ROBUST FIXTURE LAYOUT DESIGN FOR A PRODUCT FAMILY ASSEMBLED IN A MULTISTAGES RECONFIGURABLE LINE

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ABSTRACT

Reconfigurable assembly systems enable a family of products to be assembled in a single system by adjusting and reconfiguring fixtures according to each product. The sharing of fixtures among different products impacts their robustness to fixture variation and process disturbances due to frequent reconfiguration. This paper proposes a methodology to achieve robustness of the fixture layout design through an optimal distribution of the locators in a multistage assembly system for a product family. This objective is accomplished by: (1) the use of a multistage assembly process model for the product family, and (2) minimizing the combined sensitivity of the products to fixture variation. The optimization considers the feasibility of the locator layout by taking into account the constraints imposed by the different products and the processes (assembly sequence, datum scheme and reconfigurable tools workspace). A case study where three products are assembled in four stages is presented. The sensitivity of the optimal layout was benchmarked against the ones obtained using dedicated assembly lines for each product. This comparison demonstrates that the proposed approach does not significantly sacrifice robustness while allowing the assembly of three products in a single reconfigurable line.

Key Words: Fixture design, Product family, Reconfigurable systems, Multistage assembly
1. INTRODUCTION

Traditionally mass production of complex products has been done using dedicated manufacturing systems. Such systems are characterized by high productivity and low flexibility, which work well for a relatively static market. However, today’s market features rapid changes in demand and short product lifecycle. Those changes have obliged manufacturers to increase product variety and reduce lot size. Therefore, manufacturers are continuously developing new products and production systems. The development of product families has helped manufacturers to meet customer requirements in terms of variety. An example of a product family is presented in Fig.1, where three car models of different sizes form the family. The use of reconfigurable manufacturing systems and controls has given manufacturers the possibility to cost effectively produce the family of products through systematic reconfigurations.

![FIG. 1 A PRODUCT FAMILY CONSISTING OF SEDANS OF SMALL, MEDIUM AND LARGE SIZES.](image.png)

In the automotive industry, the body assembly process is the less flexible one and has been receiving a lot of attention nowadays in pursuing flexibility. The auto body is usually assembled in a multistage sequential process (up to 70 stages), where on each stage fixtures are used to locate and clamp the parts for welding and joining. These fixtures play a critical role in
controlling the position of the parts and subassemblies on each stage, and on the final product quality. Traditionally, fixtures are dedicated to one product type thus limiting the possibility to reuse them for other products. Since fabricating assembly systems for each product type in the family can be very expensive, there is a necessity for fixture flexibility to allow the assembly of a product family in a single line.

Reconfigurable assembly systems using flexible fixtures allow the assembly of different products in a single assembly line by sharing process tools. An example of such flexible fixture is the FANUC robot F-200iB [1], which can hold different part-types in automobile body assembly lines. Such robots are often called Programmable Tools (PT). As the product changes from one type to another, the robots change their positions as needed by the new part geometry, thus allowing the assembly of different product types in the same production line. The disadvantages of such systems are that assembling multiple products in a single reconfigurable line imposes additional constraints on product design, and the frequent change-over between products is an additional source of process variation, which impacts the final product quality.

Product quality is usually characterized by the fulfillment of customer’s specifications and product functionality. In the auto industry, the parameters that determine product quality are known as the Key Product Characteristics (KPC). The KPCs are, in general, quantitative features of the product such as relative position of parts, flushes and gaps. Fixtures have a key role in determining the position of the parts, and doing so, on the achievement of the KPC specifications. For this reason the fixtures form part of the Key Control Characteristics (KCC) of the process [2]. FIG. 2 represents a part (a rectangular sheet) mounted on a 3-2-1 fixture formed by three NC blocks. Two of the blocks have pins that help to restrict the in-plane motion of the part. The pins locate the part through their fitting into a hole and a slot on the part. The three
blocks also position and restrain the part in the direction normal to the plane. The 3-2-1 locating points are known as Principal Locating Points (PLP). The positions of the PLPs and their interaction with the fixture play an important role on the quality of the product (e.g. the position of the KPC points $M_1$ and $M_2$ in Fig. 2).

When dedicated fixtures are used for each product at each stage, it is possible to optimize the location of the PLPs in terms of robustness to fixture variation. However, when multiple products of the same family are assembled in a single line, the products have to share fixtures. Sharing fixtures may result in the distribution of the fixtures-PLPs not being optimal for each individual product. Therefore, it is important to determine a robust distribution of the PLPs for the product family considering fixture sharing.

This paper presents a methodology to design robust fixture layouts for a product family assembled in a single line using reconfigurable fixtures, involving rigid parts. The requirements to solve such a problem are:

1) To obtain an expression that relates the PLP layout (design variables) to the final product variation (i.e., a variation propagation model), applicable to all products in the family.
2) To define the search space for the location of the PLPs and the constraints mathematically. In the case of the product family, the constraints for the solution not only include product-parts geometry, but also consider the sharing of fixtures and the workspace of the reconfigurable fixtures.

3) To minimize the effect that fixture variation has on product variation without violating the constraints, using an appropriate optimization method.

The remainder of the paper is organized as follows: Section 2 reviews the state of the art in multistage assembly variation propagation models, fixture design and reconfigurable fixturing systems. Section 3 addresses the design problem of determining the optimal distribution of the PLPs for a product family. A case study is presented in Section 4, with the conclusions given in Section 5.

2. LITERATURE REVIEW

The literature review covers the following three areas related to the proposed research: multistage assembly models, fixture design and reconfigurable fixturing systems.

2.1 Multistage assembly modeling

To establish relations between part and process variation and the final product quality in a multistage assembly process, it is necessary to have a model of the process. Such a model was first developed for auto body assembly at the stage level [3]. The modeling of a multistage assembly process was first attempted by Shiu et al [4], where a kinematics-base model of the process was developed. One of the main contributions was the identification of the “relocation” effect that occurs in multistage assembly processes. This effect occurred when subassemblies are
located again in downstream stages where the PLPs may not be the same as in prior stages. Figure 3 illustrates the effects of fixture deviation and the relocation, where Fig 3(a) presents the effect that a displacement of the 2-way pin (P2) has on the part, and especially in the location of points M1 and M2. Figure 3(b) shows the relocation effect on a subassembly as it moves from stage k to stage k+1. Then, variation in stage k is transmitted to stage k+1 due to relocation. The existence of the relocation is the major difference between the single stage and the multistage variation modeling.

**FIG. 3 EFFECT OF THE FIXTURE DEVIATION AND RELLOCATION [2]**

A formal representation of the multistage assembly process was developed by Jin and Shi [5]. They developed a state space representation of the assembly process to determine the final product variation given the variation of the incoming parts and fixtures for the case of 2-D rigid parts. Another multistage modeling method was proposed by Mantripragada and Whitney [6]. They used the state transition model to predict the variation propagation and to perform assembly corrections. Since the variation propagation model is fundamental to establishing the relation between KCCs and the KPCs, the state space model is described next.
A schematic of a multistage assembly process is presented in Fig. 4. Observing this figure, it is possible to understand how the subassemblies are transferred from one stage to another, accumulating variation along the process. The variation accumulated up to stage $k$ is represented by the variable $x_k$ in Eq. (1). This variable depends on the deviation accumulated up to stage $k-1$ plus the deviation of the fixtures $u_k$ and other un-modeled deviation or disturbances sources $w_k$. The relocation effect of the subassembly coming from stage $k-1$ in stage $k$ is represented by matrix $A_{k-1}$. This matrix relates the fixture layout of two adjacent stages and determines the re-positioning necessary for the subassembly entering stage $k$ (see Fig. 3(b)). The impact of fixture deviations in stage $k$ is determined by matrix $B_k$. On the other hand, the measurements or outputs at stage $k$, $y_k$, depend on the position of the selected measurement points for the assembly (normally they correspond to the KPC of the assembly). The relation between the variation of the part and the measurement points is given by matrix $C_k$. Usually the measurements are not perfect and they are corrupted by noise represented by $v_k$. Jin and Shi [5] and Ding et al. [7] provide detailed derivation of the matrices.

The complete state space representation of the dimensional relationships is given below

\[
\begin{align*}
x_k &= A_{k-1} \cdot x_{k-1} + B_k \cdot u_k + w_k \\
y_k &= C_k \cdot x_k + v_k
\end{align*}
\]  

(1)
Based on the linear properties of the model, it is possible to write the deviation of the measurement points in the last stage $N$ as,

$$
y_N = \sum_{k=1}^{N} C_N \cdot \Phi_{N,k} \cdot B_k \cdot u_k + C_N \cdot \Phi_{N,0} \cdot x_0 + \sum_{k=1}^{N} C_N \cdot \Phi_{N,k} \cdot w_k + v_N
$$

(2)

where $\Phi$ is the state transition matrix and can be calculated as

$$
\Phi_{k,i} \equiv A_{k-1} \cdot A_{k-2} \cdot A_{k-3} \cdot \ldots \cdot A_i
$$

$$
\Phi_{i,i} \equiv I
$$

(3)

Equation (2) can be simplified to:

$$
y_N = \sum_{k=1}^{N} \Gamma_k \cdot u_k + \Gamma_0 \cdot x_0 + \sum_{k=1}^{N} \Psi_k \cdot w_k + v_N
$$

(4)

where

$$
\Gamma_k = C_N \cdot \Phi_{N,k} \cdot B_k, \quad \Gamma_0 = C_N \cdot \Phi_{N,0} \quad \text{and}
$$

$$
\Psi_k = C_N \cdot \Phi_{N,k}
$$

(5)

Since the process analyzed involves a serial assembly line with only one station per stage, the words stage and station are used interchangeably in the remaining of the paper.

2.2 Fixture Design

Early design of fixtures did not consider the existence of external variation sources [8-9]. Later, researchers considered the existence of errors in fixtures and/or parts. In this area the research is divided two categories based on whether the workpiece is considered rigid or compliant. In both categories, the common approach is to determine the position of the locators
and clamps that ensures a correct location of the workpiece and minimizes the effect of external variation sources.

In the case of rigid parts, the research has been focused on robust layout design of fixtures and clamps. Cai et al, [10] proposed a variational method for robust fixture configuration design of 3-D rigid parts. Wang and Pelinescu [11] developed an algorithm for fixture synthesis for 3-D workpiece by selecting the positions of the clamps from a collection of discrete candidate locations called point set.

In the design of fixture for compliant parts, Lee and Haynes [12] used finite element methods to model and analyze workpiece behavior including the effect of friction forces. Menassa and Devries [13] used optimization to assist in the evaluation and selection of the 3-2-1 fixtures and clamps for prismatic parts aiming to minimize workpiece deflection. Cai et al, [14] studied the use of more complex fixture scheme, the “N-2-1” fixture, to hold compliant parts by over-constrain the part, and used optimization to distribute the fixtures in order to reduce the part’s deformation. Camelio et al, [15] determined the optimal fixture location to hold sheet metal parts considering variation of fixtures and welding guns position, and the springback effect of the subassembly after it is released from the stage.

All the previous works are based on single stage synthesis of locator layout. The problem of distributing the PLPs in a multistage process is more challenging due to relocation. This problem was first addressed by Kim and Ding [16]. They determined the distribution of PLPs for rigid parts that is robust to fixture variation for a single product assembled in a multistage process. To do so, they develop a sensitivity index that relates PLP layout to final product variation (KPC) and used several optimization methods (e.g. sequential quadratic programming, simplex, basic exchanged, revised exchange and the modified Fedorov method) to determine the
distribution. Kim and Ding centered their effort on reducing the impact of fixture variation on the final product quality. Following this approach, Eq. (4) can be simplified as,

\[
y_N = \sum_{k=1}^{N} C_N \cdot \Phi_{N,k} \cdot B_k \cdot u_k = \sum_{k=1}^{N} \Gamma_k \cdot u_k = D \cdot u, \tag{6}
\]

where \( u \) is the stack up vector of all the fixture deviation, and matrix \( D \) is calculated as

\[
D = [\Gamma_1 \quad \Gamma_2 \quad \ldots \quad \Gamma_N]. \tag{7}
\]

In their model, Kim and Ding ignored the last term in matrix \( D \) (\( \Gamma_N \)) because it is the final measurement stage, which has fixtures with tighter tolerances and a better maintenance policy. Using those simplifications, they proposed the calculation of a sensitivity index that relates the deviation sum squares of the output measurements \( y^T y \) as presented in Eq. (7). The sub index \( N \) in Eq. (6) is now dropped for simplification.

\[
y^T \cdot y = u^T \cdot D^T \cdot D \cdot u \tag{8}
\]

Then, the input/output sensitivity \( S \) can be calculated as the ratio of the sum of output variance of the KPCs to input variance as presented in Eq. (9),

\[
S = \frac{\hat{y}^T \cdot \hat{y}}{u^T \cdot u} = \frac{u^T \cdot D^T \cdot D \cdot u}{u^T \cdot u}. \tag{9}
\]

When analyzing the sensitivity index, it is possible to observe that if product \( D^T D \) is “small”, then the effect of the fixture variation is minimized. This is precisely the objective of a robust locator layout: minimizing the impact that fixture variation has on the KPC. To achieve this goal there are several criteria, most of which have an origin in optimal design of experiments:

- A-optimality, which is to minimize the trace of \( D^T D \).

- D-optimality, which is to minimize the determinant of \( D^T D \).
- E-optimality, which is to minimize the extreme (maximum or minimum) eigenvalue of $D^T D$.

The A-optimality criterion is equivalent to minimizing the sum of all the eigenvalues and can be understood as minimizing the sum of all the sensitivities of the process. The D-optimality criterion corresponds to minimizing the multiplication of the eigenvalues. This criterion has been widely used in design of experiments due to its clear interpretation, which is the minimization of the uncertainty on the parameters estimated using least squares. However, this criterion cannot be used in fixture design because matrix $D$ is singular due to the singularity of the A’s matrices used to form it [17].

The E-optimality criterion is equivalent to minimizing the squared root of the 2-norm of $D$. In practice, this is equivalent to minimizing the worst possible deviation in the process, which is associated to the maximum eigenvalue of $D$. Using the E-optimality criterion, the optimization problem can be stated as determining the location of the locators $\varphi$ which minimizes the sensitivity and do not violate the constraints $G(\varphi)$, as presented in Eq. (10).

$$\min_{\varphi} S_{\max} \equiv \lambda_{\max} (D^T \cdot D)$$
subject to $G(\varphi) = 0$ \hspace{1cm} (10)

The constraints $G(\varphi)$ consider that the locators have to be located in the feasible region inside the parts.

To solve the optimization problem just presented in Eq. (10), Kim and Ding used several different methods, such as sequential quadratic programming, simplex, basic exchange, modified Fedorov and revised exchange. Since the problem in Eq. (10) is nonlinear and may have several local minima, the global optimality of the solution cannot be guaranteed.
2.3 Reconfigurable fixturing systems

There have been many attempts to use reconfigurable fixturing systems in manufacturing aiming to reduce cycle time, fixture costs and process variation [18]. The first automatically reconfigurable assembly fixture was developed by Asada and By [19]. They studied reconfigurable or adaptive fixture systems using a kinematical and mechanical approach. Since then, research has been done in the area such as assembly flexibility [20], and on quality by error compensation [21].

In machining, Walczyk and Longtin [22] studied the use of reconfigurable fixtures for compliant parts. They analyzed the performance of a reconfigurable system formed by a matrix of extendable pins used to locate a workpiece in terms of the forces applied and the system accuracy. More recently, Shen et al [23] developed a reconfigurable fixturing system that can be relocated in the pallet as different parts gets into the machining stage.

The aforementioned efforts were mainly focused on the design of reconfigurable fixture devices. However, they do not consider the layout of the fixture (e.g. distribution and selection of reconfigurable devices). The single station layout design for a family of products was studied by Lee et al, [18]. They investigated the use of reconfigurable equipment to fixture a family of sheet metal parts using the N-2-1 scheme. The problem addressed was to determine the feasible position of the fixturing robots in the station to ensure that all the parts can be processed. They also determined the minimal size of the required working spaces in order to use small robots by using genetic algorithms.

Table 1 summarizes the methodologies presented in this review section. Previous work in robust design and reconfigurable fixtures has been based on single machine (stage) level for a single or multiple products. On the other hand, the multistage approach has only been considered
for a single product. Therefore, there is a necessity to develop a methodology to design a robust reconfigurable fixture layout for a product family assembly in a single line.

### TABLE 1. COMPARISON OF MODELING & FIXTURE DESIGN METHODOLOGIES

<table>
<thead>
<tr>
<th></th>
<th>Single product</th>
<th>Multiple products</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multistage level</td>
<td>Modeling</td>
<td>Shiu et al 1996; Jin and Shi 1999; Mantripragada and Whitney 1999; Camelio et al 2004</td>
</tr>
<tr>
<td></td>
<td>Fixture design</td>
<td>Kim and Ding 2004</td>
</tr>
</tbody>
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3. **OPTIMAL LOCATOR LAYOUT FOR A PRODUCT FAMILY**

This section presents a methodology to solve the problem of distributing the locators for a product family in which the products share fixtures. This problem can be formulated as a constrained optimization problem, including the determination of the objective function, the definition of the constraints and the optimization method to search the solution.

**3.1 Objective function**

The criterion used to determine the optimal location of the PLPs for a product family is based on minimizing the multiple product version of the sensitivity index (Eq. (10)). In the case that the family, consisting of $R$ products or models, shares the same assembly line, the objective function is the weighted sum of the maximum eigenvalues of each model matrix set $\mathbf{D}^T \mathbf{D}$ as presented in Eq. (11).
The use of scalar weights \( w_i \) allows one to incorporate design information such as the relative importance that each product has on the family. Since the selection of the weights is arbitrary, a possible criterion is to consider the expected demand for each product (product with higher expected demand can have a higher weight). Exploring possible selections of the weights is not the scope of this work; therefore, it is assumed that the weights are all equal to one.

Again the term \( G(\phi^i) \) denotes the set of constraints that limit the location of the \( 2M \) PLP required to hold the \( M \) parts of product \( i \) in the X-Z plane. Only the locations of the pins are considered since the locations of the locating blocks do not impact the in-plane variation. The pin locations are denoted by \( \phi^i = [P_{1i}^i \ P_{2i}^i \ P_{3i}^i \ ... \ P_{2Mi}^i] \), where the odd sub index corresponds to holes (4-way pins) and the even corresponds to slots (2-way pins). Each pin \( (P^i_k) \) has 2 coordinates in the plane (see Fig. 2), one in \( X \) and one in \( Z \); therefore the design variables can be rewritten for part \( i \) as: \( \phi^i = [(X_{1i}^i \ Z_{1i}^i) \ (X_{2i}^i \ Z_{2i}^i) ... (X_{Mi}^i \ Z_{Mi}^i)] \).

Since the objective is to minimize the maximum eigenvalue of \( D^T_i D_i \), it is important to analyze the sensitivity of the eigenvalue calculation to modeling and computational errors. Model errors are caused by errors in the generation of the system matrices A’s, B’s and C’s. On the other hand, computational errors are inherent in calculations with floating point arithmetic [24]. In this research, both errors can be seen as perturbations of the true matrix product \( D^T_i D_i \). The eigenvalue condition quantifies the sensitivity of the eigenvalues to those types of perturbations [24].

\[
\begin{align*}
\text{Min}_{\phi} \quad & \quad S_{\text{max}} \equiv \sum_{i=1}^{R} w_i \cdot \lambda_{\text{max}}(D_i^T \cdot D_i) \\
\text{s.t.} \quad & \quad G(\phi^i) = 0 \quad \forall i \ 1 \ldots R
\end{align*}
\]
Now considering the symmetric matrix product $D^T_i D_i$ (the multiplication of a non-symmetric matrix by its transpose results in a symmetric matrix), it is possible to decompose it using the eigenvalue decomposition given by

$$D^T_i D_i = V_i \cdot \Lambda \cdot V_i^T$$  \hspace{1cm} (12)

where $V_i$ is the eigenvector matrix (left eigenvectors matrix), and $\Lambda_i$ is the eigenvalue matrix of $D^T_i D_i$.

An upper bound for the eigenvalue variation is presented in Eq. (13) [24-25].

$$\| \Delta \Lambda_i \|_2 \leq \left\| V_i^{-1} \right\|_2 \left\| V_i \right\|_2 \cdot \left\| \Delta D^T_i D_i \right\|_2 = \kappa(V_i) \cdot \left\| \Delta D^T_i D_i \right\|_2,$$  \hspace{1cm} (13)

where $\Delta \Lambda_i$ represents a variation of the eigenvalue matrix, $\Delta D^T_i D_i$ the variation or perturbation of $D^T_i D_i$ and $\kappa(V_i)$ is the condition number of matrix $V_i$.

The sensitivity as obtained in Eq. (10) is dependent on the matrix variation and the amplificatory factor $\kappa(V_i)$, which is greater or equal to one. A special property of symmetric matrices is that the conditioning number of their left eigenvector matrix is one [24-25]. Doing so, the sensitivity of the eigenvalues of $D^T_i D_i$ to the variation of the system matrices and computation errors is not amplified; it is directly proportional to their variations (best possible case). Therefore, the selected criterion, based on the minimizing the maximum eigenvalue (Eq. (10) and Eq. (11)) is robust to modeling and computation errors.

### 3.2 Constraints definition

The constraints define the feasible space where the PLPs can be located as well as the necessary conditions to ensure that the assembly is feasible. Thus, they define the viability of the assembly. Before describing the constraints for the product family design problem, it is necessary
to present some process conditions or considerations that make the problem addressed in this research closer to the reality; those are:

1) Each part has only one set of PLPs. This implies that in later stages each subassembly must be held using some of the previously used locating points available on the parts. The use of only one set of locators per part is a common practice in industry because it helps to maintain the cost of the parts low.

2) Each PT carries the set of fixture elements (blocks and pins) necessary to hold a part or subassembly. This condition avoids the use of multiple PTs to carry each part or subassembly, to save cost and space.

3) To avoid increasing the mechanical complexity and cost of the PT, it is considered that the distance between the pins installed on the PT is constant. This distance is a design variable, therefore, it has to be the same for all the products, and can not vary from product to product.

Considering the aforementioned conditions, it is possible to define the constraints for the product family as follows:

a) All the PLPs must be positioned within the feasible area of an individual part. This area includes all the part and excludes the internal holes on the part. A safety margin of 30 mm is defined along all the part contours (internal and external) to ensure that the locators are not too close to the edges. The verification of the belonging or not of a point to the feasible region of a part was done using an image-matrix of the geometric shape of every part. Then, a value of 0 was assigned to the “in” or feasible region and 1 to the outer or infeasible region (including cavities on the parts). Doing so, the verification of the in/out location of a point was done by checking if the
coordinates of the point correspond to 0 or 1 in the appropriate image (part). The advantages of this method are that it is simple to check, and the image has to be calculated only once, then stored and used every time it is required. The generation of the image requires information of the position of the vertex that defines the part (external and internal), and an algorithm to check if a point belongs or not to a certain region. There are many algorithms to perform this type of verification, one of those is the point inclusion test widely use in the CAD-CAM and the computational geometry field.

b) The distance between the locators on each part-type (p) and subassembly-type (s) should be the same for all models. This means that the distance between the two locators used to hold the same type of part or subassembly is fixed. However, the position of the pins in the stage can be adjusted using the PT to accommodate the different products. If the distance between the locators used to hold a given part-type or a subassembly-type are not the same for all the models, then one or more assemblies are not feasible because the parts or subassemblies do not fit into the fixtures. Fig. 5(a) presents graphically the constraint for the part-type (products A, B and C), and Fig. 5(b) presents the constraints for the subassembly-type (only products A and B are shown).
c) The PT has to be able to locate the fixture elements in the appropriate position; therefore, at least one point in between both pins has to belong to the workspace of the PT (e.g. the middle point between the pins). Graphically this can be presented in Fig. 6, where the locator’s middle points, represented by triangles, are inside the PT workspace. For the case where the workspace is circular; the radius of the minimum circle that contains all the middle points must be smaller than the workspace radius. The problem of determining the circle with minimum radius that contains a set of points is known as the minimum circle enclosing problem. This problem has been extensively studied in the computational geometry field; a good review of the available methods used to solve it can be founded in [26].

d) Another constraint that can be included is that the PLPs on each part have to be aligned along one of the principal axes of the part. This prevents the coupling of the errors in the three axes. Therefore, having the PLPs aligned with the principal axis of the part is a recommended practice. Mathematically, the constraints can be...
represented as the product of the differences in location of the hole and the slots in the X and Z directions, which has to be equal to zero to ensure the correct alignment.

![Robot workspace diagram](image)

**FIG. 6 WORKSPACE VERIFICATION**

### 3.3 Optimization and optimality

Due to the non-linear nature of the problem and the constraints, sequential quadratic programming was chosen to perform the optimization. This optimization method is frequently used for fixture design [14], [16] and [27]. One of the properties of the gradient-based method is that it tends to converge rapidly because it uses derivatives to look for a solution following the steepest descent path. Usually this method finishes searching for a minimum when the rate of change with respect to the previous iteration is less than a threshold, or when a given amount of function evaluations is reached. A disadvantage of this method is that it can be easily entrapped in a local optimum. Therefore, different initial conditions can be used to perform the search for a good locator layout.

Due to the complexity of the objective function and the constraints, solving the problem as proposed in Eq. (11) is difficult. On top of this, obtaining a feasible initial condition that satisfies all the constraints is also challenging. Therefore, the problem was solved first using the relaxed formulation (Lagrange relaxation), which is, in general, easier to solve compared with
the original one [28]. Eq. (12) presents the formulation, where the objective function directly includes the squares of the constraints multiplied by a constant factor $\beta$ (Lagrange multiplier).

$$J = \text{Min}_{\varphi} \sum_{i=1}^{R} w_i \cdot \lambda_{\text{max}} (D_i^T \cdot D_i) + \beta \cdot [G(\varphi')^T \cdot G(\varphi')]$$ (12)

The relaxed form of the problem has the advantage of allowing a slight violation of the constraints. Therefore, it can be used as a starting point for the solution of the constrained problem Eq. (11). The selection of the multiplier $\beta$ is done to ensure a reasonable solution (low value of $\beta$), and then it is increased to look for a solution that is closer to the one of the real problem. Finally the true problem Eq. (11) can be solved starting from the result of the one with the highest factor $\beta$.

4. CASE STUDY

The case study selected is the assembly of the side frame of the family of sedans presented in Fig. 1. The side frames are composed of four parts each and are assembled in the process pictured in Fig. 7. The process consists of three assembly stages and a final measurement stage, where the location of the KPCs defined for this process are measured (points M).

![FIG. 7 ASSEMBLY SEQUENCE OF THE SEDAN SIDE FRAME](image-url)
The datum scheme defined for this process is the following: in stage 1 the locators used are \{P_1, P_2, P_3, P_4\}, this means that the first part is held using locators P1 and P2, the second part using locators P3 and P4. In stage 2 the locators used are \{(P_1, P_4), (P_5, P_6)\}, in stage 3 \{(P_1, P_6), (P_7, P_8)\} and in the measurements stage \{(P_1, P_8)\}.

The PTs used in the assembly were assumed to be robots with three degrees of freedom in the plane as presented in Fig. 8(a), which corresponds to a Revolute-Revolute-Revolute type robot. Due to the robot characteristics, they have a circular working space as shown in Fig. 8(b). The radius of the work space was selected to be \(r = 500\) mm.

The relative sizes of the frames compared to the small one were selected as 1.06 and 1.12 for the medium and large frames respectively. The location of the KPC’s defined for the products are presented in Appendix I. The locations are reported for the small sedan, the ones for the medium and small can be obtained using the corresponding scale factors.

![PT side view](a) PT side view  
![PT top view and workspace](b) PT top view and workspace

**FIG. 8 VIEWS OF A PROGRAMMABLE TOOL AND ITS WORKSPACE**

4.1 Results

The results of the locator layout for a product family are presented next. The results are benchmarked with the optimal solutions obtained for each product as it were assembled in a dedicated assembly line (dedicated line for each product). This comparison provides information
of the performance compromised, in terms of robustness to fixture variation, by using a single reconfigurable line.

Due to the existence of several local minima, in accordance with the results obtained by Kim and Ding, 100 random initial conditions were used to search for a good layout of the PLPs for both the product family and the single products (case of dedicated lines). In the product family case the multiplier $\beta$ was first set to 5. Later, the layout with lower $J$ was optimized after increase $\beta$ to 50.

a) Fixture layout for a dedicated line

In the optimization for each single model, considering dedicated lines, two cases were analyzed. Case one has no constraint in the alignment of the locators and case two impose constraints on the alignments of the locators. In both cases the optimization was performed 100 times starting from random initial conditions of the locators for each model independently. Figure 9 presents the location of the fixtures for each model for the cases with and without locator’s alignment. Table 2 presents the values of $\lambda_{\text{max}}$ for each configuration. The table also includes the sum of the $\lambda_{\text{max}}$ for later comparison with the product family solution.

**TABLE 2. RESULTS OF THE OPTIMIZATION FOR EACH SINGLE MODEL**

<table>
<thead>
<tr>
<th></th>
<th>Dedicated lines</th>
<th>Dedicated lines (aligned pins)</th>
<th>Reconfigurable line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small car</td>
<td>18.04</td>
<td>20.28</td>
<td>24.06</td>
</tr>
<tr>
<td>Medium car</td>
<td>18.01</td>
<td>18.56</td>
<td>25.14</td>
</tr>
<tr>
<td>Large car</td>
<td>18.02</td>
<td>19.98</td>
<td>21.92</td>
</tr>
<tr>
<td><strong>Sum $\lambda$</strong></td>
<td><strong>54.07</strong></td>
<td><strong>58.70</strong></td>
<td><strong>71.12</strong></td>
</tr>
</tbody>
</table>
For the case of the aligned pins, the solution obtained in height (z) is close to the “center of gravity” of the sensor points in the same direction. Therefore, for this case where the pins have to be aligned, their locations tend to be equally distant to the “upper” and “lower” set of measurement points. In that way, the effect of fixture variation will be minimized in average. The final locations of the locators are reported in Appendix II.

b) Fixture layout for a reconfigurable line

Figure 10 presents the final location of the PLPs for the family, where the distances of the hole and the slot are the same across the three products. The values of the upper bound of sensitivity (\(\lambda_{\text{max}}\)) obtained for the each model and for the product family (summations of the \(\lambda_{\text{max}}\)) are presented in Table 2.

The difference between the value of \(\lambda_{\text{max}}\) for the reconfigurable line and the dedicated lines (non-aligned and aligned cases) are 17.05 and 12.42 respectively, which corresponds to an increment of a 31.5% and 21.1% for each case. Those increases can be judged as reasonable considering the complexity of the geometries, the amount of constraints that the single line imposes in the assembly and the differences in sizes of the cars. It is important to note that the value obtained through the use of \(\lambda_{\text{max}}\) corresponds to the upper bound on the sensitivity. Therefore, it corresponds to the worst case scenario. The increase in the sensitivity of the product family can be compensated through an appropriate distribution of the tolerances in the fixtures and locators and a good maintenance strategy that keeps the variation low. The final location of the locators is reported in Appendix III.
FIG. 9 OPTIMAL LOCATIONS OF THE PLPs FOR SINGLE PRODUCTS WITH AND WITHOUT THE ALIGNMENT CONSTRAINT
FIG. 10 LOCATION OF THE PLPs FOR THE PRODUCT FAMILY  
(Note that the distance between the hole and the slot remains the same for each part type across the three models)

No results are presented for the case of the product family with aligned pins since there is no feasible solution to that problem for the cases considered here.

5. CONCLUSIONS

This paper proposes a new approach for fixture configuration design for a family of products assembled in a single reconfigurable line. The problem is formulated as a constrained optimization by considering part geometry, fixture workspace and the alignment of the pins. Sequential quadratic programming was used to solve the optimization problem, and a relaxed formulation of the problem allowed searching for a robust layout. The resulting fixture layout using a reconfigurable line is compared with the case of single product dedicated lines in term of the quality of the solution. Two different scenarios were analyzed: no alignment restriction on the
PLPs, and the PLPs has to be aligned (in X or Z directions). The result obtained for the product family is feasible; however, the sensitivity is 31.5 % (worst case) higher than the one for dedicated lines. This increment does not imply that the product family assembly is in general worse than the single lines. Obviously there is a trade-off between the achievement of production flexibility by using a reconfigurable line, and the robustness of the system to fixture variation for the product family. Using separated PTs for the each pin will significantly improve the robustness; however, at a significant cost. An enterprise level evaluation of the pros and cons of both approaches (reconfigurable-dedicated) seems to be an appropriate method to decide which production scheme is better considering expected demands, product and process costs, flexibility and quality among other factors. It is the aim of this research to help that type of decision through the development of tools that help to perform such evaluation, and also help designers on the development of this type of assembly process.

ACKNOWLEDGMENTS

This work has been supported in part by the General Motors Collaborative Research Laboratory at the University of Michigan and the Korean Research Foundation (Grant KRF-2005-013-D00005). The authors would like to thank Hui Wang and Dr. Meng Li for their contributions and discussions.
APPENDIX I: LOCATION OF THE MEASUREMENT POINTS

<table>
<thead>
<tr>
<th>Measurement point</th>
<th>Position (x/z) in mm</th>
<th>Measurement point</th>
<th>Position (x/z) in mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>100 / 450</td>
<td>M11</td>
<td>1500 / 100</td>
</tr>
<tr>
<td>M2</td>
<td>1070 / 800</td>
<td>M12</td>
<td>2150 / 1260</td>
</tr>
<tr>
<td>M3</td>
<td>1100 / 300</td>
<td>M13</td>
<td>3550 / 700</td>
</tr>
<tr>
<td>M4</td>
<td>1100 / 600</td>
<td>M14</td>
<td>3350 / 250</td>
</tr>
<tr>
<td>M5</td>
<td>1360 / 940</td>
<td>M15</td>
<td>3300 / 100</td>
</tr>
<tr>
<td>M6</td>
<td>2000 / 1200</td>
<td>M16</td>
<td>2500 / 100</td>
</tr>
<tr>
<td>M7</td>
<td>2300 / 1200</td>
<td>M17</td>
<td>3800 / 685.8</td>
</tr>
<tr>
<td>M8</td>
<td>2200 / 1000</td>
<td>M18</td>
<td>4430 / 650</td>
</tr>
<tr>
<td>M9</td>
<td>2200 / 400</td>
<td>M19</td>
<td>4050 / 0</td>
</tr>
<tr>
<td>M10</td>
<td>2000 / 100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

APPENDIX II: OPTIMAL LOCATION OF THE LOCATORS  
(DEDICATED LINES)

<table>
<thead>
<tr>
<th>Non-Aligned PLPs (x/z) in mm</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>866.8/ 382.3</td>
<td>975.2 / 435</td>
<td>823 / 412.7</td>
</tr>
<tr>
<td>P2</td>
<td>94.5 / 321.8</td>
<td>53.5 / 156.7</td>
<td>210.2 / 60.8</td>
</tr>
<tr>
<td>P3</td>
<td>1103.6 / 410.9</td>
<td>1167 / 369.9</td>
<td>1241.9 / 13.2</td>
</tr>
<tr>
<td>P4</td>
<td>2260.5 / 36.2</td>
<td>2397.5 / 868.8</td>
<td>2509.1 / 02.4</td>
</tr>
<tr>
<td>P5</td>
<td>3098.9 / 12.5</td>
<td>3137.3 / 56.4</td>
<td>2665.7 / 339.8</td>
</tr>
<tr>
<td>P6</td>
<td>3519.5 / 76.5</td>
<td>3538 / 1005.6</td>
<td>3844 / 852.9</td>
</tr>
<tr>
<td>P7</td>
<td>4327.9 / 82</td>
<td>3822.6 / 512.4</td>
<td>4125.8 / 70.5</td>
</tr>
<tr>
<td>P8</td>
<td>4003.4 / 529.1</td>
<td>4587.5 / 540.3</td>
<td>4931.2 / 61.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aligned PLPs (x/z) in mm</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>459.8 / 46.5</td>
<td>529.7 / 572.5</td>
<td>1079.2 / 554.5</td>
</tr>
<tr>
<td>P2</td>
<td>721.2 / 46.5</td>
<td>908.8 / 572.5</td>
<td>386.1 / 553.5</td>
</tr>
<tr>
<td>P3</td>
<td>1240.1 / 46.3</td>
<td>1305.5 / 572.5</td>
<td>1320.1 / 554.5</td>
</tr>
<tr>
<td>P4</td>
<td>2269.5 / 46.5</td>
<td>2407.5 / 572.5</td>
<td>2546 / 554.5</td>
</tr>
<tr>
<td>P5</td>
<td>2331.8 / 546.3</td>
<td>2483.9 / 572</td>
<td>2658.4 / 554.2</td>
</tr>
<tr>
<td>P6</td>
<td>3503.4 / 546.5</td>
<td>3727.9 / 572</td>
<td>3911.3 / 554.2</td>
</tr>
<tr>
<td>P7</td>
<td>3767.2 / 46.5</td>
<td>3938.8 / 572</td>
<td>4168.5 / 554.2</td>
</tr>
<tr>
<td>P8</td>
<td>4174.4 / 46.4</td>
<td>4497.4 / 72</td>
<td>4884.1 / 554.2</td>
</tr>
</tbody>
</table>
APPENDIX III: OPTIMAL LOCATION OF THE LOCATORS

(RECONFIGURABLE LINE)

| Location of the PLPs (x/z) in mm |
|---|---|---|
|   | Small       | Medium     | Large       |
| P1 | 610.4 / 477.7 | 557.9 / 413.8 | 1062.9 / 439.4 |
| P2 | 328.3 / 91.1  | 176 / 153.9  | 584.4 / 427.5  |
| P3 | 1119.1 / 90.2 | 1187.7 / 600.9 | 1393.5 / 945.1 |
| P4 | 2269.5 / 207.6 | 228.8 / 97.5  | 2477.6 / 1347.4 |
| P5 | 3337.4 / 78.9  | 2495.8 / 615.9 | 2806.7 / 1352.5 |
| P6 | 3256.9 / 997   | 3255.8 / 93.4  | 3684.9 / 1071 |
| P7 | 4354.5 / 151.3 | 4525.7 / 314.1 | 4835.6 / 199.4 |
| P8 | 3821.4 / 667.4 | 3794.4 / 439.5 | 4270.5 / 680.3 |

REFERENCES


