Engineering-Driven Factor Analysis for Variation Sources Identification in Multistage Manufacturing Processes

Jian Liu
Department of Industrial and Operations Engineering, The University of Michigan

Jianjun Shi
H. Milton Stewart School of Industrial and Systems Engineering, Georgia Institute of Technology

S. Jack Hu
Department of Mechanical Engineering, The University of Michigan

Abstract

Variation source identification in a multistage manufacturing process (MMP) is greatly enabled by the fusion of engineering domain knowledge with multivariate measurements of product quality. However, existing approaches, including the quantitative engineering-model-based methods and the data-driven methods, provide limited capabilities in variation sources identification. This paper proposes a new methodology that does not depend on accurate quantitative engineering models. Instead, engineering domain knowledge about the interactions between potential variation sources and product quality variables are represented as qualitative indicator vectors. These indicator vectors guide the rotation of the factor loading vectors that are derived from factor analysis of the multivariate measurement data. Based on this engineering-driven factor analysis, a procedure is presented to identify multiple variation sources that present in an MMP. The effectiveness of the proposed methodology is demonstrated in a case study of a three-stage assembly process.

Key words: Multistage manufacturing processes, variation sources identification, indicator vector, spatial pattern vector, engineering-driven factor analysis,

Nomenclature:

$\mathbf{y}$: a $p\times 1$ vector containing random deviations of p KPC's

$\mathbf{u}_s$: an $s\times 1$ vector containing random deviations of s process variation sources that present in the process

$\mathbf{\Gamma}_s$: a $p\times s$ matrix with each column a spatial pattern vector of a variation source

p: the number of KPC's

s: the number of variation sources that present in the process

$\mathbf{L}$: a $p\times s$ initial loading matrix with each column vector, $\mathbf{l}_j$, an initial factor loading vector

$\mathbf{l}_j$: an $p\times 1$ initial factor loading vector, and $\mathbf{l}_j$ is orthogonal to $\mathbf{l}_t$ when $j\neq t$, j,t=1,2,…, s

$\mathbf{L}^*$: a $p\times s$ rotated loading matrix with each column vector, $\mathbf{l}_j^*$, an rotated factor loading vector

$\mathbf{l}_j^*$: a $p\times 1$ rotated factor loading vector, j=1,2,…, s

$\mathbf{T}$: a $p\times M$ indicator matrix with each column vector, $\boldsymbol{\tau}_m$, an indicator vector derived from engineering knowledge representation

$\boldsymbol{\tau}_m$: a $p\times 1$ indicator vector, m=1,2,…, M

M: the number of potential variation sources

$F_{k:i_k}$: the i_k^{th} feature in stage k, i_k=1, 2, …, n_k

n_k : the number of features in stage k

$FX_{k:l_k}$: the l_k^{th} fixtures used in stage k, l_k=1, 2,…, t_k

t_k: the number of fixtures used in stage k

1 Introduction

Variation source identification is an important task of quality assurance in multistage manufacturing processes (MMP's) [1]. The variation sources are special causes of variation in key product characteristics (KPC's). Since KPC's variation affects the final product quality as well as the manufacturing system productivity [2], it is essential to conduct variation reduction by detecting process variation changes and identifying the underlying variation sources. This is especially challenging for MMP's, where multiple stages are involved in generating designated KPCs or functionality of a product. As illustrated in Figure 1, in stage k of an MMP, special causes [3] (e.g., excessive variation of the locations of locating pins) will increase the variation of some KPC's measurements to a level that exceeds their tolerances. Compounded with the input quality transmitted from preceding stages, these quality problems will be further propagated to the downstream stages and accumulated to the final product. In order to trace the variation propagation and monitor an MMP, it is ideal to take measurements in every stage. However, to reduce the inspection cost, it is customary to measure KPC's only after the final stage, e.g., stage N, as shown in Figure 1. Therefore, the complex variation propagation and the lack of in-process measurements make it extremely difficult to identify the variation sources in an MMP [4].

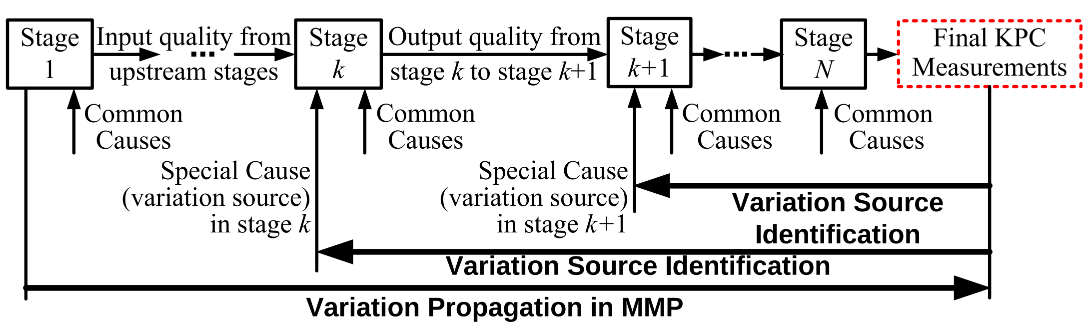


Figure 1 Complex variation propagation scenario in an MMP.

Variation source identification can be accomplished by investigating multiple KPC's and their spatial patterns, since the spatial patterns reflect the characteristics of the variation sources. As shown in Figure 2 (a), part A and part B are fixed by fixture locating pins, P_1 to P_4, and are assembled in stage 1. P_1 and P_3 are 4-way pins that restrain the degree of freedoms of the parts along X and Z axes, whereas P_2 and P_4 are 2-way pins that restrain the parts from rotating around the axis perpendicular to the XZ plane. The subassembly is then assembled with Part C in stage 2. Five (5) features, two locating holes, F_1 and F_3, and three corner points, F_2, F_4 and F_5, are considered and their position along X and Z axes are the KPC's. The variations of the locating pins are the variation sources since they cause excessive variation in the KPC's. For instance, in stage 1, the variation of P_4 along z direction, defined as a variation source P_4_z, leads to orientation variation of Part B and thus causes the position variation of F_4. KPC's are grouped together to form spatial pattern vectors (SPV's), which describe the effects of a variation sources. As shown in Figure 2 (c), P_4_z's SPV is a 10×1 vector corresponding to 10 KPC's. In stage 1, only F_4 are affected by P_4_z, thus $F_4(x)$ and $F_4(z)$ in the SPV will be non-zero values, denoted as "#". Because of the variation propagation caused by parts'

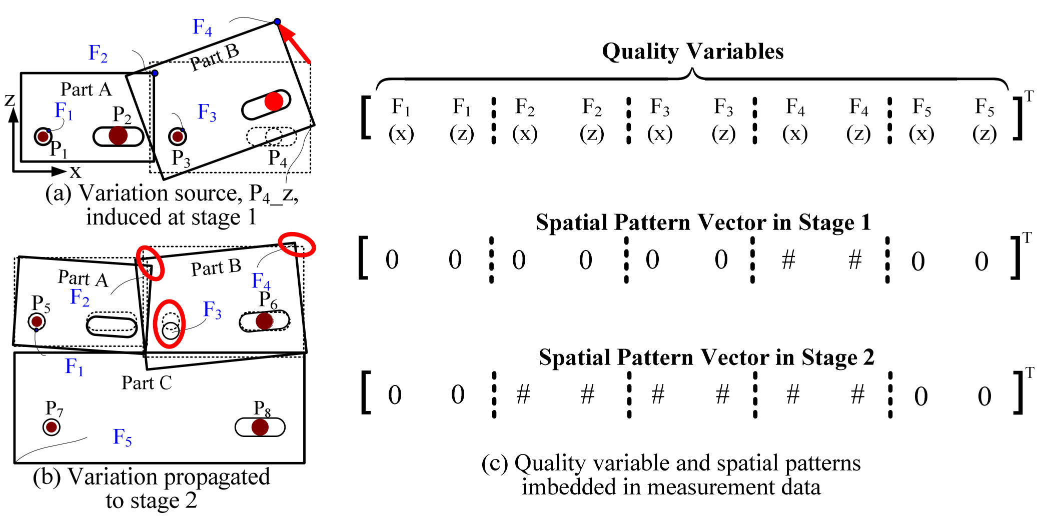


Figure 2 KPC's, variation source and their SPV's

reorientation, P_4_z's SPV in stage 2 is different, indicating that F_2, F_3 and F_4 are all affected by P_4_z introduced in preceding stage 1. Figure 2 shows that each individual potential variation source in a stage of an MMP will have its particular SPV. This one-to-one relationship makes it possible to identify the variation sources by investigating the SPV's, which are either derived from engineering domain knowledge about product/process design, or estimated from multivariate measurements of KPC's.

Progresses have been made in recent years in developing methodologies to implement this basic idea in MMP's [4]. Existing approaches can be divided into two categories: engineering-driven approaches and data-driven approaches, as summarized in Table 1.

Table 1 Summary of reported variation source identification approaches

	Engineering-driven [5-14]	**Data-driven [15-20]**
Features	• SPV's are derived from accurate *a priori* engineering knowledge of product/process design. • SPV's are defined with exact values. • Variation sources are identified by engineering-model-based direct estimation.	• SPV's are estimated from multivariate quality measurement data. • Exact values of true SPV's are unknown. • Variation sources are identified by pattern interpretation according to engineering knowledge.
Limitations	• Comprehensive and accurate engineering knowledge is mandatory. • Diagnostic results are not robust to unknown tooling position due to adjustments or worn out.	• Engineering knowledge is not directly involved in the estimation of SPV's. • Achieved spatial patterns may not be the best estimates of true SPV's.

Engineering-driven approaches intend to directly link the engineering knowledge of the process variation sources with the KPC measurements through mathematical modeling. Jin and Shi [5] developed the state space models to represent the geometrical relationships between KPC and process key control characteristics (KCC's) according to product/process design information. Mantripragada and Whitney proposed a "datum flow chain (DFC)" concept for an assembly [6] and explicitly defined it in a discrete state transition model to describe the variation propagation in assembly process [7]. State space modeling techniques are further investigated and applied in assembly processes [8, 9] and machining processes [10-12]. These modeling approaches lead to a generic linear model:

$$\mathbf{y} = \mathbf{\Gamma u} + \mathbf{v}, \tag{1}$$

where $\mathbf{y}$ ($\mathbf{y} \in \Re^{p\times 1}$) is a vector of dimensional deviations of p KPC's. Vector $\mathbf{u}$ ($\mathbf{u} \in \Re^{M\times 1}$) consists of the deviations of M potential process variation sources. $\mathbf{\Gamma}$ ($\mathbf{\Gamma} \in \Re^{p\times M}$) in engineering-driven approaches is a constant coefficient matrix derived from the product/process design and it describes the linear interactions between process (potential variation sources) and product (KPC's). The negligible un-modeled factors and measurement noise are represented by vector $\mathbf{v}$ ($\mathbf{v} \in \Re^{p\times 1}$). Based on the linear model (1), diagnostic approaches have been developed [13, 14]. These engineering-model-based techniques fundamentally improve the capability of variation sources identification. However, significant effort is needed to derive and validate the exact values of the coefficients of the model, i.e., the values of the elements in matrix $\mathbf{\Gamma}$. The dependence on comprehensive and accurate *a priori* engineering knowledge also impacts the robustness of the diagnosis approach. When unknown tooling adjustments are performed, the engineering knowledge will be either incomprehensive or inaccurate. Thus the model will no longer reflect the true linear interactions between process and quality and may lead to unreliable or misleading diagnostic results.

Data-driven approaches avoid the dependence on accurate *a priori* engineering knowledge and complex model derivation. Wolbercht *et al.* [15] developed a system to implement real time monitoring an diagnosis of MMP's using Bayesian networks. The method depends on in-process measurement data collected at several points throughout the process. When the KPC measurements are only available at the final stage, diagnosis is conducted by directly estimating the SPV's imbedded in the multivariate measurements of KPC's and matching them to the expected SPV's of potential variation sources. This is equivalent to estimating a matrix $\boldsymbol{\Gamma}_s$ whose s column vectors are the SPV's of the s variation sources that actually present in an MMP. Ceglarek and Shi [16] employed a principal component analysis (PCA) technique to extract a single spatial pattern from KPC covariance matrix and compare them with predefined spatial patterns of potential variation sources. Liu *et al.* [17] proposed a factor analysis (FA) based method to diagnose an MMP through investigating multiple orthogonal SPV's. The diagnosis capability of those approaches [16, 17] is confined by the assumptions of orthogonality. Apley and Shi [18] developed an FA method to extract and interpret the SPV's of the variation sources. An assumption a ragged lower-triangular $\boldsymbol{\Gamma}$ matrix is mandatory. Independent component analysis techniques were also used by Apley and Lee [19] to separate variation sources "blindly," with constraints on auto-correlation and distribution conditions. Jin and Zhou [20] developed a method to identify variation sources through analyzing fault space, which is spanned by the eigenvectors of the covariance matrix of measurement data. However, when there is more than one variation sources present in a process, the estimated SPV's will be different from the true SPV's and thus insufficient to guide corrective actions. This is because that the existing data-driven methods do not directly use the engineering knowledge when analyzing the multivariate statistics of KPC measurements.

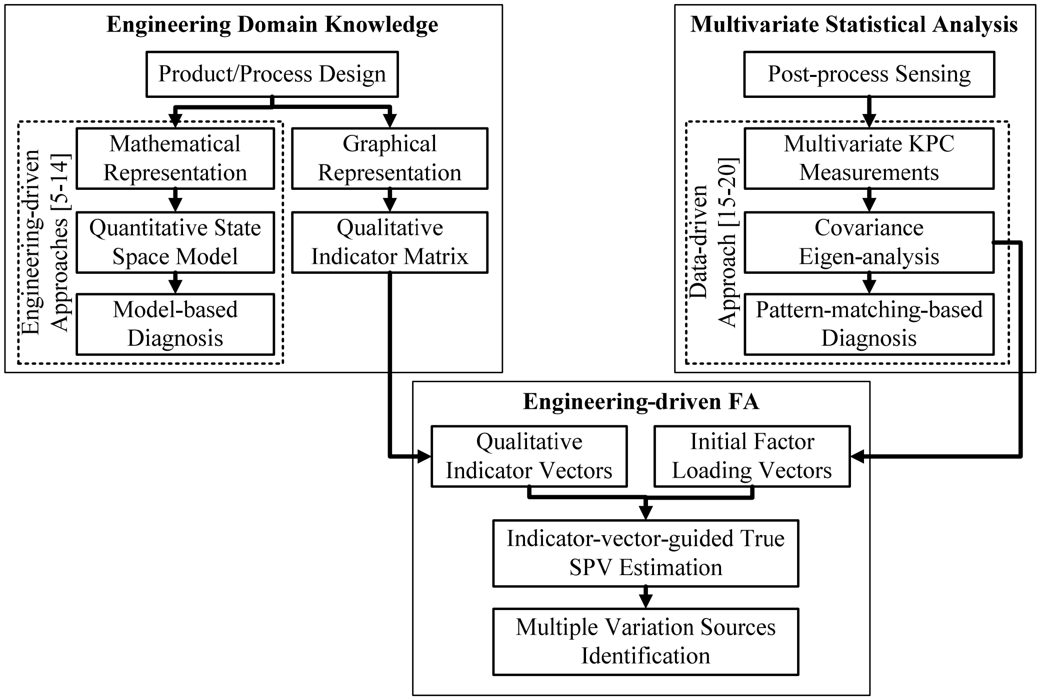


Figure 3 Overview of the proposed approach

In order to improve the interpretation of data-driven approaches and avoid dependence on accurate models, it is important to directly use the engineering knowledge to direct statistical data analysis. Liu and Hu [21] and Camelio and Hu [22] developed designated component analysis (DCA) approach based on mutually orthogonal designated variation patterns that are derived directly from knowledge of product/process design. This approach facilitates the diagnosis of multiple fixture faults that present in a single stage of an assembly process. However, it has limited potential to be applied in MMP's. This is because that variation propagation along stages will make the designated variation patterns more complicated and thus un-orthogonal to each other. And the orthonomalization of those patterns will change their physical interpretations and result in misleading diagnostic conclusion. In this paper, an engineering-driven FA method is proposed for estimating multiple non-orthogonal true SPV's and identifying their underlying variation sources in MMP's. This method involves four steps: (i) converting engineering domain knowledge into qualitative indicator vectors, (ii) deriving the initial principal factor loading vectors, (iii) rotating the initial factor loading vectors to estimate the true SPV's, and (iv) identifying of multivariate variation sources, as shown in Figure 3. Different from existing data-driven methods, the engineering knowledge of the impacts of potential variation sources is systematically represented in a set of qualitative indicator vectors to directly guide the factor rotation. The rotated factor loading vectors are the best estimation of true SPV's and therefore, can be used to best interpret the true nature of the variation sources and to direct corrective actions. Compared to the engineering-driven methods, it is robust to process changes since no exact model coefficient values are needed. This method effectively integrates the engineering knowledge with statistical analysis of multivariate KPC measurements. It combines the advantages of the engineering-driven approaches and data-driven approaches, and overcome their limitations.

The remainder of this paper is organized as follows. Section 2 provides an overview of the proposed methodology and presents the necessity of the engagement of engineering domain knowledge in guiding SPV estimation. Section 3 introduces the qualitative representation of engineering domain knowledge and engineering-driven factor analysis for variation sources identification. Case study results are presented in Section 4 to demonstrate the capability of the proposed method. Conclusions and future works are discussed in Section 6.

2 Overview of the methodology

This paper focuses on directly incorporating the engineering knowledge in guiding the estimation of SPV's. The variation sources identification starts from statistically analyzing multivariate KPC measurements, collected at the final stage of an MMP. The objective is to detect large variations, estimate their SPV's, and interpret or map them with the expected spatial patterns. This can be achieved by analyzing the covariance matrix of KPC measurements with the FA method [23] that adopts the linear model defined in Eq. (1). Following assumptions about Eq. (1) are made:

(i) $\mathbf{y}$ follows p-dimensional multivariate normal distribution, i.e., $\mathbf{y}\sim N_p(\boldsymbol{\mu}_\mathbf{y},\boldsymbol{\Sigma}_\mathbf{y})$. Since only the quality problem manifested as increased KPC variations are considered in this paper, it is assumed that $\boldsymbol{\mu}_\mathbf{y}=\mathbf{0}$.

(ii) $\mathbf{u}_s$ is an $s\times 1$ ($s < p$) unknown vector containing the deviations of s variation sources that present in an MMP. It follows an s-dimensional multivariate normal distribution, i.e., $\mathbf{u}_s\sim N_s(\mathbf{0}, \boldsymbol{\Sigma}_{\mathbf{u}_s})$. Since the variation sources, e.g., fixture locators, are often fabricated, installed and maintained separately for different stations, it is reasonably to assume that $\boldsymbol{\Sigma}_{\mathbf{u}_s}$ is a diagonal matrix.

(iii) Different from that defined in Eq. (1), FA assumes that $\boldsymbol{\Gamma}_s$ ($\boldsymbol{\Gamma}_s=[\gamma_1\ \gamma_2\ \ldots\ \gamma_s]$) is an *unknown* constant $p\times s$ matrix that reflects the linear impacts of $\mathbf{u}_s$ on $\mathbf{y}$. Each column vector γ_i is called a loading vector corresponding to the i^{th} element in $\mathbf{u}$, and in this paper, the column vector γ_i is an SPV of the i^{th} variation source. The objective of the proposed method is to estimate the γ_i's, $i=1,2,\ldots,s$, in $\boldsymbol{\Gamma}_s$ to investigate the natures of the variation sources and identify them.

(iv) $\mathbf{v}$ follows a p-dimensional multivariate normal distribution, i.e., $\mathbf{v}\sim N_p(\mathbf{0},\boldsymbol{\Sigma}_\mathbf{v})$. In this paper, it is assumed that all KPC's are measured by the same measuring device. Thus, measurement noises are independent of each other, i.e., $\boldsymbol{\Sigma}_\mathbf{v}=\sigma^2\mathbf{I}$, where σ^2 is a scalar and $\mathbf{I}$ is an identity matrix with an appropriate dimension. It is also reasonable to assume that $\mathbf{v}$ is independent of $\mathbf{u}_s$, i.e., the measurement noises are independent of the variation sources.

With the above assumptions, the covariance matrix of $\mathbf{y}$ can be computed as

$$\mathbf{\Sigma}_{\mathbf{y}} = \mathbf{\Gamma}_s \mathbf{\Sigma}_{\mathbf{u}_s} \mathbf{\Gamma}_s^{\mathrm{T}} + \mathbf{\Sigma}_{\mathbf{v}}. \tag{2}$$

The first step in estimating $\mathbf{\Gamma}_s$ is to determine the number of variation sources present in an MMP, i.e., s. Based on the eigenvalue-eigenvector pairs derived from eigen-decomposition of $\mathbf{\Sigma}_{\mathbf{y}}$, s can be determined by Akaike Information Criterion (AIC) and Minimum Description Length (MDL) criterion, as defined in Eq. (3) and Eq.(4),

$$\mathrm{AIC}(q) = n(p-q)\log(a_q / g_q) + q(2p-q), \text{ and} \tag{3}$$

$$\mathrm{MDL}(q) = n(p-q)\log(a_q / g_q) + q(2p-q)\log(n)/2, \tag{4}$$

where n is the sample size, and q is the number of the largest eigenvalues considered. a_q and g_q are the arithmetic mean and the geometric mean of the rest (p-q) smallest eigenvalues of $\mathbf{\Sigma}_{\mathbf{y}}$, respectively. In order to determine s, AIC(q) and MDL(q) are iteratively evaluated for q=1, …, p-1, and s is equal to the q^* that minimizes AIC(q^*) or MDL(q^*). As recommended by Apley and Shi [18], when small magnitudes of variations are expected, adopting AIC will result in a high probability of correct number estimation. Otherwise, MDL criteria should be adopted to achieve consistent estimation of q^*.

Apley and Shi [18] showed that if s variation sources are present in the process, the enginvalues of $\mathbf{\Sigma}_{\mathbf{y}}$, λ_i (i=1, 2, …, p), will have a relationship such that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_s > \sigma^2 = \lambda_{s+1} = \ldots = \lambda_p$. In addition, Jin and Zhou [20] showed that the s eigenvectors associated with the largest s eigenvalues of $\mathbf{\Sigma}_{\mathbf{y}}$ span the same linear space of the s SPV's in $\mathbf{\Gamma}_s$. Therefore, it is applicable to estimate the true SPV's by performing an eigen-decomposition of $\mathbf{\Sigma}_{\mathbf{y}}$,

$$\mathbf{\Sigma_y} = \mathbf{L}\mathbf{L}^{\mathrm{T}} + \mathbf{\Sigma_v} = \sum_{i=1}^{s}\left(\lambda_i - \sigma^2\right)\mathbf{e}_i\mathbf{e}_i^{\mathrm{T}} + \sigma^2\sum_{i=1}^{p}\mathbf{e}_i\mathbf{e}_i^{\mathrm{T}} = \mathbf{E}_s\left[\mathbf{\Lambda}_s - \sigma^2\mathbf{I}\right]\mathbf{E}_s^{\mathrm{T}} + \sigma^2\mathbf{I}, \quad (5)$$

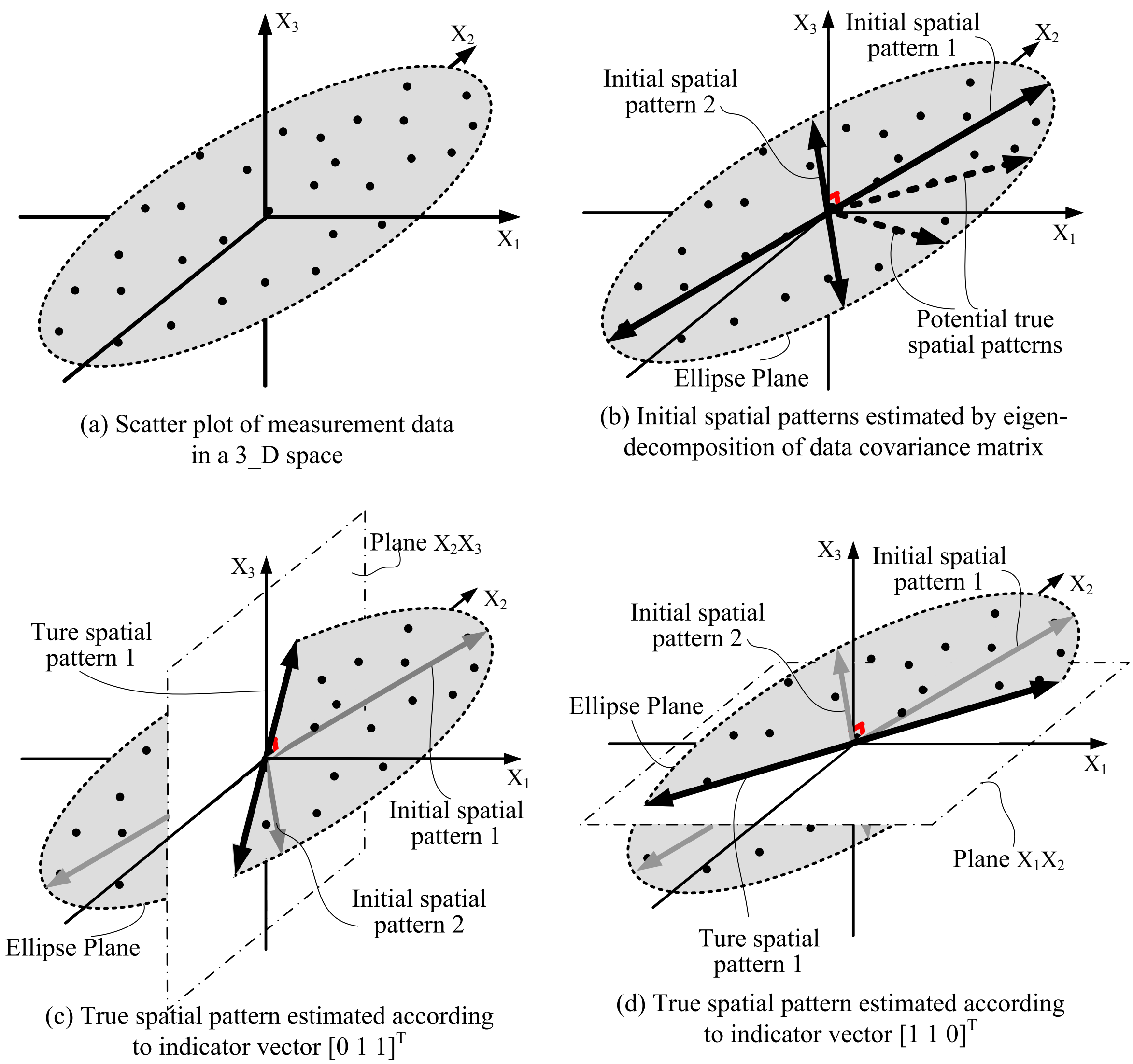
where $\mathbf{E}_s = \begin{bmatrix}\mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_s\end{bmatrix}$ and $\mathbf{\Lambda}_s = diag\{\lambda_1, \lambda_2, ..., \lambda_s\}$ are the matrix composed of eigenvectors $\mathbf{e}_i$ and λ_i, respectively. $\mathbf{L} = \begin{bmatrix}\mathbf{l}_1 & \mathbf{l}_2 & \ldots & \mathbf{l}_s\end{bmatrix} = \mathbf{E}_s\left(\mathbf{\Lambda}_s - \sigma^2\mathbf{I}\right)^{1/2}$ is the loading matrix in FA, where $\mathbf{l}_i$'s, i=1,2,…, s, are initial loading vectors that are orthogonal to each other. Compared to Eq.(2), $\mathbf{l}_i$'s are the initial estimates of the scaled γ_i's since $span\{\mathbf{l}_1\ \mathbf{l}_2 \ldots \mathbf{l}_s\} = span\{\gamma_1\ \gamma_2\ ...\ \gamma_s\}$. $\sigma^2\mathbf{I}$ is assumed to be the covariance matrix of measurement noise, i.e., $\mathbf{\Sigma_v}$. However, in practice, the value of σ^2 is often not available, or the $\mathbf{\Sigma_v}$ is not in such a simple structure. Therefore, an approximation of $\mathbf{L}$ can be achieved by

$$\mathbf{L} \approx \mathbf{E}_s\mathbf{\Lambda}_s^{1/2}. \quad (6)$$

This approximation will make the initial factor loading vectors deviate from their true values. According to the investigation conducted by Li, *et al.* [23], when σ^2 is much smaller than λ_i, i=1,2,…,s, the deviations will be negligible.

When there is a single variation source present in an MMP, i.e., s=1, the initial estimation, $\mathbf{l}_1$, is an effective estimation of the scaled true SPV of that variation source [16]. However, when there are multiple variation sources, i.e., $s > 1$, the eigenvectors may not be coincident with SPV's of the s variation sources, unless those true SPV's are orthogonal to each other. Treating the eigenvector-based loading vectors as estimates of true SPV's may yield non-interpretable or even misleading results and consequently provide limited information for diagnosis and corrective actions. Thus, the orthogonal factor loading vectors should be rotated, obliquely, to estimate true SPV's of the variation sources. Various oblique factor rotation methods have been developed based on explanatory criteria to achieve simple structure and improved factor interpretability [24].

However, these criteria have limited justification for the diagnostic purpose, since the rotated loading vectors may not be effective in estimating the true SPV's of variation sources. Target rotation [23] provides directional information for factor rotation with hypothesized patterns. This technique is adopted in this paper, with target patterns systematically defined by the engineering domain knowledge. Thus, the rotated factor loading vectors have the right structures that best estimate the true SPV's of the variation sources.



(a) Scatter plot of measurement data in a 3_D space

(b) Initial spatial patterns estimated by eigen-decomposition of data covariance matrix

(c) True spatial pattern estimated according to indicator vector $[0\ 1\ 1]^T$

(d) True spatial pattern estimated according to indicator vector $[1\ 1\ 0]^T$

Figure 4 Conceptual illustration of indicator vector guided factor rotation

The underlying idea of the target guide rotation is illustrated in Figure 4. For a case where three KPC's, X_1, X_2, and X_3, are measured, the three-dimensional (3-D) measurement data can be shown in a 3-D scatter plot. Suppose that there are two (2) variation sources present in the process, the measurement data points will be scattered in a 2-D plane and within an ellipse, as shown in Figure 4 (a). Orthogonal factor loading vectors can be achieved from eigen-analysis of data covariance matrix. These orthogonal loading vectors and the true SPV's of the two variation sources will span the same 2-D plane. And the true SPV's could be any linear combinations (rotation) of the orthogonal loading vectors, as shown as the "potential true spatial patterns" in Figure 4 (b). Information that indicates the direction of the true SPV's is critically important for the rotation. For instance, if it is known that a certain variation source affects only X_2 and X_3, its true SPV must span the plane defined by axis X_2 and axis X_3, i.e., plane X_2X_3, and simultaneously span the ellipse plane, as shown in Figure 4 (c). This means that the true SPV is the intersection of the plane X_2X_3 and the ellipse plane. The direction information of each potential variation source can be represented with an indicator vector, e.g., $[0\ 1\ 1]^T$, where those three elements correspond to three KPC's, X_1, X_2, and X_3, and their values indicate whether the corresponding KPC is affected (1) or not affected (0) by the variation source. Therefore, in this case, if the direction information corresponding to the two potential variation sources are $[0\ 1\ 1]^T$ and $[1\ 1\ 0]^T$, respectively, the scaled true SPV's can be estimated, as shown in Figure 4 (c) and Figure 4 (d), respectively.

As illustrated in Figure 4, in order to estimate the true SPV's of variation sources from multivariate statistical analysis, it is crucial to (i) obtain engineering domain knowledge on interactions between product KPC's and potential variation sources and represent it in an appropriate form, and (ii) use the engineering knowledge in guiding the factor rotation to estimate the true SPV's. The variation sources are identified based on the interpretation of the estimated true SPV's.

3 Engineering-driven factor analysis for variation sources identification

This section introduces the proposed new methodology following the ideas illustrated in Section 2. Basically, the methodology includes three components: (i) qualitative engineering domain knowledge representation, (ii) indicator-vector-guided factor rotation for estimating the scaled true SPV, and (iii) the procedure of multiple variation sources identification.

3.1 Engineering knowledge representation

A successful engineering knowledge representation for variation sources identification needs a thorough description of an MMP, and a mechanism to interrelate potential variation sources with the KPC's. More importantly, it is necessary to provide directional indication of true SPV's.

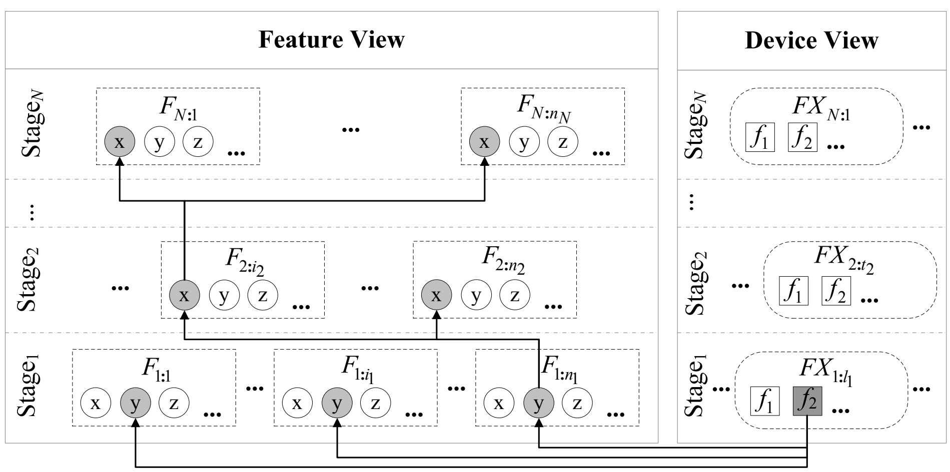


Figure 5 Graphical description of an MMP

Engineering knowledge about an MMP comes from the product and process design, which includes the process flow, the manufacturing datum schemes selected for each operation, the features generated in a series of operations, their precedence relationships, and the potential variation sources. The description of an MMP can be represented in a diagram, as shown in Figure 5.

This diagram contains a "Feature View" and a "Device View", and includes four categories of information about an MMP: (i) process flow and precedence relationships, (ii) potential variation sources, (iii) relationships between potential process variation sources and KPC's, and (iv) deviation propagation scenario.

The *process flow and precedence relationships* include process sequence information, operations information and datum flow information. As shown in Figure 5, process sequence is presented as N layers in the two views of the diagram, corresponding to the N stages of an MMP. Operations information indicates the features generated in each stage. Denoted as dashed boxes in Figure 5, $F_{k:i_k}$ represents the i_k^{th} feature (i_k=1, 2, …, n_k) generated in stage k, and n_k is the number of features generated in stage k, k=1,2,…,N. For each feature, its KPC's are denoted as circles nodes, e.g., z, following vector feature representation [12]. Datum flow information is shown through the linkage between features. In feature view, features in a particular stage k are linked with some features generated in upstream stages. This means that those upstream features are used as the datum features in stage k.

Potential variation sources are in the machine tool, fixtures and datum features, as discussed by Jin and Shi [5], and Zhou *et al.* [12]. In Figure 5, fixtures are denoted as dashed blocks, and are organized in different layers in the device view, where $FX_{k:l_k}$ represents the l_k^{th} fixtures (l_k=1, 2,…, t_k) used in stage k, and t_k is the total number of fixtures used in stage k. This paper considers the most commonly used 3-2-1 fixturing scheme. Vectorial notation for fixture components used in [5, 12] are adopted and its elements are represented as the square nodes within the blocks, e.g., $\boxed{f_1}$.

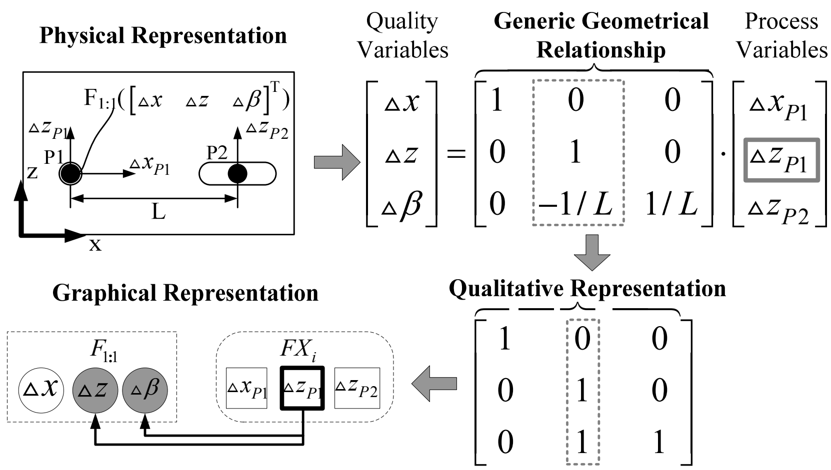


Figure 6 Qualitative representation of quality/process interaction

The *relationships between potential process variation sources and KPC's* describe the impacts of potential variation sources on KPC's. These impacts are denoted as the connections linking the process variables in the device view, e.g., $\boxed{f_2}$ of $FX_{1:l_1}$, and the KPC's in the feature view, e.g., y of $F_{1:1}$. With the fixtures and features represented in a vector form, the linear relationship between them can be described with a qualitative indicator matrix, as illustrated by Figure 6. The interactions between three KPC's, Δx, Δz, and $\Delta\beta$, of $F_{1:1}$ and three variation sources, Δx_{P1}, Δz_{P1}, and Δz_{P2}, of FX_i are considered. Their generic geometric relationship can be described in a matrix developed in [5, 12], as shown in Figure 6. Qualitative representation matrix can be achieved by keeping zero elements in the generic geometric relationship matrix and replacing the non-zero elements with 1's. The remaining 0's in the pattern matrix are called *specified elements*, and the 1's are *unspecified elements*. In the qualitative indicator matrix, the i,j^{th} element indicates whether the j^{th} variation source has an impact on the i^{th} quality variable. When it is 1, there will be a connection between the two nodes in the graphical representation, and vise versa. As shown in Figure 6, according to the qualitative representation, Δz_{P1} of fixture FX_i affects Δz, and $\Delta\beta$ of $F_{1:1}$.

Deviation propagation is caused by the faulty datum features generated in upstream stages. It can be described by linking the datum features with the features generated based on them. As shown in Figure 5, feature $F_{1:n_1}$ is used as the datum in stage 2. Thus, if the f_2 of $FX_{1:k_1}$ causes an deviation of $F_{1:n_1}$ in the y direction, this deviation will be propagated to the x's of $F_{2:i_2}$ and $F_{2:n_2}$, and further propagated to the x's of $F_{N:1}$ and $F_{N:n_N}$, since $F_{2:i_2}$ is selected as the datum feature in stage *N*. Therefore, the graphical representation of deviation propagation is a path that connects the potential variation sources with all the affected KPC's. The rationale underlying the connections is the same as that of the state transition matrices proposed in [5, 12].

3.2 Indicator matrix definition

The graphical description of an MMP in Figure 5 should be transformed to a qualitative indicator matrix to guide the SPV transformation. The feature view of an MMP contains *p* KPC's in **y** of Eq. (1), whereas the device view contains *M* potential variation sources in **u**. Although matrix $\boldsymbol{\Gamma}$ is not known since the exact values of its elements are not derived, we can still use a $p \times M$ indicator matrix, $\mathbf{T} = [\tau_{im}]$, to link potential variation sources with KPC's, where

$$\tau_{im} = \begin{cases} 1, & \text{When there is a connection linking the } m^{\text{th}} \text{ potential} \\ & \text{variation source and the } i^{\text{th}} \text{ quality variable;} \\ 0, & \text{Otherwise;} \end{cases} \tag{7}$$

for i=1,2,…,p, and m=1,2,…,M. The column vectors, $\boldsymbol{\tau}_m$, in matrix **T** are indicator vectors to indicate the direction information of true SPV's of variation sources, where $\boldsymbol{\tau}_m=[\tau_{1m}\ \tau_{2m}\ \ldots\ \tau_{pm}]^{\mathrm{T}}$. The column vectors in $\boldsymbol{\Gamma}$ of Eq. (1) are the true SPV's of all potential variation sources, whereas the columns in pattern matrix **T** only partially reflect $\boldsymbol{\Gamma}$.

3.3 Indicator-vector-guided true SPV estimation

Factor rotation is a transformation technique to improve the interpretability of factor loadings, i.e., the initial estimates of true SPV's. The objective is to find a rotation matrix **R** to transform the orthogonal initial factor loading vectors that are defined in Eq. (6), to

$$\mathbf{L}^* = \mathbf{L}\mathbf{R} \approx \mathbf{E}_s\boldsymbol{\Lambda}_s^{1/2}\mathbf{R}, \tag{8}$$

which best estimates the scaled true SPV's of underlying variation sources. In this paper, indicator matrix **T** works as a target that guides the factor rotation. Two assumptions are necessary to make the target factor rotation applicable [23]:

(i) Each indicator vector, $\boldsymbol{\tau}_m$, should contain at least s-1 specified elements, i.e., 0's;

(ii) The indicator vectors are different from each other. That is, if an operator, $\otimes$, is defined such that $1\otimes 1=0$, $0\otimes 0=0$, $1\otimes 0=1$, and $0\otimes 1=1$ hold for scalars, and $\boldsymbol{\tau}_k{}^T \otimes \boldsymbol{\tau}_l = \sum_{t=1}^{p}(\tau_{tk}\otimes\tau_{tl})$, $k \neq l$, hold for vectors, the indicator vectors should satisfy

$$\boldsymbol{\tau}_k^{\mathrm{T}} \otimes \boldsymbol{\tau}_l \neq 0, \text{ for } k \neq l. \tag{9}$$

The initial $p\times s$ orthogonal factor loading matrix can be denoted as $\mathbf{L}=\left[l_{ij}\right]$, whereas the rotated factor loading matrix can be denoted as $\mathbf{L}^*=\left[l_{ij}^*\right]$, where i= 1,2, …, p, and j=1, 2, …, s. Let $\mathbf{r}_j$ denote the j^{th} column of the rotation matrix **R**, then the j^{th} column of $\mathbf{L}^*$ can be achieved by

$$\mathbf{l}_j^* = \mathbf{L}\mathbf{r}_j, \; j = 1, 2, ..., s. \tag{10}$$

The objective of the indicator-vector-guided factor rotation is to achieve maximum agreement between the estimated SPV's and the spatial patterns specified by *a priori* knowledge. This means that, corresponding to the specified elements, $\tau_{i^*m_j}$, of a given $\boldsymbol{\tau}_{m_j}$ in **T,** (i.e., $\tau_{i^*m_j}$ =0, $i^* \in I_{m_j}$, $I_{m_j} \subset \{1,2,...,p\}$ and $m_j \in \{1,2,...,M\}$), the elements, $l_{i^*j}^*$, of the rotated loading vector, $\mathbf{l}_j^*$, should also have values that are close to zero. This can be evaluated by the sum of squares of the elements corresponding to the specified elements in $\boldsymbol{\tau}_{m_j}$,

$$ag_j(m_j) = \sum_{i^* \in I_{m_j}} (l_{i^*j}^*)^2, \; j = 1, 2, \ldots, s, \text{ and } m_j \in \{1, 2, ..., M\}, \tag{11}$$

where $ag_j(m_j)$ is the agreement index of loading vector j with respect to indicator vector $\boldsymbol{\tau}_{m_j}$. In order to achieve best agreement, $ag_j(m_j)$ should be minimized. In other words, the factor rotation can be formulated as a problem to find a rotation vector $\mathbf{r}_j$ to minimize Eq. (11), subject to the sum of squares of all elements being held constant. This is equivalent to maximizing the sum of squares of the unspecified elements [23]. Given an indicator vector, $\boldsymbol{\tau}_{m_j}$, $\mathbf{L}_{m_j}^r$ denotes a restricted loading matrix that can be achieved by replacing all the elements in the $i^{*\text{th}}$ row of **L** with zeros (i = 1,2,…, p) whenever $\tau_{i^*m_j}$ =0 and keeping all the rows corresponding to unspecified elements in $\boldsymbol{\tau}_{m_j}$ unchanged. Consider vector $\mathbf{l}_j^r(m_j) = \mathbf{L}_{m_j}^r \mathbf{r}_j(m_j)$, $\mathbf{l}_j^r(m_j)$ and $\mathbf{r}_j(m_j)$ are the restricted rotated loading vector and rotation vector corresponding to $\boldsymbol{\tau}_{m_j}$, respectively. $\mathbf{l}_j^r(m_j)$ can be treated as the result of replacing all the specified elements in $\mathbf{l}_j^*$ with zeros. Thus, the sum of squares of the unspecified elements can be calculated accordingly and the factor rotation can be formulated as an optimization problem, i.e., for a given $\boldsymbol{\tau}_{m_j}$, $m_j \in \{1,2,...,M\}$,

$$\begin{aligned} &\max_{\mathbf{r}_j(m_j)} \quad \left(\mathbf{L}^r_{m_j}\mathbf{r}_j(m_j)\right)^{\mathrm{T}}\left(\mathbf{L}^r_{m_j}\mathbf{r}_j(m_j)\right) \\ &\text{s.t.} \quad \mathbf{l}_j^{*\mathrm{T}}\mathbf{l}_j^* = \mathbf{l}_j^{\mathrm{T}}\mathbf{l}_j. \end{aligned} \tag{12}$$

Lagrange multiplier method for solving (12) indicates that the optimal rotation vector $\mathbf{r}_j(m_j)$ is the eigenvector associated with the largest eigenvalue of matrix $\mathbf{H}_j(m_j)$, where

$$\mathbf{H}_j(m_j) = \left(\mathbf{L}^{\mathrm{T}}\mathbf{L}\right)^{-1}\left(\mathbf{L}^r_{m_j}{}^{\mathrm{T}}\mathbf{L}^r_{m_j}\right). \tag{13}$$

Based on this relationship, for a given $\boldsymbol{\tau}_{m_j}$, the rotated loading vector that achieves maximum agreement can be defined as

$$\mathbf{l}_j^*(m_j) = \mathbf{L}\mathbf{r}_j(m_j),\ \ j = 1,2,...,s, \tag{14}$$

and the associated agreement coefficient, $ag_j(m_j)$, can be calculated.

3.4 Procedure of multiple variation sources identification

The indicator-vector-guided factor rotation defines the derivation procedure that determines the rotation vector to achieve a single rotated loading vector that maximizes the agreement with the given $\boldsymbol{\tau}_{m_j}$. Corresponding to the s variation sources that are present in an MMP, the s rotation vectors $\mathbf{r}_j(m_j)$ can be calculated successively for j = 1, 2,…, s. The s indicator vectors, $\boldsymbol{\tau}_{m_j}$, are selected from $\mathbf{T}_j$, where $\mathbf{T}_j$ is the indicator matrix containing the indicator vectors for rotating $\mathbf{l}_j^*$. This means that m_j is also a decision variable. Rotation vectors $\mathbf{r}_j(m_j)$ ($m_j \in \{1,2,...,M\}$) are determined and their agreement coefficients $ag_j(m_j)$ will be calculated for different $\boldsymbol{\tau}_{m_j}$. For each rotated loading vector, $\mathbf{l}_j^*$, j=1,2,…, s, the index m_j^* that minimizes agreement coefficient is determined by

$$m_j^* = \underset{m_j}{\arg\ \min} \left[ag_j(m_j) \right]. \tag{15}$$

Let h be a predefined threshold value that reflects the desired agreement of the rotated loading vectors with respect to the indicator vectors. If $ag_j(m_j^*) < h$, it indicates that the $m_j^{*\ \mathrm{th}}$ process variation source is identified. Otherwise, it indicates that an unknown variation source is present and its SPV is not pre-specified in **T**. According to Eq. (6), $\mathbf{l}_j^*$ reflects the combined impact of the variation sources and the measurement noise. For the elements (of $\mathbf{l}_j^*$) corresponding to the specified elements (0's) of indicator vectors, their magnitudes should be in the same scale of measurement noise. Thus, the threshold value h can be determined as, $\bar{\sigma} < h < p\,\bar{\sigma}$, where $\bar{\sigma}$ is the average of the standard deviations of measurement noises, and p is the number of KPC's.

After identifying the j^{th} process variation source, its indicator vectors will be removed from $\mathbf{T}_j$. Thus, m_{j+1}^* can only be chosen from the reduced pattern matrix, $\mathbf{T}_{j+1}$, where $\mathbf{T}_{j+1} = \left\{ \boldsymbol{\tau}_k \mid k \neq m_1, m_2, \ldots, m_j \right\}$. For each m_j^*, j = 1,2,…,s, the optimal rotation vectors, $\mathbf{r}_j(m_j^*)$, are determined and used to form rotation matrix $\widehat{\mathbf{R}}$, where $\widehat{\mathbf{R}} = \left[\mathbf{r}_1(m_1^*) \quad \mathbf{r}_2(m_2^*) \quad \ldots \quad \mathbf{r}_s(m_s^*) \right]$. In practice, it is convenient to rescale the columns of $\widehat{\mathbf{R}}$ to make the rotation vectors have unit variances. This rescaling is accomplished by

$$\mathbf{R} = \widehat{\mathbf{R}}\mathbf{D}, \tag{16}$$

where $\mathbf{D}$ is a diagonal matrix with positive diagonal elements, defined by $\mathbf{D}^2 = diag\left[\left(\widehat{\mathbf{R}}^{\mathrm{T}} \widehat{\mathbf{R}} \right)^{-1} \right]$, and $diag(\mathbf{X})$ denotes the diagonal part of matrix $\mathbf{X}$. Therefore, based on Eq. (8) through Eq. (16), rotated factor loading vectors of $\mathbf{L}^*$ give the best estimations of the true SPV's of the variation sources.

The procedure for process variation sources identification is illustrated in Figure 7. The four major steps are summarized as follows:

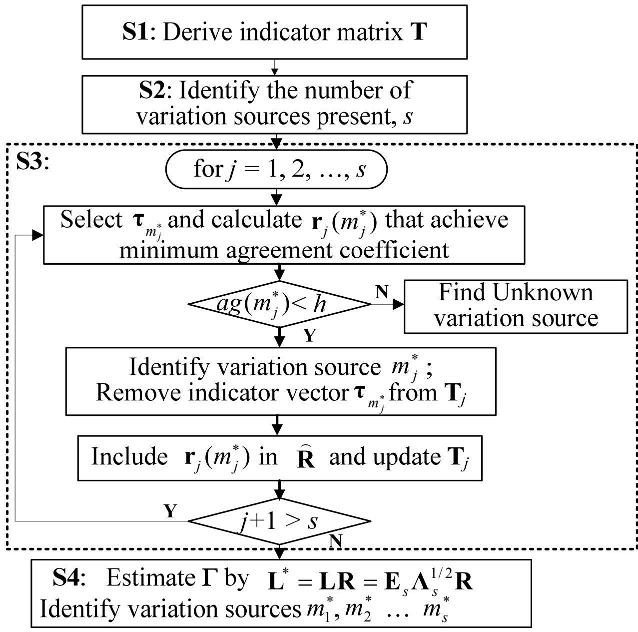


Figure 7 Procedure of multiple variation sources identification for an MMP

S1: An indicator matrix, **T**, is obtained from the engineering domain knowledge to represent the relationships between the potential variation sources and KPC's.

S2: The number of variation sources that present in an MMP is determined by using the eigen-decomposition of $\boldsymbol{\Sigma}_{\mathbf{y}}$ and AIC or MDL criteria.

S3: All the s initial estimates of SPV's are sequentially rotated based on indicator matrix, $\mathbf{T}_j$, $j = 1,2,\ldots,s$. The rotation results indicate that either the known variation sources are identified or unknown sources are found. Index m_j^* and rotation vectors $r_j(m_j^*)$, $j = 1,2, \ldots, s$, are recorded and in indicator matrix $\mathbf{T}_j$ is updated.

S4: Process variation sources $m_1^*, m_2^*, \ldots, m_s^*$ are identified and their true SPV's, vectors in $\mathbf{L}^*$, are estimated based on normalized rotation matrix **R**.

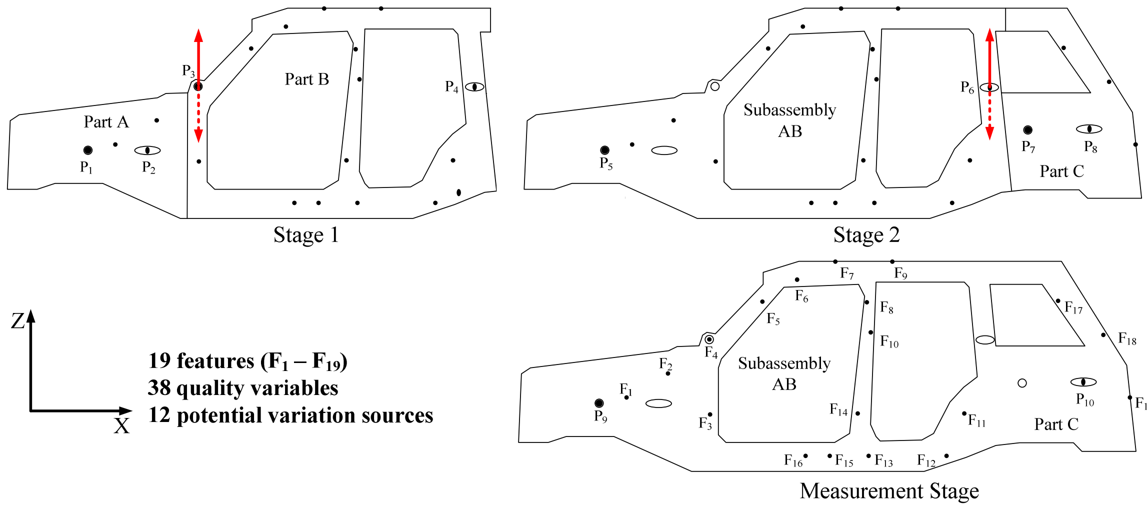


Figure 8 A three-stage assembly processes

4 Case study

A case study of a three-stage assembly process, as shown in Figure 8, is conducted to demonstrate the effectiveness of the proposed methodology. In this process, three panel parts are assembled in the first two stages to form an automotive side aperture. As summarized in Table 2, in each stage, the parts and/or subassembly are fixed by fixtures with a 4-way pin (e.g., P_1) and a 2-way pin (e.g., P_2). In the third stage, nineteen (19) KPC features, F_1 through F_{19}, are measured to monitor the dimensional quality of the final side-aperture ABC. These features are measured in terms of X-direction and Z-direction deviations from their nominal positions. Thus, there are totally 38 KPC's considered.

Table 2. Summary of the three-stage assembly process

Stage	Fixture	Operations
1	Fix part A by $FX_{1:1}$ (P_1 and P_2) Fix part B by $FX_{1:2}$ (P_3 and P_4)	Assemble part A and part B
2	Fix subassembly AB by $FX_{2:1}$ (P_5 and P_6) Fix part C by $FX_{2:2}$ (P_7 and P_8)	Assemble subassembly AB and part C
3	Fix side aperture ABC by $FX_{3:1}$ (P_9 and P_{10})	Measure 19 features (F_1~F_{19}) on side aperture ABC

As a preparation of variation source identification, graphical description of this 3-stage assembly process is developed, as shown in Figure 9. The nineteen KPC features in different stages are denoted in different layers in the Feature View. In the Device View, every fixture contains three elements, where f_1 and f_2, represent the potential X-direction and Z-direction variation sources of 4-way pins, respectively, and f_3 represents the potential Z-direction variation source of 2-way pins. The connections link the potential variation sources to some KPC's, indicating the product/process interactions. For instance, Figure 9 shows that if the potential variation source f_2 of $FX_{1:2}$ actually present in the process, all the features on part B ($F_{1:3}$ through $F_{1:16}$) will be affected in stage 1 and will deviate from their nominal positions. According to the process design, these random deviations will be propagated to stage 2 and reflected in the measurements. For simplicity, only the fixtures used in stage 1 and stage 2 are considered and thus there are 12 potential variation sources. Accordingly, a 38×12 qualitative indicator matrix **T** for the 12 potential variation sources is established. For instance, the indicator vector for f_2 of $FX_{1:2}$ is $\boldsymbol{\tau}_{FX_{1:2\_f_2}} = [\mathbf{0}_4^{\mathrm{T}} \quad \mathbf{1}_{28}^{\mathrm{T}} \quad \mathbf{0}_6^{\mathrm{T}}]$, where $\mathbf{0}_m$ is a m×1 column vector with all elements equal to 0, and $\mathbf{1}_n$ is a n×1 column vector with all element equal to 1.

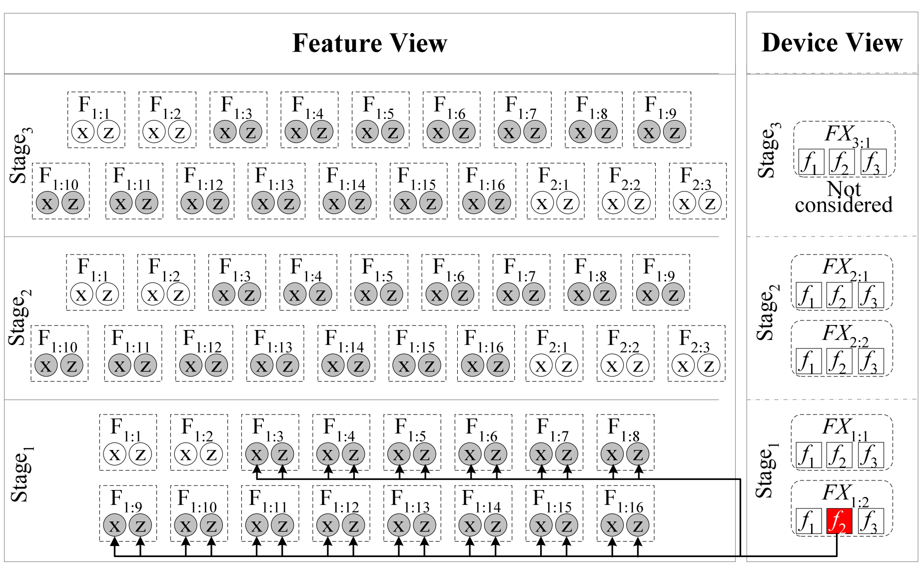


Figure 9 Graphical description of 3-stage assembly process

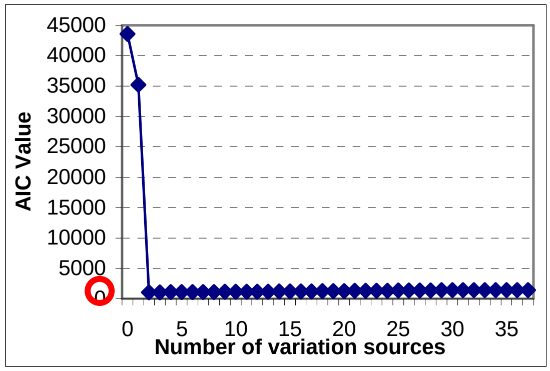
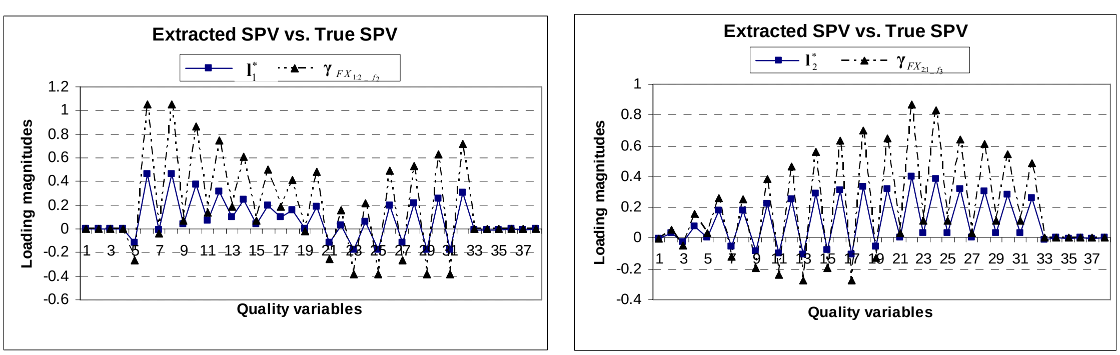


Figure 10 Determining the number of variation sources according to AIC

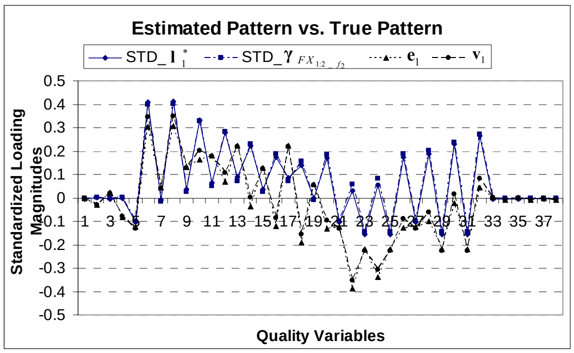
Monte Carlo simulation is performed to generate measurement data based on the linear model (1), where the matrix $\mathbf{\Gamma}$ is derived by state space model introduced in [5]. The $\mathbf{\Gamma}$ matrix contains 12 true SPV's, denoted as $\boldsymbol{\gamma}_{FX_{k:j\_f_t}}$, corresponding to the 12 potential variation sources of f_t of fixture j used in stage k. The input vector $\mathbf{u}$ follows 12-variate normal distribution, i.e., $\mathbf{u}\sim N(\mathbf{0}_{12},\mathbf{\Sigma_u})$. In this case, f_2 of $FX_{1:2}$ (i.e., P_3, Z-direction, denoted as $FX_{1:2\_f_2}$) and f_3 of $FX_{2:1}$ (i.e., P_6, Z-direction, denoted as $FX_{2:1\_f_3}$) are simulated as the variation sources, as shown in Figure 8. Thus, the 5th and the 9th diagonal elements of $\mathbf{\Sigma_u}$ are 0.6 and 0.4, respectively, whereas the other diagonal elements of $\mathbf{\Sigma_u}$ are set to be 0.01. The measurement noise vector $\mathbf{v}$ follows 38-variate normal distribution, i.e., $\mathbf{v}\sim N(\mathbf{0}_{38},\mathbf{\Sigma_v})$, where $\mathbf{\Sigma_v}$ is a 38×38 diagonal matrix with its diagonal elements ranging from 0.001 to 0.01. There are 150 samples simulated to generate KPC measurements.



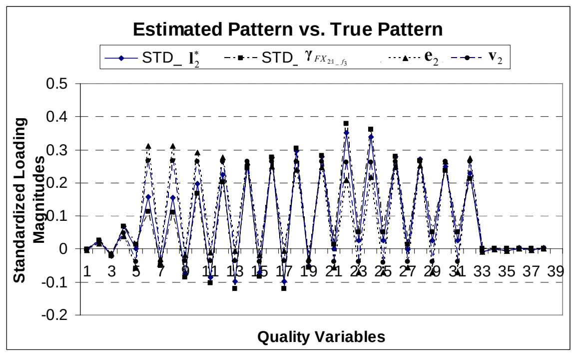
(a) (b)

Figure 11 Comparison of the rotated loading vectors with the true SPV

Following the procedure proposed in Section 3, the engineering-driven factor analysis was conducted. AIC value indicates that there are s=2 variation sources present, as shown in Figure 10. According to the agreement coefficients, the two aforementioned variation sources are identified. The two rotated factor loadings, $\mathbf{l}_1^*$, and $\mathbf{l}_2^*$, are compared with $\boldsymbol{\gamma}_{FX_{1:2}\_f_2}$ and $\boldsymbol{\gamma}_{FX_{2:1}\_f_3}$, respectively, in Figure 11.



(a)



(b)

Figure 12 Comparison of standardized SPV's

Figure 12 shows the comparison of the standardized rotated loading vectors and the standardized true SPV's. This visual comparison shows that the rotated factor loading vectors agree with the true SPV's very well. Also shown in Figure 12 are the standardized SPV's estimated with methods in [20] ($\mathbf{v}_1$ and $\mathbf{v}_2$) and in [17] ($\mathbf{e}_1$ and $\mathbf{e}_2$). There are significant discrepancies between $\mathbf{e}_1$, $\mathbf{v}_1$ and $\boldsymbol{\gamma}_{FX_{1:2}\_f_2}$, and between $\mathbf{e}_2$, $\mathbf{v}_2$ and $\boldsymbol{\gamma}_{FX_{2:1}\_f_3}$. This is because the angle between $\boldsymbol{\gamma}_{FX_{1:2}\_f_2}$ and $\boldsymbol{\gamma}_{FX_{2:1}\_f_3}$ is 51.58°, the orthogonal factor loading rotation cannot give an acceptable estimation of the true SPV's.

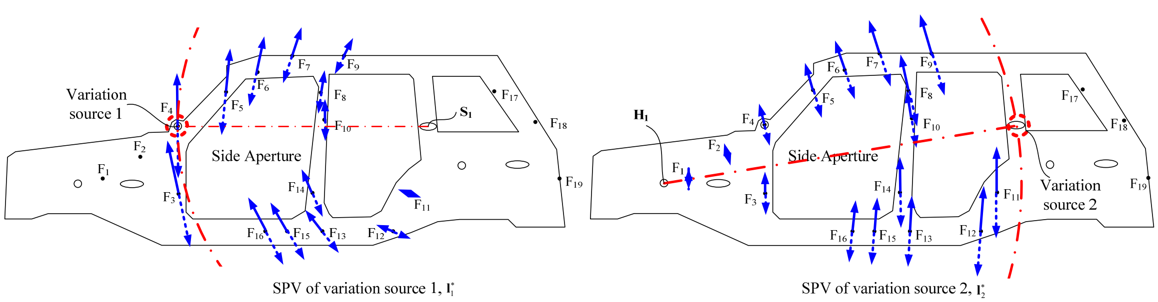


Figure 13 Visualization of estimated SPV's

The estimated SPV's, $\mathbf{l}_1^*$, and $\mathbf{l}_2^*$, are visualized in Figure 13. The SPV of variation source 1, $\mathbf{l}_1^*$, shows that all the features on part B, F_3~F_{16}, deviate from their nominal positions along a circle centered at the slot S_1. This SPV indicates that P_3 used in stage 1 has an abnormally large variation along Z direction. The SPV of variation source 2, $\mathbf{l}_2^*$, shows that all the features on subassembly AB, F_1~F_{16} , deviate from their nominal positions along a circle centered at the hole H_1. This SPV indicates that P_6 used in stage 2 has an abnormally large variation along Z direction. In practice, although their true SPV's are unknown, the variation sources can still be identified by visualizing and interpreting the geometric implications of the estimated SPV's.

A mathematical way to justify the agreement between loading vectors with true SPV is to calculate the angle between these two vectors, when the true SPV's in $\mathbf{\Gamma}$ are known. The smaller the angle, the better the two vectors match. Ding *et al*. [13] and Li *et al.* [25] used a method to define the boundary of the perturbation caused by measurement noise. This boundary is represented as an angle of the cone surrounding an SPV and can be calculated according to the eigenvalues associated with the SPV's, and eigenvalues of $\mathbf{\Sigma_v}$. The maximum and minimum eigenvalues of the matrix $\mathbf{A}$ are denoted as $\lambda_{max}(\mathbf{A})$ and $\lambda_{min}(\mathbf{A})$, respectively. According to the simulation conducted in the case study, $\lambda_{max}(\mathbf{\Sigma_v})$ = 0.01, $\lambda_{max}(\mathbf{\Sigma_v})/ \lambda_{min}(\mathbf{\Sigma_v})$ is 10, and the ratio of eigenvalues associated with SPV's over $\lambda_{max}(\mathbf{\Sigma_v})$ is approximately equal to 20.350. Based on the equation (7) in [25], the boundary angle will be approximately equal to 11.28°. According to the standardized rotated loading vectors and true SPV's, the angle between $\mathbf{l}_1^*$ and $\boldsymbol{\gamma}_{FX_{12}\_f2}$ is 4.05°, and that between $\mathbf{l}_2^*$ and $\boldsymbol{\gamma}_{FX_{21}\_f3}$ is 6.21°, which means the rotated loading vectors fall in the boundary of true SPV's and thus, identify variation sources and give a best estimation of the true SPV's. It should be emphasized that these diagnosis results are obtained following the method proposed in this paper, which does not require the accurate state space model. In other word, the proposed method is more robust to unknown process adjustments or tooling worn out and has more appealing capability in variation sources identification.

Although, for the sake of simplicity, a three-stage assembly process is used to demonstrate the effectiveness in identifying multiple variation sources in MMP's, the proposed methodology is applicable for more complex MMP's with more stages. For such a complex process, the graphical description of the MMP will contain more layers for more stages. In each layer, more feature nodes and fixture nodes may be considered. Their interactions will still be denoted as links connecting nodes in different views and different layers, as shown in Figure 5. More qualitative indicator vectors will be derived from the graphical knowledge presentation, corresponding to more potential variation sources that may present in the process. The dimension of the measurements of KPC's, $\mathbf{y}$, may also increase. However, these changes will not affect the effectiveness of the proposed methodology. This is because that, the two necessary assumptions for indicator vectors are more likely to be satisfied, as the number of stages increases. For instance, if the process is properly designed, the random deviation of a locating pin used in stage 10 will less likely affect the features generated in stage 1. The efficiency of the proposed methodology will not be affected dramatically either. According to the procedure illustrated in Figure 7, if there are s variation sources that present in an MMP that contains M potential variation sources, $sM - \sum_{q=1}^{s}(q-1)$ iterations are needed to identify them. Whereas the method introduced in [20] considers all those s eigenvectors together to form a fault space and $\begin{pmatrix} s \\ M \end{pmatrix}$ iterations to finish the variation sources identification. This will substantially increase the computational load.

5 Conclusion

The identification of process variation sources in an MMP demands the integration of engineering domain knowledge with appropriate multivariate statistical analysis. This paper presents a methodology to implement this integration in estimating the true SPV's of the variation sources, without complex quantitative modeling of the interactions between process variation sources and KPC's. Instead, the engineering knowledge about those relationships is represented as a qualitative indicator matrix. The key element of this methodology is the indicator vectors defined based on product/process knowledge to guide the factor rotation, which significantly improves the diagnostic interpretability of factor loadings and thus ensures the applicability of FA in variation sources identification. A procedure based on this factor rotation technique is developed for diagnosing MMP's by identifying multiple process variation sources, whose SPV's are non-orthogonal. Although the effectiveness of the proposed methodology is demonstrated through a case study of dimensional variation sources identification in a manufacturing process, the method can also be used in other applications where the measurements of observable variables can be linearly linked with a set of latent variables, as defined in Eq. (1).

In order to implement the proposed methodology in a complex MMP, various resources are necessary. For instance, software is needed to aids engineers in converting product/process design information into graphical and qualitative representations. The estimation of SPV's also needs substantial amount of multivariate data. Thus, advanced measurement system, such as in-line Optical Coordinate Measuring Machines, are desirable to create a data-rich environment to reduce the uncertainty caused by the random samples. The developed methodology could be sensitive to data uncertainty or missing data. More research needs to be done to investigate the impacts of sample uncertainty on the effectiveness of the proposed methodology and improve its robustness.

Acknowledgement

The authors gratefully acknowledge the financial support of the Engineering Research Centre for Reconfigurable Manufacturing Systems (NSF Grant EEC-9529125) at the University of Michigan.

Reference

[1] Ferrell, W.G. and Elmaghraby, S.E., 1990, "Quality assurance and stage dynamics in multi-stage manufacturing, Part I," International Journal of Production Research, **28**(5), pp. 853 - 877.

[2] Heikes, R.G. and Montgomery, D.C., 1981, "Productivity Is Enhanced with Statistical Quality-Control," Industrial Engineering, **13**(3), pp. 52-5, 75.

[3] Montgomery, D.C., 2004, *Introduction to Statistical Quality Control*, ed. 5. New York: John Wiley & Sons.

[4] Shi, J., 2006, *Stream of Variation Modeling and Analysis for Multistage Manufacturing Processes*. Boca Raton, FL: CRC Press, Taylor & Francis Group. 469.

[5] Jin, J. and Shi, J., 1999, "State Space Modeling of Sheet Metal Assembly for Dimensional Control," ASME J. Mfg. Sci. Eng., **121**, pp. 756-762.

[6] Mantripragada, R. and Whitney, D.E., 1998, "The datum flow chain: A systematic approach to assembly design and modeling," Research in Engineering Design-Theory Applications and Concurrent Engineering, **10**(3), pp. 150-165.

[7] Mantripragada, R. and Whitney, D.E., 1999, "Modeling and controlling variation propagation in mechanical assemblies using state transition models," IEEE Transaction on Robotics and Automation, **15**(1), pp. 124-140.

[8] Ding, Y., Ceglarek, D., and Shi, J., 2002, "Design Evaluation of Multi-station Assembly Processes by Using State Space Approach," ASME Transactions, Journal of Mechanical Design, **124**(3), pp. 408-418.

[9] Camelio, J., Hu, S.J., and Ceglarek, D., 2003, "Modeling Variation Propagation of Multi-Station Assembly Systems with Compliant Parts," ASME Transactions, Journal of Mechanical Design, **125**, pp. 673-681.

[10] Huang, Q., Zhou, N., and Shi, J. *Stream of Variation Modeling and Diagnosis of Multi-station Machining Processes*. in *ASME Int. Mech. Eng. Congress & Exposition - MED*. 2000. Orlando, FL.

[11] Djurdjanovic, D. and Ni, J., 2001, "Linear stat space modeling of dimensional machining errors," Transaction of NAMRI/SME, **29**, pp. 541-548.

[12] Zhou, S., Huang, Q., and Shi, J., 2003, "State space modeling of dimensional variation propagation in multistage machining process using differential motion vectors," IEEE Transactions on Robotics and Automation, **19**(2), pp. 296-309.

[13] Ding, Y., Ceglarek, D., and Shi, J., 2002, "Fault diagnosis of multistage manufacturing processes by using state space approach," ASME Transactions, Journal of Manufacturing Science and Engineering, **124**(2), pp. 313-322.

[14] Zhou, S., Chen, Y., and Shi, J., 2004, "Statistical estimation and testing for variation root-cause identification of multistage manufacturing processes," IEEE Transactions on Automation Science and Engineering, **1**(1), pp. 73-83.

[15] Wolbrecht, E., et al., 2000, "Monitoring and diagnosis of a multistage manufacturing process using Bayesian networks," Artificial Intelligence for Engineering Design, Analysis and Manufacturing, **14**(1), pp. 53-67.

[16] Ceglarek, D. and Shi, J., 1996, "Fixture Failure Diagnosis for Auto Body Assembly Using Pattern Recognition," ASME Transactions, Journal of Engineering for Industry, **118**, pp. 55-65.

[17] Liu, J., Li, J., and Shi, J., 2005, "Engineering Driven Cause-Effect Modeling and Statistical Analysis for Multi-Operational Machining Process Diagnosis," Transactions of NAMRI/SME, **33**(65-72).

[18] Apley, D.W. and Shi, J., 2001, "A Factor-Analysis Method for Diagnosing Variability in Multivariate Manufacturing Processes," Technometrics, **43**(1), pp. 84-95.

[19] Apley, D.W. and Lee, H.Y., 2003, "Identifying Spatial Variation Patterns in Multivariate Manufacturing Processes: A Blind Separation Approach," Technometrics, **45**(3), pp. 187-198.

[20] Jin, N. and Zhou, S., 2006, "Signature construction and matching for fault diagnosis in manufacturing processes through fault space analysis," IIE Transactions, Quality and Reliability Engineering, **38**, pp. 341-354.

[21] Liu, Y.G. and Hu, S.J., 2005, "Assembly fixture fault diagnosis using designated component analysis," ASME J. Mfg. Sci. Eng., **127**(2), pp. 358-368.

[22] Camelio, J.A. and Hu, S.J., 2004, "Multiple fault diagnosis for sheet metal fixtures using designated component analysis," ASME J.Mfg. Sci. Eng., **126**(1), pp. 91-97.

[23] Lawley, D.N. and Maxwell, A.E., 1971, *Factor Analysis as a Statistical Method* London: Butterworths.

[24] Darton, R.A., 1980, "Rotation in Factor Analysis," The Statistician, **129**(3), pp. 167-194.

[25] Li, Z., Zhou, S., and Ding, Y., 2007, "Pattern Matching for Root Cause Identification of Manufacturing Processes with Consideration of General Structured Noise," IIE Transactions on Quality and Reliability Engineering, **39**, pp. 251-263.