Integration of Dimensional Quality and Locator Reliability in Design and Evaluation of Multi-station Body-In-White Assembly Processes

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Abstract: Research efforts have been made in the development of a Quality and Reliability Chain (QR-Chain) model to integrate manufacturing system component reliability and product quality in multi-station manufacturing processes. Based on a previously developed state space model, which captures the variation propagation throughout all stations, a general QR-Chain model is newly developed for Body-In-White (BIW) assembly processes. The effectiveness of the QR-Chain modeling strategy is demonstrated by thoroughly studying the relationship between locator performance and product quality in assembly processes. Based on the analytical result of system reliability obtained from the QR-Chain model, optimal locator wear rate assignment is further investigated. A case study is conducted to demonstrate the effectiveness and potential usage of the QR-Chain model for the BIW assembly system design evaluation and optimization.

Key words: state space model, BIW assembly process, reliability modeling, QR-Chain, quality and reliability interaction

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1. Introduction

An automotive body without doors, hood, fenders and trunk lid is called a "Body in White" (BIW). In a BIW assembly line, depending on the complexity of the product, there are typically 80 to 130 assembly stations that assemble 150 to 250 sheet metal parts. System reliability of the BIW assembly process is one of the key factors affecting the productivity and product quality.

In general, the system failure of a BIW assembly process includes both the tooling component catastrophic failure and the unsatisfactory product quality. Tooling component catastrophic failures such as locating pin broken and loosen directly lead to an immediate stop of the automation process. Nonconforming product quality such as large product variation is an indication that the process has lost its capability of producing products with the specified quality.

In addition, real process data have shown significant interactions between the locating tool reliability and the product quality propagated through each station of a BIW assembly process. For example, previous research indicates that 72% of the root causes of dimensional errors of a BIW are due to locating tool malfunction (Ceglarek and Shi, 1995), which indicates significant effects of locating tool reliability on the dimensional product quality. On the other hand, large dimensional errors of the locating-holes on the incoming product may lead to locating tool catastrophic failures such as locating pin broken during the part loading process, part stuck at pins, or part unable to be positioned correctly by the locators. In this paper, these kinds of locating tool failures are called locating tool failures induced by incoming product quality. Therefore the catastrophic failure rates of locating tools are affected by the dimensional accuracy of the incoming product, which is determined by the propagation of the dimensional product quality from the previous stations. Based on a previous study in Yang et al. (2000), the locating tool failure induced by incoming product quality corresponds to about 44% of all locating tool catastrophic failures. Therefore, the real process data have shown strong interactions between locating tool reliability and product quality in a multi-station BIW assembly process. In this paper, reliability of a BIW assembly process with 3-2-1 fixtures and rigid parts is studied. The rigid part assembly covers 68% of the total parts in a typical autobody (Shiu et al., 1997).

The product variation is propagated in a multi-station assembly process (Jin and Shi, 1999; Ding et al., 2000). The final product quality is affected by the accumulation or stack up of all variations generated at previous stations. The variation propagation in product quality will lead to the propagation of the interaction between the locating tool reliability and the product quality, which is called as the QR-Chain effect. This paper will study the QR-Chain effect in a multi-station BIW assembly process.

A general QR-Chain modeling framework has been recently proposed by Chen et al. (2001) based on the *process model*, which is in a general form of a linear regression model describing the relationship between process variables and product quality in manufacturing processes. The *process model* plays a critical role in analyzing the QR-Chain effect of a multi-station manufacturing process (MMP), which is assumed available in Chen et al. (2001). However, it is sometimes not easy to obtain a *process model* when Design of Experiments (DOE) is not applicable, especially for a complex multistage manufacturing process. This paper will propose a procedure to build the *process model* for a multi-station BIW assembly process based on the first principle of engineering. With the *process model* obtained based on the process knowledge, the QR-Chain model by Chen et al. (2001) can be successfully applied to the BIW assembly process. All parameters used in system reliability analysis based the QR-Chain model will have the corresponding physical meanings. In addition, an application of the QR-Chain model for optimal assignment of locator wear rate is studied in this paper for design improvement of BIW assembly processes.

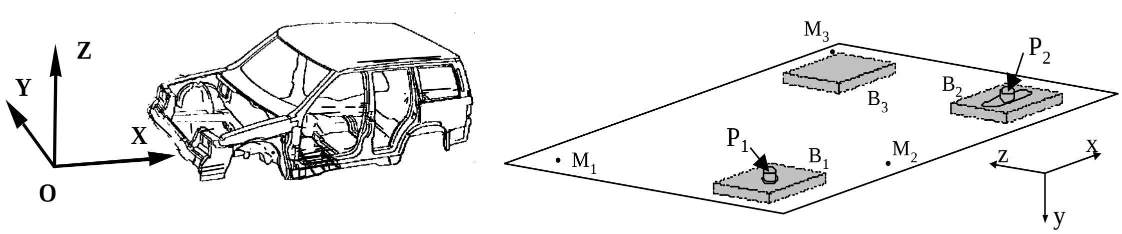
The *process model* of an assembly process should be obtained from the product and process design information and the physical model of specific processes with consideration of variation propagation. In recent years, fixture systems and variation propagation in assembly processes have been studied and significant results have been achieved. The statistical description of variation patterns and the diagnostic issues of a fixture system have been addressed (Hu and Wu, 1992; Ceglarek et al., 1994; Ceglarek and Shi, 1996; Apley and Shi, 1998). For multiple station assembly processes, Jin and Shi (1999) first developed a state space model to describe the product variation propagation across different stations. Based on this state-space model development, many research progresses have been recently made in fault diagnosis (Ding et al., 2000; Ding et al., 2002; Zhou et al., 2003), optimal sensor distribution (Ding et al., 2003b), and optimal process tolerance design (Ding et al., 2003a). However, none of the literature above studied the assembly system reliability and the interactions between product quality and locating tool reliability. In a recent research work (Jin and Chen, 2001), the interactions between product quality and locating tool reliability are studied for a single assembly station. The propagation of the product quality and the QR-interaction in a multi-station assembly process are not captured in Jin and Chen (2001). In general, the QR interaction in a multi-station assembly process is essentially much more complex than that in a single station process. With the aid of the state space model developed in Jin and Shi (1999) to study the variation propagation, this paper will develop the corresponding *process model* for a multi-station BIW assembly process and apply the QR-Chain model accordingly.

The paper is organized as follows. The process variables and product quality characteristics in the context of BIW assembly processes are specified in Section 2 of this paper. In Section 3, the process model and other elements of the QR-Chain model are provided and the physical meanings of the model parameters are discussed in the context of the BIW assembly process. The system reliability is evaluated in Section 4 based on the QR-Chain model for a BIW assembly process. The application of the QR-Chain model is discussed in Section 5 to optimize the pin wear rate assignment. A case study is conducted in Section 6 to demonstrate the proposed model and methodology. The paper is concluded in Section 7.

2. Review of BIW Assembly Processes

2.1 Fixture Layout and Locating Principle in BIW Assembly Processes

In this paper, a body coordinate system shown in Figure 1 is used. The origin of the body coordinate system is defined in the front center of a vehicle and below its underbody. The X-Y-Z axes are shown in the figure. This definition of the body coordinate system has been widely used in the automotive industry for product and process design.



(a) Body coordinate system (b) 3-2-1 fixturing principle for a rigid part

Figure 1. Automotive body and its assembly fixture

An assembly station typically consists of two or more assembly fixtures. Each fixture holds a single part to be assembled with other parts. Locating pins and blocks are locating tools widely used in fixtures to determine the part location and orientation. In this paper, a 3-2-1 fixture for rigid parts is assumed for each station. As shown in Figure 1(b), a typical 3-2-1 fixture contains several key components: (1) a four-way pin/hole (P_1) to precisely locate a part in the X and Z directions; (2) a two-way pin/slot (P_2) to locate a part in the Z direction; these two pins constrain the part rotation and translation in the X-Z plane together; and (3) three shaded locating blocks (B_1, B_2, B_3) to locate a part in the Y direction. The combination of the locating tools (pins and blocks) constrains all six degrees-of-freedom of a rigid part. Since the degree of wear-out of locating blocks is very slight compared to that of locating pins, in this paper we only consider the locating pins in BIW assembly processes.

2.2 Major Elements in the QR-Chain Framework of a BIW Assembly Process

2.2.1 Process Variables Associated with Locating Pins

A general modeling procedure focusing on the X-Z plane is presented in this paper for rigid part assembly and the four-way and two-way locating pins are considered as system components. Suppose there are n_i locating pins at the i^{th} station, i=1, 2, …, L, where L is the number of stations in a BIW assembly process. The total number of locating pins in all stations is $n = \sum_{i=1}^{L} n_i$. The changes of the pin diameter will change the clearance between the locating pin on the fixture and the locating-hole on the part, which will affect the product quality. Thus the accumulated decrement in the pin diameter due to pin wear-out is considered as the process variable. Let $P_{i,j}$ denote the j^{th} locating pin at the i^{th} station and $X_{i,j}(t)$ denote its accumulated diameter decrement at time t.

2.2.2 Product Quality Characteristics in a BIW Assembly Process

In assembly processes, product quality is generally defined by the dimensional accuracy of the Key Product Characteristic (KPC) points on the product. The KPC points on the outgoing product of each station include the locating-holes used for part locating in the next station and the points whose dimensional accuracy is specified in the product design. The measurements of these KPC points are treated as the product characteristics in a BIW assembly process. Let $Y_{i,j}(t), i=1,\ldots,L,\ j=1,\ldots,m_i$ denote the j^{th} product characteristic on the outgoing product of station i at time t, where m_i is the number of KPC measurements on the outgoing product of station i.

3. QR-Chain Model for BIW Assembly Processes

There are several key relationships in the QR-Chain model of a BIW assembly process: the locating pin wear is the major cause of KPC deviations; product quality of each station is defined based on the KPC deviations; and the outgoing product quality of a station impacts on the locating pin catastrophic failure of the next station. The following diagram is used to summarize these relationships:

Pin Degradation→KPC Deviations→Product Quality→Pin Catastrophic Failure.

The presentation of this section will follow the diagram shown above.

3.1 Locating Pin Degradation Model

The mechanism of the locating pin wear is discussed in Jin and Chen (2001). According to Jin and Chen (2001), the aggregated wear of the pin diameter is increased with the number of operations, which can be described by a stochastic process model with independently lognormal distributed increments:

$$X_{i,j}(t) = X_{i,j}(t-1) + \Delta_{i,j}(t)$$

where $\Delta_{i,j}(t)$ is the random wear increment due to operation t, $X_{i,j}(0)$ is the initial clearance between locating pin $P_{i,j}$ and its corresponding locating-hole, t is the operation index. It is assumed that $X_{i,j}(0) \sim N(\mu_{i,j}(0), \sigma^2_{i,j}(0))$ and

$$\Delta_{i,j}(t) \sim \text{LOGNOR}(\mu_{i,j}(\Delta), \sigma_{i,j}^2(\Delta)),$$

where $\mu_{i,j}(\Delta)$ *and* $\sigma_{i,j}(\Delta)$ are the mean and standard deviation of the lognormal random variable $\Delta_{i,j}(t)$. Let $\boldsymbol{\mu}_0 \equiv [\mu_{1,1}(0) \quad \mu_{1,2}(0) \quad \ldots \quad \mu_{L,n_L}(0)]^T$ and

$$\boldsymbol{\Sigma}_0 \equiv \begin{bmatrix} \sigma_{1,1}^2(0) & & & \mathbf{0} \\ & \sigma_{1,2}^2(0) & & \\ & & \ddots & \\ \mathbf{0} & & & \sigma_{L,n_L}^2(0) \end{bmatrix}.$$

In this paper, we choose one production day as the time interval to discretize the time scale for a BIW assembly process. Let h denote the number of operations during each production day, and t_k denote the end time of the k^{th} production day. Since the time is measured by the number of operations, t_k is the total number of operations until the end of production day k. So $t_k=kh$ and $t_0=0$. The BIW assembly operations are discrete in nature. The time of sliding wear when the part is positioned on the pin is much shorter than the cycle time of an operation (more time is spent on welding, clamping operations and part handling and moving). And the accumulated wear of a locating pin is much smaller than the pin diameter and has little impact on future wear mechanism. So it is reasonable to assume that the wear amount during an operation is independent of that of previous operations. In addition, a BIW assembly process can produce 500-1,500 car bodies during each day of production. Therefore the accumulated wear of such a large number of operations can reasonably be approximated as Normally distributed based on the central limit theorem. Thus the following equation can be used to model the pin wear:

$$\mathbf{X}(t_k) = \mathbf{X}(t_{k-1}) + \boldsymbol{\varepsilon}_k, k = 1, 2, \ldots \tag{1}$$

where $\boldsymbol{\varepsilon}_k \sim N(\boldsymbol{\mu}_\varepsilon, \mathbf{Q})$, $\boldsymbol{\mu}_\varepsilon = h \cdot [\mu_{1,1}(\Delta) \quad \ldots \quad \mu_{1,n_1}(\Delta) \quad \ldots \quad \mu_{L,1}(\Delta) \quad \ldots \quad \mu_{L,n_L}(\Delta)]^T$,

$\mathbf{Q} = h \cdot \mathrm{diag}\left(\sigma_{1,1}^2(\Delta) \quad \ldots \quad \sigma_{1,n_1}^2(\Delta) \quad \ldots \quad \sigma_{L,1}^2(\Delta) \quad \ldots \quad \sigma_{L,n_L}^2(\Delta)\right)$, and $\mathbf{X}(t_0) \sim N(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0)$. In this paper, increasing wear is assumed, that is, for any $1 \le j \le n$, $\Pr\{[\mathbf{X}(t_0)]_j < 0\}$ and $\Pr\{[\mathbf{X}(t_{k+1}) - \mathbf{X}(t_k)]_j < 0\}$ can be ignored.

3.2 Relationship between Process Variables and KPC Deviations

In Chen et al. (2001), the following process model is used to describe the relationship between process variables and product quality characteristics:

$$Y_{i,j}(t) = \eta_{i,j} + \boldsymbol{\alpha}_{i,j}^T \mathbf{X}(t) + \boldsymbol{\beta}_{i,j}^T \mathbf{z}_t + \mathbf{X}(t)^T \boldsymbol{\Gamma}_{i,j} \mathbf{z}_t, \; i = 1,2,\ldots,L, \; j = 1, 2, \ldots, m_i \tag{2}$$

where $\mathbf{X}(t) \equiv \left[X_{1,1}(t) \quad X_{1,2}(t) \quad \ldots \quad X_{L,n_L}(t)\right]^T \in R^n$ is the vector of process variables in the system; $\mathbf{z}_t \equiv [z_{1t}, z_{2t}, \ldots, z_{lt}]^T \in R^l$ is the random vector of noise variables, with mean E($\mathbf{z}_t$) and covariance matrix Cov($\mathbf{z}_t$) independent of the time index t; l is the total number of noise variables in the BIW assembly process; $\eta_{i,j}, i = 1,2,\ldots,L, \; j = 1,2,\ldots,m_i$ are constants, $\boldsymbol{\alpha}_{i,j}$ and $\boldsymbol{\beta}_{i,j}$ are vectors characterizing the effects of $\mathbf{X}(t)$ and $\mathbf{z}_t$, and $\boldsymbol{\Gamma}_{i,j}$ is a matrix characterizing the effects of the interaction between $\mathbf{X}(t)$ and $\mathbf{z}_t$. For BIW assembly processes, $\mathbf{z}_t$ describes the random pin-hole contact orientations.

The physical model of a BIW assembly process needs to be studied to obtain the coefficients $\boldsymbol{\alpha}_{i,j}$, $\boldsymbol{\beta}_{i,j}$, $\boldsymbol{\Gamma}_{i,j}$, and $\eta_{i,j}, i = 1,2,\ldots,L, \; j = 1,2,\ldots,m_i$ in (2). The relationship between process variables and product quality characteristics for a single fixture station is given in Jin and Chen (2001) by studying the relationship among locating pin diameters, part locating errors, and KPC deviations. An overall multi-station process model addressing the variation propagation across different stations is required in the development of the process model for a multi-station BIW assembly process. The state space model developed in Jin and Shi (1999) for multi-station sheet metal assembly processes will be used in this paper to obtain the coefficients of the process model. In the following subsections, the state space model and the physical knowledge of an assembly process will be reviewed to obtain the coefficients of the process model for a BIW assembly process.

3.2.1 State Space Model for Part Locating Errors and KPC Deviations

From Jin and Shi (1999), the state equation at station i can be expressed by:

$$\begin{aligned}\mathbf{V}_i(t) &= \mathbf{H}_{i-1}\mathbf{V}_{i-1}(t) + \mathbf{B}_i\mathbf{F}_i(t) \\ \mathbf{Y}_i(t) &= \mathbf{C}_i\mathbf{V}_i(t)\end{aligned} \qquad i=1, 2, \ldots, L \tag{3}$$

where $\mathbf{V}_i(t)$, which is equivalent to the state vector in the state space model, is the part error vector defined in Jin and Shi (1999) characterizing the dimensional errors of all outgoing parts of station i; $\mathbf{V}_0(t)$ is the dimensional errors of the raw parts coming from stamping processes; system matrices $\mathbf{H}$, $\mathbf{B}$, and $\mathbf{C}$ encode process configuration such as the layout of locating tools and KPC points; $\mathbf{F}_i(t)$ is the vector of part locating errors which is the dimensional error of the part at the position of the locating pins of station i; and $\mathbf{Y}_i(t) \equiv \begin{bmatrix} Y_{i,1}(t) & \ldots & Y_{i,m_i}(t) \end{bmatrix}^T$. The first equation of the state space model in Eq. (3) uses a recursive relationship to characterize the propagation of product quality. Based on this relationship, it can be seen that the part dimensional errors at the current station are the accumulation of part locating errors and raw part dimensional errors at the previous stations.

3.2.2 Relationship between Pin Wear and Part Locating Errors

The locating pins' wear is reflected in the reduction of the pin diameters, which causes an increasing clearance between a locating pin and the corresponding locating-hole. And this clearance results in the part locating error. The following notations are used for the description of relationship between pin wear and part locating errors:

(a) $\Delta x_{P_{i,j}}$ and $\Delta z_{P_{i,j}}$ denote the part locating errors of pin $P_{i,j}$ in the X and Z directions;

(b) $\mathbf{F}_i(t) \equiv \begin{bmatrix} \Delta x_{P_{i,1}} & \Delta z_{P_{i,1}} & \dots & \Delta x_{P_{i,n_i}} & \Delta z_{P_{i,n_i}} \end{bmatrix}^T$ represents the vector of part locating errors at station i;

(c) $\theta_{i,j}$ represents the orientation of the contacting point between the pin $P_{i,j}$ and the locating-hole.

The relationship between the part locating errors $(\Delta x_{P_{i,j}}, \Delta z_{P_{i,j}})$ and the pin diameter reduction of a four-way locating pin can be obtained as

$$\Delta x_{P_{i,j}} = 0.5 X_{i,j} \cos\theta_{i,j}\,; \qquad \Delta z_{P_{i,j}} = 0.5 X_{i,j} \sin\theta_{i,j} \tag{4}$$

where $X_{i,j}$ is the accumulated pin diameter decrement, which is considered as the process variable corresponding to the locating pin $P_{i,j}$ in this paper. The relationship between the part locating error and the wear of the two-way locating pin can be obtained as

$$\Delta z_{P_{i,j}} = 0.5 X_{i,j} \sin\theta_{i,j} \tag{5}$$

More detailed illustration of (4) and (5) and more discussion on distribution of $\theta_{i,j}$ are given in Appendix 1.

3.2.3 Process Model for the BIW Assembly Process

By recursively substituting $\mathbf{V}_i(t), \mathbf{V}_{i-1}(t), \dots, \mathbf{V}_1(t)$ in (3), the KPC deviation on the outgoing parts of station i can be calculated based on both the part locating errors at station i and those of previous stations as below:

$$\mathbf{Y}_i(t) = \mathbf{G}^{(i)} \mathbf{F}^{(i)}(t) + (\mathbf{C}_i \mathbf{H}_{i-1} \mathbf{H}_{i-2} \cdots \mathbf{H}_0) \mathbf{V}_0(t)\,, \tag{6}$$

where matrix $\mathbf{G}^{(i)} \equiv \mathbf{C}_i \begin{bmatrix} (\mathbf{H}_{i-1}\mathbf{H}_{i-2}\cdots\mathbf{H}_1)\mathbf{B}_1 & (\mathbf{H}_{i-1}\mathbf{H}_{i-2}\cdots\mathbf{H}_2)\mathbf{B}_2 & \dots & \mathbf{H}_{i-1}\mathbf{B}_{i-1} & \mathbf{B}_i \end{bmatrix}$ and vector $\mathbf{F}^{(i)}(t) \equiv \begin{bmatrix} \mathbf{F}_1^T(t) & \mathbf{F}_2^T(t) & \dots & \mathbf{F}_i^T(t) \end{bmatrix}^T$.

Let $\mathbf{z}'_{t,i} \equiv \begin{bmatrix} \cos\theta_{i,1} & \sin\theta_{i,1} & \dots & \cos\theta_{i,n_i} & \sin\theta_{i,n_i} \end{bmatrix}^T$, $\mathbf{z}'_t \equiv \begin{bmatrix} (\mathbf{z}'_{t,1})^T & \dots & (\mathbf{z}'_{t,L})^T \end{bmatrix}^T$ and

$$\mathbf{z}_t \equiv \left[\mathbf{V}_0^T(t) \quad (\mathbf{z}'_t)^T\right]^T . \tag{7}$$

That is, the raw part error and the random orientation of the contacting point between the pin and the locating-hole are considered as noise variables in the process model. From (6), (4), and (5), with some basic algebraic manipulations, it can be seen that

$$Y_{i,j}(t) = \left[\boldsymbol{\beta}_{i,j}^{(1)} \quad \boldsymbol{\beta}_{i,j}^{(2)}\right] \mathbf{z}_t + \mathbf{X}(t)^T \left[\boldsymbol{\Gamma}_{i,j}^{(1)} \quad \boldsymbol{\Gamma}_{i,j}^{(2)}\right] \mathbf{z}_t,\ i = 1,2,\ldots,L,\ j = 1,2,\ldots,m_i \tag{8}$$

where $\boldsymbol{\beta}_{i,j}^{(1)} = \left[\mathbf{C}_i \mathbf{H}_{i-1} \mathbf{H}_{i-2} \cdots \mathbf{H}_0\right]_{(j,:)}$; $\boldsymbol{\beta}_{i,j}^{(2)}$ is a $1\times 2n$ vector whose elements are all zero; $\boldsymbol{\Gamma}_{i,j}^{(1)}$ is a zero matrix of appropriate dimension; and

$$\boldsymbol{\Gamma}_{i,j}^{(2)} = \frac{1}{2}\begin{bmatrix} \left[\mathbf{G}^{(i)}\right]_{(j,1)} & \left[\mathbf{G}^{(i)}\right]_{(j,2)} & 0 & 0 & \cdots & 0 & 0 & \mathbf{0}_{(1\times 2(n-p_i))} \\ 0 & 0 & \left[\mathbf{G}^{(i)}\right]_{(j,3)} & \left[\mathbf{G}^{(i)}\right]_{(j,4)} & \cdots & 0 & 0 & \mathbf{0}_{(1\times 2(n-p_i))} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \left[\mathbf{G}^{(i)}\right]_{(j,2p_i-1)} & \left[\mathbf{G}^{(i)}\right]_{(j,2p_i)} & \mathbf{0}_{(1\times 2(n-p_i))} \\ \mathbf{0}_{((n-p_i)\times 1)} & \mathbf{0}_{((n-p_i)\times 1)} & \mathbf{0}_{((n-p_i)\times 1)} & \mathbf{0}_{((n-p_i)\times 1)} & \cdots & \mathbf{0}_{((n-p_i)\times 1)} & \mathbf{0}_{((n-p_i)\times 1)} & \mathbf{0}_{(r\times 2(n-p_i))} \end{bmatrix}_{(n\times 2n)}$$

where $p_i \equiv \sum_{k=1}^{i} n_k$. Thus, the coefficients of the process model (2) for a BIW assembly process are $\eta_{i,j} = 0$, $\boldsymbol{\alpha}_{i,j} = \mathbf{0}$, $\boldsymbol{\beta}_{i,j} = \left[\boldsymbol{\beta}_{i,j}^{(1)} \quad \boldsymbol{\beta}_{i,j}^{(2)}\right]^T$ and $\boldsymbol{\Gamma}_{i,j} = \left[\boldsymbol{\Gamma}_{i,j}^{(1)} \quad \boldsymbol{\Gamma}_{i,j}^{(2)}\right]$.

3.3 Product Quality Assessment

The product quality can be assessed by the mean squared deviations of the quality characteristics—the KPCs. Therefore, the j^{th} quality index at the i^{th} station is

$$q_{i,j}(t \mid \mathbf{X}(t_k)) = E((Y_{i,j}(t) - \gamma_{i,j})^2 \mid \mathbf{X}(t_k)) = Var(Y_{i,j}(t) \mid \mathbf{X}(t_k)) + (E(Y_{i,j}(t) \mid \mathbf{X}(t_k)) - \gamma_{i,j})^2,\ t_k \le t < t_{k+1}$$

where $\gamma_{i,j}$ is the target value for $Y_{i,j}(t)$.

From the discussion in subsection 3.2 and Appendix 1, it can be shown that $\mathrm{E}(\mathbf{z}'_t) = \mathbf{0}$, thus $\boldsymbol{\Gamma}_{i,j}\mathrm{E}(\mathbf{z}_t) = \mathbf{0}$ and $\boldsymbol{\alpha}_{i,j} + \boldsymbol{\Gamma}_{i,j}\mathrm{E}(\mathbf{z}_t) = \mathbf{0}$. Then from (2),

$$(E(Y_{i,j}(t) \mid \mathbf{X}(t_k)) - \gamma_{i,j})^2 = \mathbf{X}(t_k)^T(\boldsymbol{\alpha}_{i,j} + \boldsymbol{\Gamma}_{i,j}E(\mathbf{z}_t))(\boldsymbol{\alpha}_{i,j} + \boldsymbol{\Gamma}_{i,j}E(\mathbf{z}_t))^T \mathbf{X}(t_k) = 0, \forall i, j, \text{ and } k$$

So the locating pin degradation of a BIW assembly process will not cause mean shifts of the KPCs. The quality index can be written as a quadratic function of **X**(k) as follows

$$\begin{aligned} q_{i,j}(t\,|\,\mathbf{X}(t_k)) &= Var(Y_{i,j}(t)\,|\,\mathbf{X}(t_k)) = (\mathbf{X}(t_k)^T\boldsymbol{\Gamma}_{i,j})\,\mathrm{cov}(\mathbf{z}_t)(\boldsymbol{\Gamma}_{i,j}^T\mathbf{X}(t_k)) + \boldsymbol{\beta}_{i,j}^T\,\mathrm{cov}(\mathbf{z}_t)\boldsymbol{\beta}_{i,j} \\ &= \mathbf{X}(t_k)^T\mathbf{B}_{i,j}\mathbf{X}(t_k) + d_{i,j} \end{aligned} \tag{9}$$

where $\mathbf{B}_{i,j} = \boldsymbol{\Gamma}_{i,j}\,\mathrm{cov}(\mathbf{z}_t)\boldsymbol{\Gamma}_{i,j}^T$ and $d_{i,j} = \boldsymbol{\beta}_{i,j}^T\,\mathrm{cov}(\mathbf{z}_t)\boldsymbol{\beta}_{i,j}$. Since $\boldsymbol{\beta}_{i,j}$ is only related to the raw part errors, $d_{i,j}$ can be interpreted as the contribution of the raw part errors to the product quality at each station.

Let $E_t^q \equiv \bigcap_{i=1}^{L}\bigcap_{j=1}^{m_i}\left(q_{i,j}(\tau) \le a_{i,j}, \forall 0 \le \tau \le t\right)$, where $a_{i,j}$ is the threshold of the specification for the j^{th} KPC at the i^{th} station. E_t^q represents the event that no quality index has exceeded its threshold by time t, i.e., no failure due to nonconforming products has occurred by time t.

3.4 Relationship between Product Quality and Locating Pin Catastrophic Failure Rate

The deviation of the locating-hole center of an incoming part may accelerate the catastrophic failures of the corresponding locating pin. The larger the variation of the locating-hole position, the higher failure chance the pin will have. The relationship between the catastrophic failure rate $\lambda_{i,j}(t)$ and the product quality index $\mathbf{q}(t)$ is described as

$$\lambda_{i,j}(t) = \lambda_{i,j}(0) + \mathbf{s}_{i,j}^T\mathbf{q}(t\,|\,\mathbf{X}(t_k))$$

where $\mathbf{q}(t\,|\,\mathbf{X}(t_k)) = [q_{1,1}(t\,|\,\mathbf{X}(t_k)) \quad q_{1,2}(t\,|\,\mathbf{X}(t_k)) \quad \ldots \quad q_{L,m_L}(t\,|\,\mathbf{X}(t_k))]^T$ and $\mathbf{s}_{i,j}$ is a vector of appropriate dimension with nonnegative elements and called as the QR-coefficient. The QR-coefficient can be calibrated by collecting catastrophic failure data and product dimensional measurements.

The catastrophic failure rate of a four-way locating pin can be affected by the locating-hole variation in both the X direction and the Z direction. The catastrophic failure rate of a two-way locating pin is affected only by the locating-hole variation in the Z direction since there is normally little contact in the X direction between a locating-hole and a two-way locating pin. Based on the relationship between the locating pin catastrophic failure rate and its corresponding locating-hole variations discussed above, if $P_{i,j}$ is a four-way pin, then

$$[\mathbf{s}_{i,j}]_r = \begin{cases} \geq 0, & \begin{array}{l}\text{if } [\mathbf{q}(t)]_r \text{ corresponds to the X or Z direction variation of the} \\ \text{locating hole located by the } j^{\text{th}} \text{ pin at the } i^{\text{th}} \text{ station}\end{array} \\ = 0, & \text{otherwise} \end{cases};$$

if $P_{i,j}$ is a two-way pin, then

$$[\mathbf{s}_{i,j}]_r = \begin{cases} \geq 0, & \begin{array}{l}\text{if } [\mathbf{q}(t)]_r \text{ corresponds to the Z direction variation of the} \\ \text{locating hole located by the } j^{\text{th}} \text{ pin at the } i^{\text{th}} \text{ station}\end{array} \\ = 0, & \text{otherwise} \end{cases}.$$

Here $[\cdot]_r$ denotes the r^{th} element of a vector.

4. System Reliability Evaluation

From the definition of $\boldsymbol{\Gamma}_{i,j}$ and $\mathbf{z}_t$ in Section 3.2.3 and the distribution of $\theta_{i,j}$ discussed in Appendix 1, it can be shown that $\mathbf{B}_{i,j} = \boldsymbol{\Gamma}_{i,j}\operatorname{cov}(\mathbf{z}_t)\boldsymbol{\Gamma}_{i,j}^T$ is diagonal. Based on Chen et al. (2001), the product quality of such a process does not have self-improvement. Therefore, following the same procedures in Chen et al. (2001), the system reliability of a BIW assembly process can be finally obtained as

$$\begin{aligned} R(t_K) &\equiv \Pr(\text{No pin catastrophic failure AND Each quality index is within specification by } t_K) \\ &= \Pr(E_{t_K}^c \text{ AND } E_{t_K}^q) = \Pr(E_{t_K}^c)\Pr(E_{t_K}^q \mid E_{t_K}^c) \\ &= R_I(t_K)\cdot R_{II}(t_K) \end{aligned} \tag{10}$$

where $t_K = Kh$ is the production mission time (the reliability evaluation lifetime), $R_I(t_K) \equiv \exp(-ct_K)\dfrac{\exp(-\rho_K)}{|\boldsymbol{\Sigma}_K|^{\frac{1}{2}}}|\tilde{\boldsymbol{\Sigma}}_K|^{\frac{1}{2}}$, $R_{II}(t_K) \equiv \displaystyle\int_{\mathbf{x}(t_K)\in\boldsymbol{\Omega}_K} dF_{\tilde{\mathbf{X}}(t_K)}(\mathbf{x}(t_K))$, and E_t^c is defined as the event that system catastrophic failures never occurred by time t. Because $R_{II}(t_K)$ is affected by the quality constraint $\mathbf{\Omega}_K$, while $R_I(t_K)$ is not, it can be seen that $R_I(t_K) = \Pr(E_{t_K}^c)$, and $R_{II}(t_K) = \Pr(E_{t_K}^q \mid E_{t_K}^c)$. The probabilities associated with $E_{t_K}^c$ and $E_{t_K}^q$ are determined by the pin catastrophic failure rate derived in Section 3.4 and the quality index derived in 3.3. The detailed derivation on how to calculate $\Pr(E_{t_K}^c \text{ AND } E_{t_K}^q)$ is shown in Chen et al. (2001). The parameters c, $\mathbf{\Sigma}_K$, $\mathbf{\Omega}_K$, ρ_K, and distribution of $\tilde{\mathbf{X}}(t_K)$ in (10) are given in Appendix 2.

The following design parameters are used as the input information to calculate system reliability for general BIW assembly processes:

(a) Layout of locating pins (used to get $\mathbf{H}_i$, $\mathbf{B}_i$, and $\mathbf{C}_i$ in the state space model)

(b) Layout of KPCs (used to get $\mathbf{C}_i$ in the state space model)

(c) Raw-part error ($\text{cov}(\mathbf{V}_0(\text{t}))$)

(d) Pin degradation rate & standard deviation ($\mu_{i,j}(\Delta)$, $\sigma_{i,j}(\Delta)$)

(e) Initial pin-hole clearance ($\mu_{i,j}(0)$, $\sigma_{i,j}(0)$)

(f) Initial catastrophic failure rate ($\lambda_{i,j}(0)$)

(g) QR co-efficient $\mathbf{s}_{i,j}$

(h) Threshold for quality index $a_{i,j}$

(i) Length of time interval (h)

In Appendix 2, the procedure to calculate system reliability based on the input information is summarized in Table 6.

5. Optimal Locator Wear Rate Assignment

The analytical solution (10) provides many potential applications in design of a multi-station BIW assembly process. For example, it can be used to determine the optimal setting of the wear rates of locating pins. Suppose that the wear rate range for each locating pin is defined as

$$0 \le \mu_{\min} \le \mu_{i,j}(\Delta) \le \mu_{\max} < \infty$$

where $\mu_{\min}$ and $\mu_{\max}$ are the lowest and highest allowable wear rates for each locating pin. And the selection of the pin wear rate directly affects its fabrication cost. Theoretically, zero is the lower bound for the wear rate. The $\mu_{\min}$ in our formulation is a practical lower bound. When $\mu_{\min} = 0$, it is equivalent to the problem without a specific practical lower bound. Therefore, the optimization without a lower bound (or lower bound is zero) is a special case of our model. We use $\mu_{\min}$ because in many situations if the wear rate is extremely small, it may not be achievable based on available tool making techniques. For these situations, a positive minimum value of $\mu_{\min}$ will be more reasonable in the optimization problem formulation.

5.1 Fabrication and Coating Costs of Locating Pins

Typically the less wear rate of a locating pin, the more fabrication and coating costs will be suffered associated with the pin. So it is reasonable to assume that the pin wear rate is inversely proportional to the fabrication and coating costs. In this paper, the costs associated with all locating pins are defined by reciprocal function of pin wear rates as

$$C_p = \sum_{i=1}^{L} \sum_{j=1}^{n_i} \frac{w_{i,j}}{\mu_{i,j}(\Delta)} \tag{11}$$

where $\mu_{i,j}(\Delta)$ is the wear rate of pin $P_{i,j}$ and $w_{i,j} \ge 0$ is the weighting coefficient associated with $\mu_{i,j}(\Delta)$.

5.2 Optimization Formulation and Optimality

The objective of the optimal assignment of locating pin wear rates is to maximize the system reliability at t_K. The optimization problem can be formulated mathematically as

$$\begin{aligned} &\boldsymbol{\mu}^* = \operatorname{Arg}\max_{\boldsymbol{\mu}}\{R(t_K)\} \\ &\text{subject to } C_p \le C_{\max}, \mu_{\min} \le \mu_{i,j}(\Delta) \le \mu_{\max}, \forall i, j \end{aligned} \tag{12}$$

where $\boldsymbol{\mu} = [\mu_{1,1}(\Delta) \quad \mu_{1,2}(\Delta) \quad \ldots \quad \mu_{L,n_L}(\Delta)]^T$ and $C_{\max}$ is the budget for the total tooling fabrication cost.

Let $\overline{E}_{t_K}^q$ denote the complement of $E_{t_K}^q$. $\overline{E}_{t_K}^q$ is the event that the failure due to nonconforming products occurs by time t_K. If we first constrain the maximum wear rate $\mu_{\max}$ such that $\Pr(\overline{E}_{t_K}^q) < \alpha$, the failure due to nonconforming products can be ignored. The selection of α depends on specific manufacturing processes. The BIW assembly process is not a highly reliable process due to its complexity. Normally the system reliability of interest for a BIW assembly process cannot be very high and the failure probability of less than 0.01 due to nonconforming products is acceptable. Therefore, for BIW assembly processes, we choose $\alpha = 0.01$. It can be seen that

$$R(t_K) = 1 - \Pr(\overline{E}_{t_K}^c \cup \overline{E}_{t_K}^q) = 1 - \Pr(\overline{E}_{t_K}^c) - \Pr(\overline{E}_{t_K}^q) + \Pr(\overline{E}_{t_K}^c \cap \overline{E}_{t_K}^q)$$

Since $0 \le \Pr(\overline{E}_{t_K}^c \cap \overline{E}_{t_K}^q) \le \Pr(\overline{E}_{t_K}^q) < \alpha$,

$$\left| R(t_K) - (1 - \Pr(\overline{E}_{t_K}^c)) \right| < \alpha$$

Therefore, if $\Pr(\overline{E}_{t_K}^q)$ can be ignored, the objective of maximizing $R(t_K)$ becomes maximizing

$$1 - \Pr(\overline{E}_{t_K}^c) = \Pr(E_{t_K}^c) = R_I(t_K)$$

Now suppose that the constrained maximum wear rate $\mu_{\max}$ is selected such that

$$\Pr(\overline{E}_{t_K}^q \mid \mu_{i,j}(\Delta) = \mu_{\max}, \forall i, j) < \alpha \tag{13}$$

The following result assures that under this condition, the failure due to nonconforming products can be ignored for all settings of locating pin wear rates within the allowable range.

Result 1. If $\mu'_{i,j}(\Delta) \le \mu''_{i,j}(\Delta), \forall i, j$, then $\Pr(E^q_{t_K} \mid \mu_{i,j}(\Delta) = \mu'_{i,j}(\Delta)) \ge \Pr(E^q_{t_K} \mid \mu_{i,j}(\Delta) = \mu''_{i,j}(\Delta))$

Proof. This result can be easily seen based on the fact that the product quality of the BIW assembly process does not have self-improvement. The detailed proof is omitted in this paper.

From (13) and Result 1, the failure due to nonconforming products can be ignored for any pin wear rate settings within the allowable ranges, and the optimization problem in (12) is equivalent to the following optimization problem:

$$\begin{aligned} &\boldsymbol{\mu}^* = \text{Arg}\max_{\boldsymbol{\mu}}\{R_I(t_K)\} \\ &\text{subject to } C_p \le C_{\max},\ \mu_{\min} \le \mu_{i,j}(\Delta) \le \mu_{\max}, \forall i, j \end{aligned} \tag{14}$$

Since c, $\boldsymbol{\Sigma}_K$, and $\tilde{\boldsymbol{\Sigma}}_K$ are independent of the decision variable $\boldsymbol{\mu}$, (14) can be further rewritten as

$$\begin{aligned} &\boldsymbol{\mu}^* = \text{Arg}\min_{\boldsymbol{\mu}}\{s_K\} \\ &\text{subject to } C_p \le C_{\max},\ \mu_{\min} \le \mu_{i,j}(\Delta) \le \mu_{\max}, \forall i, j \end{aligned} \tag{15}$$

Regarding the optimality of the above optimization formulation, the following results can be derived based on the form of the analytical solution.

Lemma 1. ρ_K is a convex function of $\boldsymbol{\mu}$.

Proof. From (19), $\rho_K = \boldsymbol{\mu}_K^T \left[\mathbf{U}_K^T \left(\mathbf{U}_K + \left(\boldsymbol{\Sigma}_K^{-1}/2\right)\right)^{-T} \left(\boldsymbol{\Sigma}_K^{-1}/2\right)\right] \boldsymbol{\mu}_K$. Both $\mathbf{U}_K$ and $\boldsymbol{\Sigma}_K$ are symmetric positive semidefinite matrices based on the discussion in Chen et al. (2001). So the summation, product, and inverse of them are all positive semidefinite. Therefore, ρ_K is positive semidefinite on $\boldsymbol{\mu}_K$. From (19), obviously ρ_K is also positive semidefinite on $\boldsymbol{\mu}$. Hence it is convex on $\boldsymbol{\mu}$.

Lemma 2. The constraint in (15) is a convex set.

Proof. Obviously C_p is a convex function of $\mu_{i,j}(\Delta)$ for $\mu_{i,j}(\Delta)>0$. So the level set $C_p \le C_{\max}$ is convex and hence $\{\mu_{i,j}(\Delta) \mid C_p \le C_{\max}, \mu_{\min} \le \mu_{i,j}(\Delta) \le \mu_{\max}\}$ is a convex set.

From Lemma 1 and Lemma 2, the result below about the optimality of (15) follows (see Avriel (1976)).

Result 2. The nonlinear optimization problem stated in (15) converges to a global minimum $\boldsymbol{\mu}^*$.

6. Case Study

A case study is conducted to illustrate the developed methodology. A side aperture inner panel assembly, as shown in Figure 2, is selected in the study.

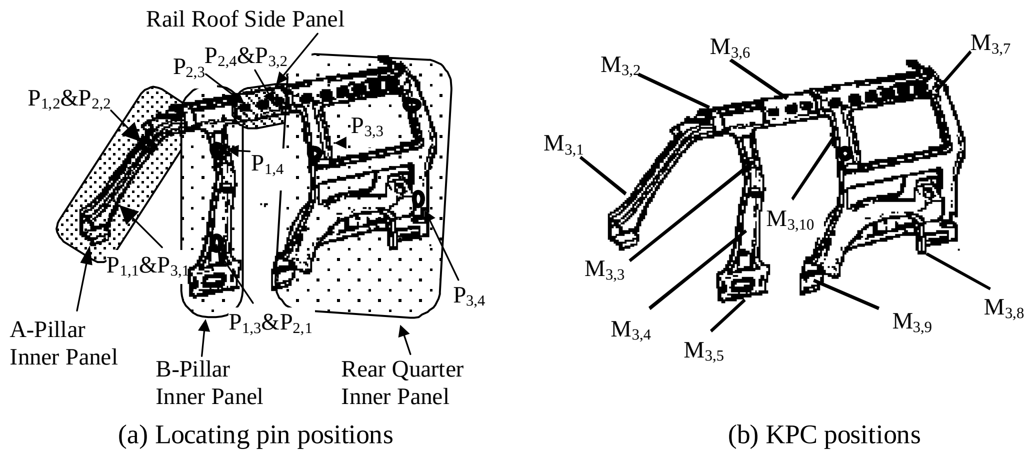


Figure 2. Layout of the tooling positions and KPC points

In this example, four parts are assembled together by three stations (Figure 3). A-Pillar and B-Pillar are assembled at station 1. The subassembly of A-Pillar and B-Pillar is then assembled with Rail Roof at station 2. At station 3, the subassembly of A-Pillar, B-Pillar, and Rail Roof are assembled with the Rear Quarter Inner Panel. The product quality is defined by 10 KPC points measured at station 3 (Figure 2(b)). The assembly sequences of these four parts are illustrated in Figure 3 and the layout of tooling positions is shown in Figure 2(a). Table 1 and Table 2 give all the dimensions of the tooling positions and the KPC points. The design parameters used for this example are shown in Table 3. The raw part dimensional errors and variation of initial pin/hole clearance is very small and ignored in this case study.

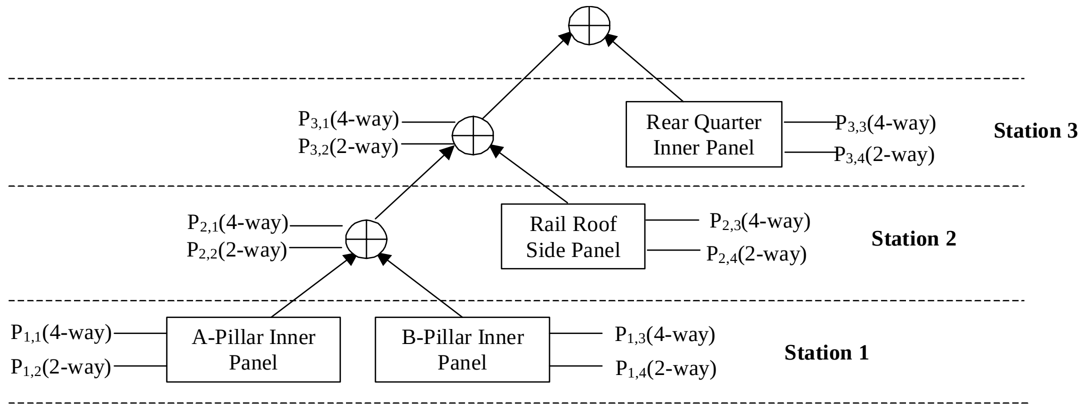


Figure 3. Assembly sequences of the side aperture inner panel

Table 1. Nominal X-Z coordinates for locating points

Locating Points		$P_{1,1}$& $P_{3,1}$	$P_{1,2}$& $P_{2,2}$	$P_{1,3}$& $P_{2,1}$	$P_{1,4}$	$P_{2,3}$	$P_{2,4}$ &$P_{3,2}$	$P_{3,3}$	$P_{3,4}$
Nominal Coordinates (mm)	**X**	367.7	667.5	1272.7	1301.0	1470.7	1770.5	2120.3	3026.3
	Z	906.1	1295.4	537.4	1368.9	1640.4	1702.6	1402.8	950.3

Table 2. Nominal X-Z coordinates for KPC points

KPC Points		$M_{3,1}$	$M_{3,2}$	$M_{3,3}$	$M_{3,4}$	$M_{3,5}$	$M_{3,6}$	$M_{3,7}$	$M_{3,8}$	$M_{3,9}$	$M_{3,10}$
Nominal Coordinates (mm)	**X**	271.5	565	1289	1306	1244	1640	2884	2743	1838	1980
	Z	905.0	1634	1227	633	85	1781	1951	475	226	1459

Table 3. Summary of the parameters used in the study

Description of the parameter	**Value**
Initial failure rate $\lambda_{i,j}(0)$	$\lambda_{i,j}(0) = \lambda(0) = 4\times10^{-7}, \forall i, j$
QR coefficient $\mathbf{s}_{i,j}$	$[\mathbf{s}_{i,j}]_r = s = 0.001, \forall i, j$, when the r^{th} element corresponds to the quality characteristics of the locating hole located by the j^{th} pin at the i^{th} station
Initial pin/hole clearance $\mu_{i,j}(0)$	$\mu_{i,j}(0) = \mu_0 = 0.04\text{mm}, \forall i, j$
Operations per time interval h	$h = 500$ operations
Unit degradation rate $\mu_{i,j}(\Delta)$	$\mu_{i,j}(\Delta) = \mu(\Delta) = 2\times10^{-6}$ mm/operation, $\forall i, j$
Threshold for quality index $a_{i,j}$	$6a_{i,j} = 6a = 0.8\text{mm}, \forall i, j$
Unit degradation s.t.d. $\sigma_{i,j}(\Delta)$	$\sigma_{i,j}(\Delta) = \sigma(\Delta) = 5\times10^{-5}$ mm/operation, $\forall i,j$

In the study, two cases are investigated to demonstrate the concepts and potential applications of the proposed methodology.

(1) *System reliability analysis with and without considering the QR-Chain effect*: The system reliability analysis is conducted under three different definitions of system failures, which are:

(a) Only consider the probability of component catastrophic failures. That is, the pin degradation and the impact of the incoming product quality on the pin catastrophic failure are not considered in the model. It is equivalent to the case of setting the QR coefficient s and the pin degradation rate $\mu(\Delta)$ to zero in the QR-Chain model;

(b) Consider both the pin catastrophic failure and the product quality deterioration due to component wear-out, but without considering the impact of the incoming product quality on the catastrophic failure rate of the locating pins. It is equivalent to the case of setting the QR coefficient s in the QR-Chain model to zero;

(c) Consider the integrated QR-Chain model proposed in this paper.

The system reliability results obtained from the formulae in Section 4 and the calculation steps in Table 6 of Appendix 2 are shown in Figure 4. The Matlab code for the numerical evaluation of the system reliability was run on an IBM PC Pentium III machine. In average, it takes about 10 seconds to evaluate system reliability at a specific time based on the QR-Chain model. Therefore, the evaluation algorithm used in this paper is quite efficient and feasible. From the comparison study, it can be seen that the system reliabilities under definitions (a) and (b) are always higher than that under definition (c). The overestimation of (a) and (b) is not surprising. Based on the real production data, as discussed in the introduction, about 44% of locating tool catastrophic failures in BIW assembly processes are induced by incoming product quality. As a result, incorrect ignorance of the impact of incoming product quality as in (a) and

(b) may lead to significant overestimation of the overall system reliability. If a scheduled maintenance policy is planned based on the predicted system reliability using definition (a) or (b), many unexpected down times could be experienced due to neglecting the interdependency between product quality and reliability of locating pins.

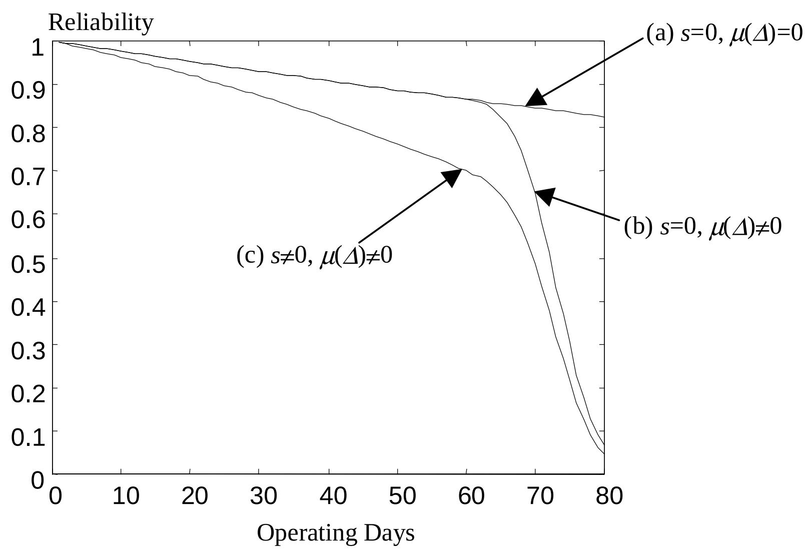


Figure 4. System reliability with and without considering the QR-Chain effect

(2) *Optimal setting of the wear rates of locating pins*: Suppose a preferred preventive maintenance cycle is every 50 production days based on the production schedule. From the simulation result in Figure 4, current system reliability at the 50th production day is only about 0.75. In order to reduce the unexpected system failure before the 50th production day, the designer needs to improve $R(t_{50})$. The overlap portion of the curves (a) and (b) in Figure 4 shows that the system failure before the 50th production day is mainly caused by the system catastrophic failure rather than the failure due to nonconforming products. So the failure due to nonconforming products by the 50th production day can be ignored (with probability of less than 1%). Furthermore, the difference between curve (c) and curves (a) and (b) shows that the incoming part quality has a significant impact on the catastrophic failures of locating pins by the 50th production day. Therefore the designer can improve $R(t_{50})$ by reducing the wear rates of the

locating pins so that the incoming product quality of each station is improved and the QR interactions are reduced.

In the original case study, the wear rate of each locating pin is set to be the same as 2×10^{-6} mm/operation. Assuming $w_{i,j}=w$, $\forall i,j$, the current pin fabricating cost can be calculated from (11) as $C_0 = \frac{12w}{2\times10^{-6}}$. Suppose the designer wants to improve $R(t_{50})$ with the available budget allowing increment of the pin fabrication cost by at most $\frac{1}{3}$. One simple way to do this is to reduce the wear rate of each locating pin from 2×10^{-6} to 1.5×10^{-6} mm/operation. However, the more efficient way is to spend more on reducing wear rates of critical locating pins determined by the QR-Chain model. For this purpose, an optimization problem can be formulated as

$$\begin{aligned} &\boldsymbol{\mu}^* = \operatorname{Arg}\max_{\boldsymbol{\mu}}\{R(t_{50})\} \\ &\text{subject to } C_p \le \frac{4}{3}C_0,\ 0 \le \mu_{i,j}(\Delta) \le 2\times10^{-6}, \forall i,j \end{aligned} \tag{16}$$

This optimization problem is a special case of (12) with K=50, $C_{\max} = \frac{4}{3}C_0$, $\mu_{\min} = 0$, and $\mu_{\max} = 2\times10^{-6}$. Also, the attention can be focused only on the system catastrophic failure because the failure due to nonconforming products can be ignored by the 50th production day.

Based on the discussion on the optimality of optimization problem (12), the optimal pin wear rate assignment is shown in Table 4. The optimization problem is solved by using the MATLAB function *fmincon* that uses a Sequential Quadratic Programming (SQP) method (MATLAB, 1999). The algorithm converges within 5 seconds, which is pretty efficient. From Table 4, it can be seen that there is no need to improve pins $P_{1,4}$, $P_{3,1}$, $P_{3,2}$, $P_{3,3}$, and $P_{3,4}$ in the optimal solution. First, we already calculated that the final product quality is satisfactory at t_{50} even if all the locating pin wear rates are not improved. So leaving these five pins not improved will not result in poor final product quality. Secondly, in terms of reducing the pin catastrophic failures before t_{50} under the constraint that the final product quality is satisfactory, the obtained result is consistent with the physical understanding of the process design. From Figures 2 and 3, Pins $P_{3,1}$, $P_{3,2}$, $P_{3,3}$ and $P_{3,4}$ are used at station 3, which is considered as the final station in this example whose output is the final product rather than the incoming part of the next station. So the degradation of these four pins will not contribute to the pin catastrophic failures of later stations. Locating pin $P_{1,4}$ contributes to the rotation movement of the B-Pillar around the four-way pin $P_{1,3}$. This movement does not affect the position of locating-hole for $P_{1,3}$, which is also used as the locating-hole for $P_{2,1}$ at station 2. So the degradation of $P_{1,4}$ does not contribute to the pin catastrophic failure of later stations either. Different degrees of improvements are performed for other locating pins based on their geometrical relationship and the part locating mechanism.

Table 4. Optimal pin wear rates $\boldsymbol{\mu}^*$ based on the QR-Chain model (10^{-6}mm/operation)

$\mu_{1,1}(\Delta)$	$\mu_{1,2}(\Delta)$	$\mu_{1,3}(\Delta)$	$\mu_{1,4}(\Delta)$	$\mu_{2,1}(\Delta)$	$\mu_{2,2}(\Delta)$
0.71	0.80	0.51	2.0	0.78	0.63
$\mu_{2.3}(\Delta)$	$\mu_{2,4}(\Delta)$	$\mu_{3,1}(\Delta)$	$\mu_{3,2}(\Delta)$	$\mu_{3,3}(\Delta)$	$\mu_{3,4}(\Delta)$
1.54	0.73	2.0	2.0	2.0	2.0

Table 5 is used to compare the original system failure probability at t_{50} (which is 1-$R(t_{50})$), the improved failure probability based on uniform 1.5×10^{-6} mm/operation wear rates for each pins, and the improved failure probability based on the optimal solution $\boldsymbol{\mu}^*$. Setting the wear rates uniformly improves the system failure probability by 9.7%. By using the optimal wear rate setting based on the QR-Chain model, the system failure probability can be improved by 23.2%, with the pin fabrication cost remaining the same as that of the uniform wear rate setting. Therefore, with the aid of the QR-Chain model developed in this paper, optimal design can be achieved to maximize the system reliability (or minimize system failure probability) under the constraints of available budget on tooling fabrication costs.

Table 5. Comparison of 1-$R(t_{50})$ for three different settings of pin wear rates

$\boldsymbol{\mu}$	2×10^{-6} mm/operation	1.5×10^{-6} mm/operation	$\boldsymbol{\mu}^*$
$F(t_{50})=1-R(t_{50})$	0.237	0.214	0.182

7. Summary

Quality and reliability are two important factors in manufacturing system design. In BIW assembly processes, real production data have shown significant interactions between locating tool reliability and product quality. This paper developed a *process model* of a multi-station BIW assembly process based on the process knowledge. The QR-Chain model is applied based on the *process model* to capture the QR interaction and its propagation through multi-station BIW assembly processes. Process and product design information of assembly systems were integrated into the QR-Chain model and a state space model was employed to study the variation propagation through all assembly stations. An analytical solution for system reliability evaluation has been obtained based on the QR-Chain model.

A case study is conducted in this paper, which shows that the system reliability will be overestimated if the QR interaction is improperly ignored. An optimization problem, as a typical application of the QR-Chain model, is formulated to optimally assign pin wear rates with constrained budget on pin fabrication costs. The optimality of the optimal solution is derived based on the analytical form of the system reliability. The optimal design obtained from the QR-Chain model significantly improves the system reliability compared with that obtained from current practice.

It should be noticed that future research is still needed on how to select and design techniques to achieve the assigned optimal wear rate for each locating pin. Another interesting future study following this paper is the integration of the maintenance decision-making and the wear rate design for locating pins, through which an overall optimal productivity of BIW assembly processes can be achieved.

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Appendix

Appendix 1. Details on Relationship between Pin Wear and Part Locating Errors

The relationship between pin wear and the part locating error in a fixture station has been studied in Jin and Chen (2001). Based on observations from lab and real autobody assembly plants, in most cases the locating pin touches the wall of the locating-hole during the assembly operations. So it is reasonable to assume that the locating-hole contacts with the pin on one side when the part is positioned by a fixture. Due to the still existing possibility that the locating pin does not touch the locating-hole, this assumption may result in slight overestimation of the product variation and conservative prediction of the reliability.

From the assumption above, the contacting orientations between the locating pin and the locating-hole for a four-way pin and a two-way pin are shown in Figure 5. The part locating error in the X-Z plane can be represented by the displacement of the locating-hole center from the center of the pin as shown in Figure 5. Based on Figure 5, the relationship between the part locating errors $\left(\Delta x_{P_{i,j}}, \Delta z_{P_{i,j}}\right)$ and the pin diameter reduction of a four-way pin can be obtained as

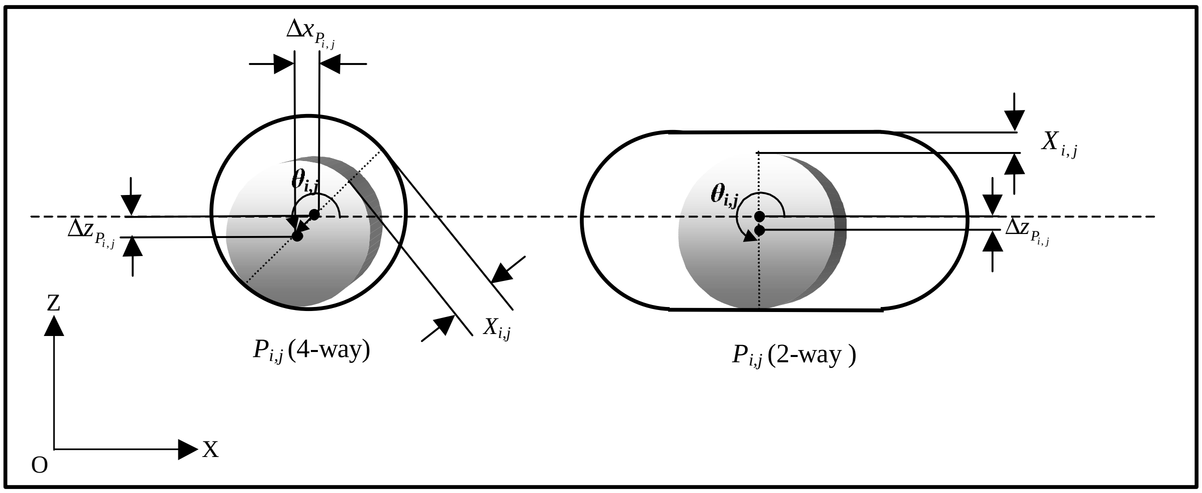

Figure 5. Part locating error due to pin wear

$$\Delta x_{P_{i,j}} = 0.5 X_{i,j} \cos\theta_{i,j}; \qquad \Delta z_{P_{i,j}} = 0.5 X_{i,j} \sin\theta_{i,j} \tag{17}$$

Here $\theta_{i,j}, \forall i, j$ are assumed to be independent random variables following uniform distribution within [0, 2π], which is denoted as $\theta_{i,j}$~U(0, 2π). It can also be obtained that $Var(\sin\theta_{i,j}) = Var(\cos\theta_{i,j}) = 0.5$ and $Cov(\sin\theta_{i,j}, \cos\theta_{i,j}) = 0$ for a four-way locating pin. Similarly, ignoring the effect of the wear of a two-way pin in the X direction, the relationship between the part locating error and the wear of the two-way pin can be obtained as

$$\Delta z_{P_{i,j}} = 0.5 X_{i,j} \sin\theta_{i,j} \tag{18}$$

Because the locating-hole contacts with the two-way pin either on the upper or the lower side in the Z direction, $\theta_{i,j}$ is a random variable having two values of -π/2 and π/2 with the same probability equal to 0.5. In this paper, it is denoted as $\theta_{i,j} \sim Unif\ \{-\pi/2, \pi/2\}$ for two-way locating pins. It can also be obtained that $Var(\sin\theta_{i,j}) = 1$ for a two-way locating pin.

Appendix 2. Parameters in (10) and procedure to calculate system reliability

Based on the reliability results for a general manufacturing process with QR-Chain effect in Chen et al. (2001), the parameters in (10) is given as

(i) $c \equiv \sum_{i=1}^{L}\sum_{j=1}^{n_i} \lambda_{i,j}(0) + \sum_{i=1}^{L}\sum_{j=1}^{n_i} \mathbf{s}_{i,j}^T \mathbf{d}$, where $\mathbf{d} \equiv [d_{1,1} \quad d_{1,2} \quad \dots \quad d_{L,m_L-1} \quad d_{L,m_L}]^T$, $d_{i,j}$ is obtained in (9);

(ii) $$\mathbf{\Sigma}_K \equiv \begin{bmatrix} \mathbf{\Sigma}(0) & \mathbf{\Sigma}(0,1) & \dots & \mathbf{\Sigma}(0,K) \\ \mathbf{\Sigma}(1,0) & \mathbf{\Sigma}(1) & \dots & \mathbf{\Sigma}(1,K) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Sigma}(K,0) & \mathbf{\Sigma}(K,1) & \dots & \mathbf{\Sigma}(K) \end{bmatrix}_{(n(K+1)\times n(K+1))}$$

From (1), $\mathbf{\Sigma}(k+i,k) \equiv \text{cov}(\mathbf{X}(t_{k+i}),\mathbf{X}(t_k)) = \mathbf{\Sigma}(k)$, $\mathbf{\Sigma}(k,k+i) = \mathbf{\Sigma}(k+i,k)^T = \mathbf{\Sigma}(k)$, and $\mathbf{\Sigma}(k) = \mathbf{\Sigma}(k-1) + \mathbf{Q}$, $k = 1,2,\dots$, $\mathbf{\Sigma}(0) = \mathbf{\Sigma}_0$;

(iii) $\mathbf{\Omega}_K$ is a domain in R^n s. t. $\mathbf{x}(t_K) \in \mathbf{\Omega}_K \Leftrightarrow \bigcap_{i=1}^{L}\bigcap_{j}^{m_i}\{\mathbf{x}^T(t_K)\mathbf{B}_{i,j}\mathbf{x}(t_K) \le a_{i,j} - d_{i,j}\}$;

(iv) The distribution of $\tilde{\mathbf{X}}(t_K)$ is $\tilde{\mathbf{X}}(t_K) \sim N(\tilde{\boldsymbol{\mu}}(K), \tilde{\mathbf{\Sigma}}(K))$, where $\tilde{\boldsymbol{\mu}}(K)$ and $\tilde{\mathbf{\Sigma}}(K)$ can be obtained by partitioning a matrix $\tilde{\mathbf{\Sigma}}_K$ and a vector $\tilde{\boldsymbol{\mu}}_K$ as

$$\tilde{\mathbf{\Sigma}}_K = \left[\begin{array}{c|c} \underset{(nK\times nK)}{\mathbf{\Sigma}11} & \underset{(nK\times n)}{\mathbf{\Sigma}12} \\ \hline \underset{(n\times nK)}{\mathbf{\Sigma}21} & \underset{(n\times n)}{\mathbf{\Sigma}22} \end{array}\right], \quad \tilde{\boldsymbol{\mu}}_K = \begin{bmatrix} \underset{(nK\times 1)}{\boldsymbol{\mu}\mathbf{1}} \\ \underset{(n\times 1)}{\boldsymbol{\mu}\mathbf{2}} \end{bmatrix},$$ and $\tilde{\boldsymbol{\mu}}(K) = \boldsymbol{\mu}\mathbf{2}$ and $\tilde{\mathbf{\Sigma}}(K) = \mathbf{\Sigma}22$. From Chen et al. (2001), $\tilde{\boldsymbol{\mu}}_K$ and $\tilde{\mathbf{\Sigma}}_K$ can be calculated by

$$\tilde{\boldsymbol{\mu}}_K \equiv \left(\mathbf{U}_K + \mathbf{\Sigma}_K^{-1}/2\right)^{-1}\left(\mathbf{\Sigma}_K^{-1}/2\right)\boldsymbol{\mu}_K, \quad \tilde{\mathbf{\Sigma}}_K^{-1} \equiv 2\mathbf{U}_K + \mathbf{\Sigma}_K^{-1};$$

where $\mathbf{U}_K = \begin{bmatrix} \mathbf{I}_{(K\times K)} \otimes \mathbf{U}' & \mathbf{0}_{(nK\times n)} \\ \mathbf{0}_{(n\times nK)} & \mathbf{0}_{(n\times n)} \end{bmatrix}$, $\mathbf{U}' \equiv h\sum_{i=1}^{L}\sum_{j=1}^{n_i}\sum_{k=1}^{i-1}\sum_{l=1}^{m_k}\left[\mathbf{s}_{i,j}\right]_{q_{k-1}+l}\mathbf{B}_{k,l}$ and $q_{k-1} \equiv \sum_{i=1}^{k-1} m_i$;

(v) ρ_K can be calculated as

$$\rho_K = \boldsymbol{\mu}_K^T\left[\mathbf{U}_K^T\left(\mathbf{U}_K + \left(\boldsymbol{\Sigma}_K^{-1}/2\right)\right)^{-T}\left(\boldsymbol{\Sigma}_K^{-1}/2\right)\right]\boldsymbol{\mu}_K > 0, \tag{19}$$

where $\boldsymbol{\mu}_K \equiv \left[\boldsymbol{\mu}^T(0) \quad \boldsymbol{\mu}^T(1) \quad \ldots \quad \boldsymbol{\mu}^T(K)\right]^T$, $\boldsymbol{\mu}(k) = \boldsymbol{\mu}(k-1) + \boldsymbol{\mu}_\varepsilon, k \geq 1,$ and $\boldsymbol{\mu}(0) = \boldsymbol{\mu}_0$.

The general procedure to calculated system reliability based on the input information listed in Section 4 is summarized in the following table.

Table 6. General procedures to calculate system reliability of BIW assembly processes

Step	Outputs	Inputs	Related formula/results in the paper
1	cov($\mathbf{z}_t$)	(c)*	(7) and Appendix 1
2	$\boldsymbol{\Gamma}_{i,j}$	(a) and (b)	Section 3.2.3 and (3)
3	$\mathbf{Q}$ and Σ_0	(d) and (e)	(1)
4	$\boldsymbol{\mu}_K$	(d) and (e)	(19)
5	$\mathbf{d}$	(a), (b), and cov($\mathbf{z}_t$) from Step 1	(8) and (9)
6	$\mathbf{B}_{i,j}$	$\boldsymbol{\Gamma}_{i,j}$ from step 2 and cov($\mathbf{z}_t$) from step 1	(9)
7	c	(f), (g), and $\mathbf{d}$ from step 5	Appendix 2(i)
8	Σ_K	$\mathbf{Q}$ and Σ_0 from step 3	Appendix 2(ii)
9	$\boldsymbol{\Omega}_K$	(h), $\mathbf{d}$ from step 5, and $\mathbf{B}_{i,j}$ from step 6	Appendix 2(iii)
10	$\mathbf{U}_K$	(g), (i), $\mathbf{B}_{i,j}$ from step 6	Appendix 2(iv)
11	ρ_K	$\boldsymbol{\mu}_K$ from step 4, Σ_K from step 8, and $\mathbf{U}_K$ from step 10	Appendix 2(v)
12	$\tilde{\boldsymbol{\mu}}_K$	$\boldsymbol{\mu}_K$ from step 4, $\mathbf{U}_K$ from step 10, and Σ_K from step 8	Appendix 2(iv)
13	$\tilde{\boldsymbol{\Sigma}}_K$	Σ_K from step 8 and $\mathbf{U}_K$ from step 10	Appendix 2(iv)
14	$F_{\tilde{\mathbf{x}}(t_K)}$	$\tilde{\boldsymbol{\Sigma}}_K$ from step 13 and $\tilde{\boldsymbol{\mu}}_K$ from step 12	Appendix 2(iv)
15	$R_I(t_K)$	c from step 7, ρ_K from step 11, Σ_K from step 8, and $\tilde{\boldsymbol{\Sigma}}_K$ from step 13	(10)
16	$R_{II}(t_K)$	$\boldsymbol{\Omega}_K$ from step 9 and $F_{\tilde{\mathbf{x}}(t_K)}$ from step 14	(10)
17	$R(t_K)$	$R_I(t_K)$ from step 15 and $R_{II}(t_K)$ from step 16	(10)

* The input numbering in parenthesis follows that of the input list in Section 4.