ADAPTIVE PREDICTIVE CONTROL AND ITS APPLICATIONS IN MISSILE INTERCEPTION PROBLEM

JIAJUN SHI[1] TAO TANG[2] ZHIFANG ZHANG[1]
1: Research center ,Beijing Institute of Managemant ,U.S.T.C.P.O.BOX3353, Beijing 101408,P.R.C.
2: Institute of Automation Chinese Academy of Science P.O.BOX 2728,Beijing 100080,P.R.C.

ABSTRACT

In this paper a new adaptive prective control in a missile interception problem is studied . The control takes not only the instantaneous state estimates but also the associated confidence level into account while being adaptive .Thus,in addition to be adaptive, this control also shows caution property.

INTRODUTION

In a missile interception problem a flight guided control system is a nonlinear , time-varying and high dimension system. Its controller design is a difficult problem. One solution to the problem is changing it into a liner model. Then,we can design its controller by means of modern control theory. However, the method has many disadvantagues. To overcome the disadvantages predictive control concepts has been used. The advantage of this approach is the simplicity of the control law. However, it ignores the confidence level of the state estimates in the deriving the adaptive scheme. As we all know there are a lot of stochastic disturbances during a missile flight.The prediction system control is sensitive to the disturbances.

If the design of the adaptive system takes not only the instantaneous state estimate but also the associated confidence level into account, it would surely result in a better system. It is objective of this paper. The control strategy developed in this paper is called as Adaption Caution Predictive(ACP) control. In order to illustrate is compared with a certain-equivalence predictive control law through some simulation.

The structure of the paper is as follow. In section 2, the controlproblem is stated. In section 3, adaptive caution predictive control algorithm is derived in detail. The simulation results presented in section 4 demonstrat some features about the algorithm. At last, some conclusion are given in section 5.

PROBLEM STATEMENT

The controlled model of the vehicle interception problem can be presented as a discrete-time nonlinear stochastic system described by:

$$X(k+1)=f(X(k),k)+g(X(k),k)U(k)+W(k)$$
$$Y(k)=c(X(k),k)+V(k) \qquad k=0,1,2,... \qquad (1)$$

where $X(k)\in R^n$, $Y(k)\in R^m$, and $U(k)\in R^1$. The vectors [X(0) ,W(k),V(k), ...,] are assumed to be mutually in dependent Gaussian random variables with knowing statistcal laws:

$$X(0)\sim N[X(0),P_x(0)];\ W(k)\sim N[0,Q_w(k)];$$
$$V(k)\sim N[0,R_v];$$

with $P_x(0)>0$, $Q_w(k) > 0$,$R_y(k)$ U 0. The notation $v\sim N[a\ ,\ b]$is used to denote that the random vector, V is Gaussian with mean a and covariance b. Furthermore we assume that f(. ,k) , g(. , k) and c(., k) are differentable

To the missile interception problem we present an optimal prediction control index function as:

$$J(U(k),k)=\tfrac{1}{2}E[\ \sum_{i=1}^{N}[X_p(k+i)-X_p^*(k+i)]^T Q(k)[X_p(k+i)-X_p^*(k+i)]+ U^T(k)R(k)U(k)] \qquad (2)$$

where $Q(k)=Q(k-1)>0$,$R(k)>0$; $X_p(k+i)$ is the predictive value of state at time k+i;$X_p^*(k+i)$ is the expected value of state at time k+i; N is the horizon of state prediction, and is positive integer; E[.] denotes expectation

It is assumed that Q(k), R(k),$X_p^*(k)$ (k=0,1,2,..., N_f;$N_f$is the control termenal time) and N are given a priori. Here ,$X_p^*(k+i)$ is determined accordin g to the control purpose .The principles of selecting according to control purpose . The principles of selecting N can be found in the reference (SHI, 1989;ZHANG, 1988).

ADPPTIVE CAUTION PREDICTIVE CONTROL

Before we derive the ACP control algorithm,let us assume that the present time is indexed by k, conrol sequence U^{k-1} = [U(0),U(1),U(2),...U(k-1)], has been applied to the system and that the observation sequence Y^k =[Y(1), Y(2),...,Y(k)] has been obtained. The condition mean $\hat{X}(k)$ and covariance P(k) are assumed available from an estimator:

$$\hat{X}(k)=E[X(k)|\ Y^k,U^{k-1}] \qquad (3)$$

$$P(k)=\mathrm{Cov}[X(k)|\,Y^k,U^{k-1}] \qquad (4)$$

The computation of $\hat{X}(k)$, $P(k)$ can be done by Extended Kalman Filter(STENGLE 1986).

$$X(k)=\hat{X}(k)+\tilde{X}(k) \qquad (5)$$

From Eq.(1) and Eq.(5),we have

$$X(k+1)=f(\hat{X}(k)+\tilde{X}(k),k)+g(\hat{X}(k)+\tilde{X}(k),k)U(k)+W(k) \qquad (6)$$

In the neighourhood of $\hat{X}(k)$,we can get Taylor series expansions of function $f(.,k)$ and $g(.,k)$ as follows:

$$f(\xi\,,\,k)=f(\hat{X}(k),K)+df(\xi,\,k)/d\xi^T\Big|_{\xi=\hat{X}(k)}(\xi-\hat{X}(k))+o'$$

$$g(\xi\,,k)=g(\hat{X}(k),k)+dg(\xi,k)/d\xi^T\Big|_{\xi=\hat{X}(k)}(\xi-\hat{X}(k))+o'' \qquad (7)$$

where o' and o'' are higher-degree terms.

Substituting $=\hat{X}(k)+\tilde{X}(k)$ into Eq.(6) and Eq.(7) and ignore the higher-degree terms in the Tayler serise expansion,we can get the linear state prediction equation:

$$X_p(k+1)=A(k)X_p(k)+A(k)\tilde{X}(k)+b(k)U(k)+B(k)\tilde{X}(k)U(k)+D(k) \qquad (8)$$

$$X_p(k)=\hat{X}(k)$$

where

$$A(k)=df(\xi\,,k)/d\xi^T\Big|_{\xi=\hat{X}(k)};$$

$$B(k)=dg(\xi\,,\,k)/d\xi^T\Big|_{\xi=\hat{X}(k)};$$

$$b(k)=g(\hat{X}(k),k);$$

$$D(k)=f(\hat{X}(k)\,,k)-A(k)\hat{X}(k).$$

Assume

$$A(k+1)=A(k);$$

$$B(k+1)=B(k);$$

$$b(k+1)=b(k);$$

$$D(k+1)=D(k);$$

$$U(k+1)=0;$$

The state predictive value at time k+i can be got from Eq.(8)

$$X_p(k+i)=A^i(k)X_p(k)+A^i(k)\tilde{X}(k)+A^{i-1}(k)(b(k)+B(k)\tilde{X}(k))U(k)+\Big(\sum_{j=0}^{i-1}A^j(k)\Big)D(k) \qquad (9)$$

Substituting Eq.(9) into Eq.(3) and let

$$A_{ik}=A^i(k)\tilde{X}(k)+\Big(\sum_{j=0}^{i-1}A^j(k)\Big)D(k)-X_p^*(k) \qquad (10)$$

$$b_{ik}=A^{i-1}(k)b(k)$$

$$B_{ik}=A^{i-1}(k)B(k)$$

we have

$$J(U(K),k)=\tfrac{1}{2}E[\sum_{i=1}^{N}[A_{ik}+A^{i}(k)\tilde{X}(k)+b_{ik}U(k)+B_{ik}\tilde{X}(k)U(k)]^{T}$$
$$Q(k)[A_{ik}+A^{i}(k)\tilde{X}(k)+b_{ik}U(k)+B_{ik}\tilde{X}(k)U(k)]$$
$$U^{T}(k)R(k)U(k)] \qquad (11)$$

Assume the state estimation is un-bias and let d. $J(U(k),k)/dU(k)=0$, then we can the ACP control law $U_{AC}(k)$:

$$U_{AC}(k)=-\tfrac{1}{2}[\sum_{i=1}^{N}[b_{ik}^{T}Q(k)b_{ik}+E(\tilde{X}^{T}(k)B_{ik}^{T}Q(k)B_{ik}\tilde{X}(k))+R(k)]]^{-1}$$
$$[\sum_{i=1}^{N}[b_{ik}^{T}Q(k)A_{ik}+A_{ik}^{T}Q(K)b_{ik}+E(\tilde{X}^{T}(k)(A^{i}(k))^{T}Q(k)B_{ik}\tilde{X}(k)$$
$$+E(\tilde{X}^{T}(k)B_{ik}^{T}Q(k)A^{i}(k)\tilde{X}(k))] \qquad (12)$$

Let a_{ts} is the t-th line, s-th row element in the matrix $B_{ik}^{T}Q(k)B_{ik}$, b_{ts} is the t-th line, s-th row element in the matrix $(A^{i}(k))^{T}Q(k)B_{ik}$; c_{ts} is the t-th line, s-th row element in the matrix $B_{ik}^{T}Q(k)A^{i}(k)$; p_{ts} is the t-th line, s-th row element in the matrix $P(k)=E[\tilde{X}(k)\tilde{X}^{T}(k)]$. Then we have

$$E[\tilde{X}^{T}(k)B_{ik}^{T}Q(k)B_{ik}^{T}\tilde{X}(k)]=\sum_{t=1}^{n}\sum_{s=1}^{n}a_{ts}p_{ts}$$
$$E[\tilde{X}^{T}(k)(A^{i}(k))^{T}Q(k)B_{ik}\tilde{X}(k)]=\sum_{t=1}^{n}\sum_{s=1}^{n}b_{ts}p_{ts} \qquad (13)$$
$$E[\tilde{X}^{T}(k)B_{ik}^{T}Q(k)A^{i}(k)\tilde{X}(k)]=\sum_{t=1}^{n}\sum_{s=1}^{n}c_{ts}p_{ts}$$

Substituting the Eq.(13) into Eq.(12), the ACP control law is:

$$U_{AC}(k)=-\tfrac{1}{2}[\sum_{i=1}^{N}[b_{ik}^{T}Q(k)b_{ik}+\sum_{t=1}^{n}\sum_{s=1}^{n}a_{ts}p_{ts}]+R(k)]^{-1}$$
$$[\sum_{i=1}^{N}[b_{ik}^{T}Q(k)A_{ik}+A_{ik}^{T}Q(k)b_{ik}+\sum_{t=1}^{n}\sum_{s=1}^{n}b_{ts}p_{ts}+\sum_{t=1}^{n}\sum_{s=1}^{n}c_{ts}p_{ts} \qquad (14)$$

The ACP control strategy can be summarized as follows:

(1) At each "present moent" k a forcast is made of the system state over a arbitrary-arange horizon of sampling periods. This forcast made by means of a first-order approximation model of control action we proposed to apply forom now on.

(2) The ACP control is obtained from Eq.(14), which is based on the principle of deriving the predicted state back to the desired point in the best way according to a specified control objective.

(3) The ACP control is then applied as a control to the real system at the present moment. The whole procedure is repeated leading to an upsated control action with corrction based the latest measurements(receding strategy

Remark 1: ACP control has open-loop feedback control property. Caution control has been presented in the algorithm . In this approach ,the adaptive system takes not only the instantaneous state estimates but also the associated confidence level into account ,i.e.

$$U_{AC}(k)=U(k,\hat{X}(k),P(k)) \tag{15}$$

It should be noted that considering the estimate confidence level do not increase the calculation difficulty siqnificantly in the ACP control . This is one of the main features of ACP control compared with other open-loop feedback control methods (Richard,1973).Besides ,N>1 is permitted in the control cost function Eq.(3).Thus ACP control should be better than the neutral control (Jacobs,1981) in which N=1 is limitted.

Remark 2: If using the separation theorem (Stengel ,1986),we can get the Certainty-Equivalence Precdictive (CEP) control simply not considering the state estimate error during state prediction. Let $\tilde{X}(k)=0$ in the Eq.(8) , the CEP control can be got as follows:

$$U_{CE}(k)=-[\sum_{i=1}^{N} b_{ik}^{T} Q(k)b_{ik}+R(k)]^{-1} [\sum_{i=1}^{N}(b_{ik}^{T}Q(k)\cdot A_{ik} + A_{ik}^{T} Q(k)b_{ik})] \tag{16}$$

thus, the CEP control law has the same expression form as in the references (Shi, 1989); (Zhang, 1988).

Remark 3: There is no probing in the ACP approach. In this algorithm, its cost function has only state prediction and no real future state. Because having no probing control, the ACP control do not need large amounts of searching and optimizing at each control step. The ACP control can be statisfied with the requirement of real time control in a missile interception problem.

SIMULATION STUDIES

In this section, we shall study a missile interception example via simulation. The purpose of this study are (1) to show the caution property in the ACP control used to the missile interception problem, (2) to compare the ACP control algorithm with the CEP control algorithm used to the missile interception problem.

The controlled model of missile interception example is

$$X(k+1)=A(\theta_1,\theta_2,\theta_3)X(k)+B(\theta_4,\theta_5,\theta_6)u(k)+W(k)$$

$$Y(k)=[0,0,1)X(k)+V(k) \quad (17)$$

Where

$$A(\theta_1,\theta_2,\theta_3)=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ \theta_1 & \theta_2 & \theta_3\end{bmatrix} ; \quad B(\theta_4,\theta_5,\theta_6)=\begin{bmatrix}\theta_4\\ \theta_5\\ \theta_6\end{bmatrix} \quad (18)$$

and θ_i, i=1,2,...,6 are unknown constant parameters with normal a priori statistics having mean and variance.

$$\theta(0\ 0)=[1,-0.6,0.3,0.1,0.7,1.5]^T \quad (19)$$

$$P_\theta(0\ 0)=\text{diag}(0.1,0.1,0.01,0.01,0.01,0.1) \quad (20)$$

The true parameters are

$$\theta^*=[1.8,-1.01,0.58,0.3,0.5,1.0] \quad (21)$$

The initial state is assumed to be known as

$$X(0\ 0)=X(0)=[0,0,0]^T \quad (22)$$

The objective is to find a control law to bring the third conponent of the state to a desired value. This is expressed by the cost

$$J=\tfrac{1}{2}E[\sum_{k=1}^{N_f}[q(x_3(k)-\rho)^2+ru^2(k)]] \quad (23)$$

where is some value and is chosen to be small. In this example, ρ =20 and r is chosen to be 0.001. The noises $[W_i(k)]_{i=1}^{3}$ and [V(k)] are assumed to be independent and normally distributed with zero mean and unit variance. N_f is chosen to be 20.

The prediction control index can be descibed as

$$J(u(k),k)=\tfrac{1}{2}E[\sum_{i=1}^{N}[X(k+i)-X_p(k+i)]Q[X(k+i)-X_p(k+i)]+ru(k)] \quad (24)$$

where

$$X(k+i)=\begin{bmatrix}0\\ 0\\ 20\end{bmatrix} \qquad Q=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0.01\end{bmatrix}$$

TABLE 1 System parameter in simulation

System	q_w	q_v	q_x
S1	[1.0 1.0 1.0]	1.0	[0.001 0.001 0.001]
S2	[0.8 0.8 0.8]	0.8	[0.001 0.001 0.001]
S3	[0.6 0.6 0.6]	0.6	[0.001 0.001 0.001]

S4	[0.5 0.5 0.5]	0.5	[0.001 0.001 0.001]
S5	[0.05 0.05 0.05]	0.05	[0.001 0.001 0.001]
S6	[0.01 0.01 0.01]	0.01	[0.001 0.001 0.001]

for all system $X(0\ 0)=X(0)=0$

$\theta(0\ 0)=(1,-0.6,0.3,0.1,0.7,1.5)^T$

TABLE 2 Simulation results

System	J(CEP)	J(ACP)
S1	10.7010	10.4766
S2	10.0881	9.5277
S3	9.4091	9.0321
S4	9.0215	8.4491
S5	5.9038	5.8963
S6	5.2452	5.2531

Some simulation results are presented in table 2 with the corresponding simulation system parameters shown in the table 1. From this two tables, we can see:

(1) When system noise covariance q becomes larger, the ACP control is better than the CEP control (see S1,S2,S3,S4).

(2) When system noise covariance q becomes smaller, the ACP control is almost as better as CEP control (see S5,S6).

From this simulation and analysis, it is shown that, in general case, the performance of the ACP control is better than or not significantly different from the CEP control when both of control strategy are used in the missile interception problem.

CONCLUSION

In this paper, the adaptive caution predictive control for a missile interception problem is presented. The ACP control takes state estimate value as well as its associated confidence level into during adaptation. The algorithm is quite robust to system parameter and state estimate uncertainties.The other feather of the ACP control is simplity and short calculating time. All these make the algorithm suitable for a missile interception prob lem.

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