed integrated manufacturing-system reliability analysis can be used as an effective tool in the evaluation of different fixture layouts in the system-design stage. In addition, it can also be used for the selection of an economical component wear rate or catastrophic failure rate. The selection of these tool materials and coating will directly relate to the manufacturing cost. Detailed discussion will be given in the following subsections.

6.1. Evaluation of the effect of component wear rate on fixture system reliability

The component wear rate is normally determined by the component material and the coating process. The effect of component wear on system reliability should be investigated in the design stage. In this study, three cases with different sets of parameters

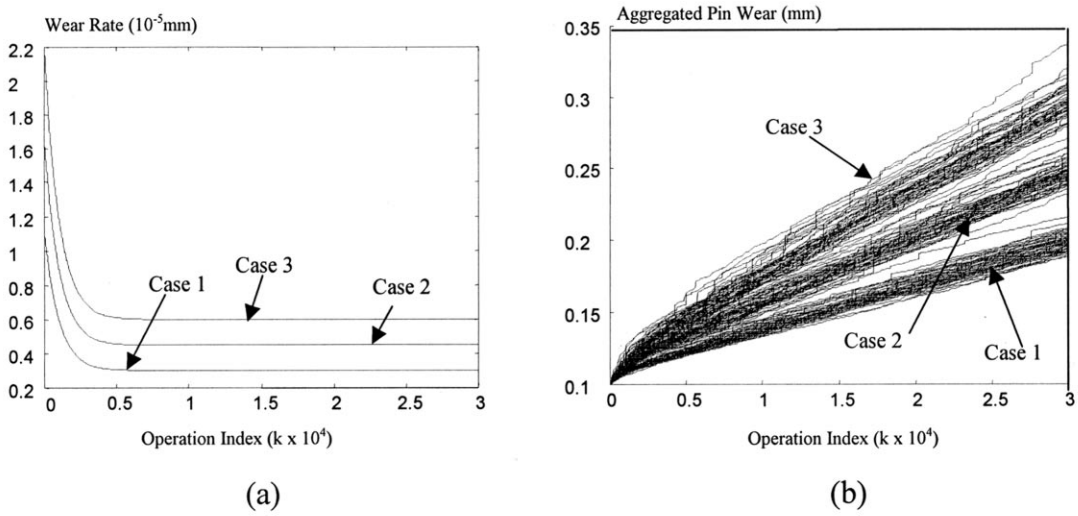

Figure 13. Comparison of the aggregated wears under different wear parameters: (a) different pin-wear parameters; (b) aggregated pin wear

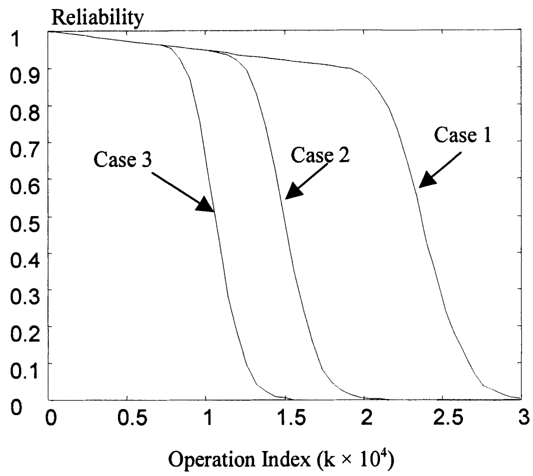

Figure 14. The effect of component wear rates on fixture-system reliability

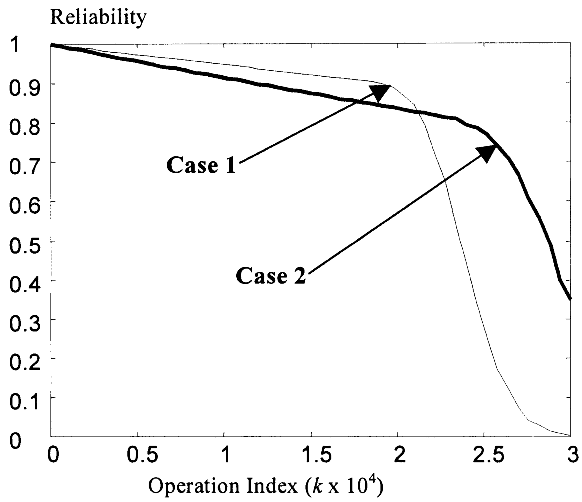

Figure 15. The joint effects of part quality and fixture-system component catastrophic failures

used in $E[\Delta(k)] = \mu_0 + \mu_1 e^{-\beta k}$ are selected for studying the effect of the wear rate on fixture-system reliability (Case 1, $\mu_0 = 3 \times 10^{-6}$ mm/operation, $\mu_1 = 8 \times 10^{-6}$ mm/operation; Case 2, $\mu_0 = 4.5 \times 10^{-6}$ mm/operation, $\mu_1 = 12 \times 10^{-6}$ mm/operation; and Case 3, $\mu_0 = 6 \times 10^{-6}$ mm/operation, $\mu_1 = 16 \times 10^{-6}$ mm/operation). Figures 13(a) and (b) give a comparison of these values of $E[\Delta(k)]$ and the corresponding aggregated component wear, where all other parameters are as listed in Table 2. The evaluated fixture-system reliability under these three wear rates is compared as shown in Figure 14. From this figure, it can be seen that after 1.2×10^4 operation, the fixture-system reliability in Case 2 and Case 3 with a faster wear rate is much lower than that in Case 1 with a slower wear rate. Thus, an optimal selection of component wear rate is very critical in fixture-system reliability design.

6.2. *Evaluation of joint effects of part quality and pin failures on fixture-system reliability*

In order to show the QR-Co-Effect of part-quality deterioration and component catastrophic failures on fixture-system reliability, two different cases with different wear rates and different failure rates are compared, that is, $\mu_{\Delta_1} > \mu_{\Delta_2}$ and $\lambda_1 < \lambda_2$ (Case 1, $\mu_0 = 3 \times 10^{-6}$ mm/operation, $\mu_1 = 8 \times 10^{-6}$ mm/operation used in μ_{Δ_1}, $\alpha = 50$ used in λ_1; Case 2, $\mu_0 = 2.5 \times 10^{-6}$ mm/operation, $\mu_1 = 6 \times 10^{-6}$ mm/operation used in μ_{Δ_2}, $\alpha = 75$ used in λ_2). The fixture system reliabilities under these two different sets of parameters are compared and shown in Figure 15. It can be seen that catastrophic failures play a major role in the fixture-system reliability at an early stage of production; thus Case 1 is more reliable than Case 2 before 21 000 operations because $\lambda_1 < \lambda_2$. However, as the fixture system degrades, tool degradation becomes more dominant. As a result, the reliability of Case 1 is much smaller than that of Case 2 after 22 000 operations because $\mu_{\Delta_1} > \mu_{\Delta_2}$.

7. CONCLUSIONS

In this paper, the interdependency between part quality and fixture-system reliability is investigated. As an example, fixture design and its reliability analysis were studied by integrating information of pin failure, pin wear and their interdependency with part quality. The proposed integrated manufacturing-system reliability model is validated by using Monte Carlo simulation results. The advantages and the effectiveness of the proposed methodology are illustrated by comparing them with other conventional reliability analysis methods. The comparison study indicates that fixture-system reliability will be biased if the QR-Co-Effect is not considered in the analysis. In addition, the fixture-system design change (e.g. tool-layout change), or its component-reliability change (different catastrophic-failure probability or wear rate) will affect fixture-system reliability significantly. Therefore, based on the proposed integrated fixture-system reliability model, a comprehensive analysis and system-level optimization could be conducted to achieve an optimized manufacturing system—in terms of both product quality and process productivity. The main objective of this paper is to provide an initial study on how to effectively integrate and utilize the rich information existing in manufacturing processes to improve manufacturing-system reliability analysis.

It should be pointed out that as an example of fixture-system reliability analysis used in the paper, the models used to represent the pin degradation and catastrophic failures are of a provisional nature, and lack comprehensive experimental calibration. Efforts will be devoted in future to improving the accuracy of the developed models. However, the model used to represent the relationship between pin wear and product quality has been validated and used in various applications in assembly process modeling [17] and assembly fixture diagnosis [13]. Furthermore, the concepts and the framework of the proposed integrated reliability analysis model are generic and applicable for many manufacturing processes.

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