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Robust Optimal Influence-Coefficient Control of Multiple-Plane Active Rotor Balancing Systems

Rotating mass imbalance causes harmful vibration of high-speed machine tools, turbomachinery, etc. Constant speed, steady-state influence coefficient control allows active balancing systems to suppress this vibration if the influence matrix is estimated accurately. An optimal strategy for multiple-plane active balancing control is presented here that improves control robustness to modeling and estimation errors. The vibration controller objectively trades off residual vibration, control effort, and control rate of change. Penalizing control effort and rate of change is shown to enhance control stability margin, with certain performance trade-offs. Experimental results illustrate the improvement in control robustness compared with traditional weighted least squares optimal control.
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Introduction

Active balancing systems incorporate balance mass actuators that can change a rotating machine's mass balance state while the machine continues operating. Steady-state vibration caused by time-varying rotating mass imbalance can often be controlled using such active balancing systems [1,2]. As noted by Bishop [3], single-plane active balancing should be sufficient for systems in which one vibrational mode dominates dynamic response at each operational speed of interest. However, many high-speed rotating machines operate at speeds in between flexible shaft bending mode frequencies. This situation, potentially combined with high bearing damping and anisotopic radial bearing stiffness, can lead to a situation in which more than one mode contributes to dynamic response at operating speed. Such scenarios do not meet Bishop's criterion and thus require multiple-plane active balancing

Multiple-plane balancing can be expected to eliminate shaft-synchronous vibration only at as many sensor points as there are balance planes. Because anisotropic bearings can cause rotor orbits to be elliptical, minimizing vibration in one radial direction will not, in general, minimize vibration in the orthogonal radial direction. Furthermore, engineers may wish to suppress imbalance-induced vibration as much as possible all along the rotor shaft. Thus, more sensors than active balance planes could be utilized and some type of optimal control of the active balancing system is necessary.

Steady-state influence coefficient-based optimal control strategies have been presented for rotating systems both with active balancers [4] and with stationary actuation schemes such as magnetic bearings [5] and piezoelectric actuators [6]. These controllers utilized weighted least-squares (WLS) vibration objective functions to determine optimal control outputs. Manchala et al. [7] also developed a constrained quadratic programming (CQP) control method to ensure minimization of least squared vibration in the event of actuator saturation.

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WLS control has been shown to be stable as long as the influence matrix estimate is accurate enough. However, it is sometimes difficult to obtain an accurate estimate of the system dynamics. Changes in bearing stiffness and damping can occur because of component degradation, thermal effects, load variations, etc. Machine foundation characteristics can vary substantially due to temperature and moisture fluctuations as the seasons change. Because critical process machinery cannot be shut down for periodic dynamic system identification, it is often necessary to operate the active balancing system with inaccurate influence matrix data. Therefore, means of improving control robustness to this estimation uncertainty are often useful or necessary.

For this work, an extended quadratic objective function is defined that includes weighted terms penalizing not only least squares vibration, but also control effort (balance correction), and control rate of change (control speed of response). The terms penalizing control effort and control rate of change are shown to enhance stability robustness in the face of influence matrix estimation errors. Experimental results are presented that validate the effectiveness of the robust optimal control on a laboratory flexible-rotor test rig.

Derivation of Optimal Control Law

At a constant shaft rotational speed, the rotation-synchronous frequency vibration at each of n vibration sensors is made up of the disturbance affecting that sensor and the sum of the influence of all m active balance correction planes. This can be described using the influence matrix equation

$$\{E\}_k=[C]\{W\}_k+\{D\} \tag{1}$$

where $\{E\}_k$ is a complex $n\times 1$ "error" vector containing the rotation-synchronous vibration phasor at each sensor during control iteration k. $\{W\}_k$ is the complex $m\times 1$ active balance correction vector at control iteration k. $[C]$ is the complex $n\times m$ influence matrix relating unbalance to steady-state vibration. $\{D\}$ is the complex $n\times 1$ vibration disturbance vector.

Past research detailed control techniques that commanded the balance correction vector to minimize the weighted least squared (WLS) vibration error during constant-speed rotation. A real, positive semi-definite $n\times n$ diagonal penalty matrix $[Q]$ can be specified to give the desired relative weighting of the n error sensors [8,9]:

$$[Q]=\begin{bmatrix} q_1 & & & 0 \\ & q_2 & & \\ & & \ddots & \\ 0 & & & q_n \end{bmatrix} \tag{2}$$

Note that q_i is the relative weight of the ith error sensor.

The robustness of the WLS control approach in the face of various types of estimation uncertainties has been previously characterized [10]. For this new research, additional terms were added to the objective function to provide more control flexibility and increase stability robustness. Real, positive semi-definite $m\times m$ weighting matrices $[R]$ and $[S]$ can be defined to penalize control effort, and control rate of change respectively.

Aside from their primary impact on control robustness, such penalty matrices also provide the opportunity for more cautious control than the traditional WLS approach. Such caution is often justified in very conservative industries utilizing critical turbomachinery in continuous processes. This is especially true because the control is meant for application at constant rotating speed conditions and is not generally fast enough to respond during fast rotor accelerations.

The $[R]$ matrix allows penalizing the control effort at each active balance plane, which could be desirable in certain cases such as during an emergency shutdown. If the source of residual unbalance was not co-located with the active balance planes, the optimal balance correction could be different at different shaft rotational speeds. The optimal balance correction at one operating speed may sometimes cause harmful vibration at another slower speed. It may be necessary to maintain fixed balance correction if an emergency shutdown resulted in deceleration through a critical speed faster than the active balancing control could track. By conservatively limiting the control effort using a nonzero $[R]$ matrix, the potential for harmful vibration during such an emergency shutdown is reduced. Obviously, this comes at the expense of higher than minimum vibration levels at the nominal operating speed.

The $[S]$ matrix provides for penalizing the speed of control response. This can have the benefit of potentially allowing enough time for operator intervention in the case of any sort of system malfunction.

An extended objective function $J(\{E\}_{k+1},\{W\}_{k+1},\{W\}_k,[Q],[R],[S])$ can be defined such that

$$J=\frac{1}{2}\{E\}^*_{k+1}[Q]\{E\}_{k+1}+\frac{1}{2}\{W\}^*_{k+1}[R]\{W\}_{k+1} + \frac{1}{2}(\{W\}_{k+1}-\{W\}_k)^*[S](\{W\}_{k+1}-\{W\}_k) \tag{3}$$

where the "*" symbol denotes the complex-conjugate transpose operator. The optimal control problem then consists of commanding the balance weight vector for the next control iteration $\{W\}_{k+1}$ so as to minimize the objective function J.

For constant speed applications, one can assume that the influence matrix $[C]$ and disturbance vector $\{D\}$ does not change over one control iteration. The error vector relationship of Eq. (1) can then be applied for two control iterations and substituted into Eq. (3). This allows evaluation of the objective function J using only the updated control vector $\{W\}_{k+1}$ and measured values $\{E\}_k$ and $\{W\}_k$:

$$J=\frac{1}{2}([C](\{W\}_{k+1}-\{W\}_k)+\{E\}_k)^*[Q]([C](\{W\}_{k+1} - \{W\}_k)+\{E\}_k)+\frac{1}{2}\{W\}^*_{k+1}[R]\{W\}_{k+1}+\frac{1}{2}(\{W\}_{k+1} - \{W\}_k)^*[S](\{W\}_{k+1}-\{W\}_k) \tag{4}$$

To find the stationary points of the objective function J with respect to the updated control vector $\{W\}_{k+1}$, one can take the corresponding partial derivative, set it equal to zero and solve for the control vector. This stationary point is guaranteed to be a minimum if the objective function J is truly a quadratic function of $\{W\}_{k+1}$. To ensure that J is a quadratic function, at least one of the matrices $[Q]$, $[R]$, or $[S]$ must be positive definite. Furthermore, if only $[Q]$ is positive-definite, then the matrix $[C]$ must also be full rank (rank m in this case) for J to be quadratic. To ensure that the influence matrix $[C]$ is full rank, care must be taken to install the active balancing devices in appropriate planes. Two issues should be considered when choosing the active balance planes: 1) avoid placing balancers at nodal locations of any vibrational modes to be controlled; and 2) make sure that the influences from each balance plane on the error vector are linearly independent for all speeds at which the active system will be operated.

Using well-known matrix calculus techniques [11] to take the partial derivative, the formula for the minimum objective function is found to be

$$\frac{\partial J}{\partial\{W\}_{k+1}}=0=[C]^*[Q]([C](\{W\}_{k+1}-\{W\}_k)+\{E\}_k)+[R] \times\{W\}_{k+1}+[S](\{W\}_{k+1}-\{W\}_k) \tag{5}$$

The optimal control vector update $\{W\}_{k+1}$ can be isolated algebraically as follows:

$$\{W\}_{k+1}=([C]^*[Q][C]+[R]+[S])^{-1}(([C]^*[Q][C]+[S]) \times\{W\}_k-[C]^*[Q]\{E\}_k) \tag{6}$$

At steady-state, assuming that control has converged, the optimal control vector is then given by

$$\{W\}_{\infty,opt}=-([C]^*[Q][C]+[R])^{-1}[C]^*[Q]\{D\} \tag{7}$$

and the subsequent steady-state error is given by

$$\{E\}_{\infty,opt}=([I_{n\times n}]-[C]([C]^*[Q][C]+[R])^{-1}[C]^*[Q])\{D\} \tag{8}$$

where $[I_{n\times n}]$ is the $n\times n$ identity matrix.

Note that the practical realization of the optimal control law in Eq. (6) requires use of an estimated influence matrix $[\hat{C}]$ instead of the generally unknown or time-varying actual influence matrix. Analytical or numerical modeling is usually not sufficiently accurate to estimate influence matrices. However, influence matrices can be estimated by selectively exercising the active balance actuators while the machine dwells at a constant operating speed. This process can be repeated for many discrete speeds throughout the operating speed range and the data stored in a table.

After this *a priori* system identification, a rotor speed sensor can then be used during operation to allow selection of the influence estimate corresponding to the current rotational speed. The performance and stability of the control depends on maintaining a relatively constant rotor speed while the control converges. Even with *a priori* system identification, however, actual influence matrices can change due to the various factors mentioned previously. Furthermore, since the identification process can be expensive for critical machinery it usually is not possible to perform periodic data updates. Thus, influence matrix estimates can be inaccurate, leading to control performance and stability problems.

Optimal Control Performance and Stability Analysis

Aside from practical control benefits discussed previously, both the $[R]$ and $[S]$ penalty matrices have the added advantage of enhancing the control stability margin. By limiting the control inputs or slowing down the control response, an implementation that might originally be unstable due to an erroneous influence matrix estimate can be made stable. An analysis of the optimal control stability follows.

Replacing the actual influence matrix $[C]$ in the optimal control law of Eq. (6) with the estimated influence matrix $[\hat{C}]$ and substituting in the error vector response relationship of Eq. (1), we obtain the recursive relationship

$$\{W\}_{k+1}=([\hat{C}]^*[Q][\hat{C}]+[R]+[S])^{-1}([\hat{C}]^*[Q]([\hat{C}]-[C])+[S])\{W\}_k-([\hat{C}]^*[Q][\hat{C}]+[R]+[S])^{-1}[\hat{C}]^*[Q]\times\{D\} \tag{9}$$

For stable control, the control input $\{W\}_{k+1}$ must converge to a constant value. Only the first term multiplied by $\{W\}_k$ is germane to the stability question. The second term multiplying $\{D\}$ is constant and only affects the correction vector to which the steady-state control converges. It then follows that the optimal active balancing control is stable for a constant disturbance vector $\{D\}$ and constant influence matrices $[C]$ and $[\hat{C}]$ if and only if:

$$\alpha=\bar{\sigma}[([\hat{C}]^*[Q][\hat{C}]+[R]+[S])^{-1}([\hat{C}]^*[Q]([\hat{C}]-[C])+[S])]<1 \tag{10}$$

where α is a stability metric and the symbol $\bar{\sigma}$ signifies the maximum singular value. Setting $[R]=[S]=[0]$ results in the stability criterion derived by Knospe et al. [10] for traditional WLS control.

Assuming stable control, and that the disturbance vector $\{D\}$ and actual influence coefficient $[C]$ are constant during control convergence, it can be shown that the balance correction vector will converge at steady state to

$$\{W\}_\infty=-([\hat{C}]^*[Q][C]+[R])^{-1}[\hat{C}]^*[Q]\{D\} \tag{11}$$

The corresponding steady-state error vector will be

$$\{E\}_\infty=([I_{n\times n}]-[C]([\hat{C}]^*[Q][C]+[R])^{-1}[\hat{C}]^*[Q])\{D\} \tag{12}$$

One effect of the matrix $[R]$ on stability is evident from inspecting Eq. (11). Regardless of the estimation error ($[\hat{C}]-[C]$), an $[R]$ matrix with large enough norm will render the control "stable" in one sense by effectively preventing the control effort, and hence vibration error, from increasing unbounded. This same type of bounding would naturally occur if the output of the active balance actuators saturated. However, for accurate enough influence matrix estimates, an $[R]$ matrix will exist for which

$$\bar{\sigma}([I_{n\times n}]-[C]([\hat{C}]^*[Q][C]+[R])^{-1}[\hat{C}]^*[Q])<1 \tag{13}$$

In this case, not only will the control be stable, but the weighted sum of squared (WSS) vibration error will also be reduced. Thus, the $[R]$ matrix can provide stable control for some situations in which traditional WLS control would not be stable. This improved stability robustness cannot be achieved by simply allowing actuators to saturate.

The effect of the control rate penalty matrix $[S]$ on stability can also be clarified by inspection of Eq. (10). Note that since $[\hat{C}]^*[Q][\hat{C}]$ will always be positive semi-definite, if only the real part of $[\hat{C}]^*[Q][C]$ is positive definite, and if both $[\hat{C}]$ and $[C]$ are full rank (rank m), then there will exist a positive semi-definite real matrix $[S]$ for which the stability criterion of Eq. (10) is met. For the special case of single-plane, single error sensor control, this criterion will be met when the phase angle of the influence coefficient estimate is within $\pm 90°$ of the actual value. Past research has noted that single-plane active balancing control could be stabilized using a small enough control gain [12]. This is analogous to a multiple-plane balancing penalty matrix $[S]$ with "large enough" norm.

If one applies nonzero $[R]$ or $[S]$ penalty matrices purely to enhance stability robustness of the non-adaptive control, certain performance trade-offs become necessary. By inspecting Eqs. (11) and (12), it is evident that as the norm of $[R]$ approaches infinity, the steady-state balance correction approaches the zero vector and the steady-state controlled error vector approaches the disturbance $\{D\}$. Thus the stability robustness associated with a large $[R]$ matrix norm comes at the expense of reduced WSS error control performance at steady-state.

If the real part of the term $[\hat{C}]^*[Q][C]$ happens to be positive definite (and $[C]$ and $[\hat{C}]$ are full rank), then the $[S]$ matrix can be increased to enhance stability margin. Although this comes at the expense of slowing down the control convergence rate, there is no detrimental effect on the steady-state WSS control performance.

Experimental Results

The optimal control law was implemented in the laboratory for a two-plane active balancing system on a flexible rotor test rig. The test rig consisted of a 16.5 mm (0.65 in.) shaft supported over a span of 762 mm (30 in.) on two ball bearings and driven by a DC motor through a narrow "quillshaft" coupling. Active balancing devices were mounted to the shaft at approximately the third-span locations. These balance actuators each contained two counterweighted rotors that could be independently positioned in discrete angular locations even as the shaft continued to rotate.

The devices, described in more detail by Pardivala et al. [2], used permanent magnets to lock the balance rotors in position and non-contacting stationary coils to move the rotors electromagnetically when re-positioning was desired at each active balancing control iteration. Each balance actuator and disk assembly weighed approximately 4.5 kg (10 lb).

Eddy current proximity probes mounted close to, and outboard of each of the two balancing disk locations were used to measure

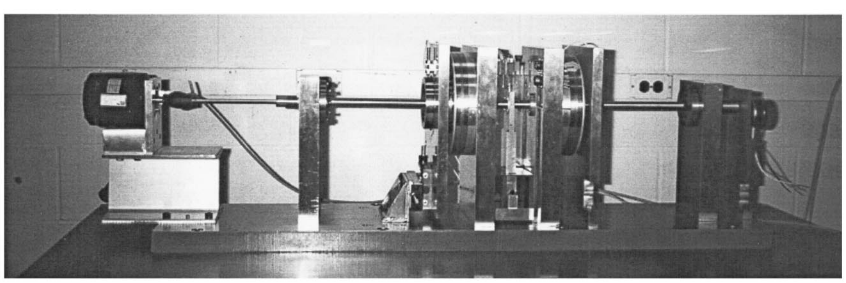

Fig. 1 Experimental flexible rotor test stand with active balancing devices installed in two axial planes

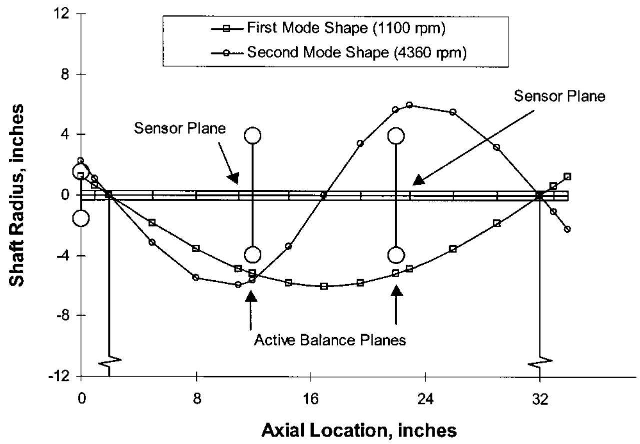

Fig. 2 Predicted mode shapes of flexible rotor test rig for first two critical speeds

radial shaft deflection in two orthogonal (i.e., x and y) directions. The apparatus used in the tests of the optimal control strategy is shown in Fig. 1. The first two critical speeds of the rotating test rig were measured to be approximately 1100 rpm and 4360 rpm. The predicted mode shapes for these two critical speeds are shown in Fig. 2.

To test the active balancing system for a plant with ‘‘rich’’ rotordynamics, the rotor was run at a constant speed of 3100 rpm during the experiments. This speed was in between the first two critical speeds. Therefore, both the first and second modes contributed significantly to dynamic response. Figure 3 shows the predicted forced-response deflected shape of the rotor due to a 245 g-mm (0.34 oz-in) unbalance at the drive-end (left end) active balancing plane. This unbalance represented the maximum correction capacity of the active balancing device.

Figure 3 could theoretically be used to predict the influence coefficients at 3100 rpm from the drive-end active balancing device to the two sensors. The combination of contributions of the two mode shapes at 3100 rpm provided for a relatively interesting response situation.

Note that unbalance at the drive end balance plane had a much greater influence on shaft response at the outboard sensor than on the drive-end sensor. Since the rotor configuration was highly symmetrical, the outboard balance plane also had the greatest influence on the drive-end sensor. The actual experimental results matched this prediction fairly well in a qualitative sense. The actual measured influence matrix from both balance planes on the four vibration sensors at 3100 rpm is presented in Table 1.

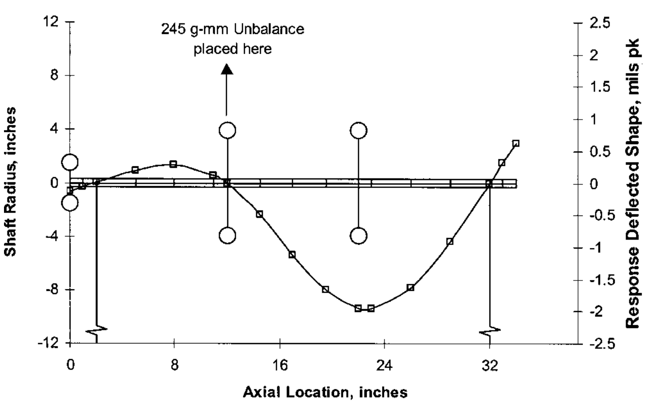

Fig. 3 Predicted forced response deflected shape of rotor at 3100 rpm due to 245 g-mm unbalance at drive-end active balancing plane

Table 1 Measured active balancing influence matrix for full balance capacity at 3100 rpm

Sensor / Influence (microns p-p) / Active Balance Plane

$$\begin{Bmatrix} \text{DriveEnd}_y \\ \text{Outboard}_y \\ \text{DriveEnd}_x \\ \text{Outboard}_x \end{Bmatrix} = \begin{bmatrix} 30.5\angle 178^\circ & 107\angle 228^\circ \\ 124\angle 343^\circ & 33\angle 152^\circ \\ 25.4\angle 152^\circ & 132\angle 142^\circ \\ 107\angle 78^\circ & 35.5\angle 226^\circ \end{bmatrix} \begin{Bmatrix} \text{DriveEnd} \\ \text{Outboard} \end{Bmatrix}$$

Multiple-Plane Versus Single-Plane Active Balancing Results. Because two modes contribute significantly to response at 3100 rpm, one active balance plane in general would not be sufficient to balance the rotor. This case represents the exception to the conclusion of Bishop [3] that only one active balancing device is necessary to balance a flexible rotor. His conclusion was based on the assumption that only one mode dominates response at any given speed. Figure 4 shows the results of active balancing using one balance plane (drive-end) compared with the results using two balance planes. The control penalty weighting matrices had the same values $[Q]=[I]$, and $[R]=[S]=[0]$ for both the single and dual plane cases. The vibration disturbance $\{D\}$ for both tests was the same. Note that the initial vibrations for the one and two-plane tests were not the same because the initial active balance correction in the single-plane balancing experiment was not zero.

It is evident from Fig. 4 that the single-plane optimal control does not match the performance of the dual-plane control. In fact, the single-plane control is only able to improve a small amount over the initial WSS vibration error. This is an indication that: 1) residual unbalance excited more than one mode; and 2) the disturbance vector lay mostly outside the span of the single-plane ‘‘controllability’’ space. The latter statement implies that the residual unbalance was not concentrated at the active balance plane and, thus, did not excite the two modes in the same way as the active balance correction.

Penalized Control Effort Results. As noted above, there may be occasions where balance correction should be restricted to allow for safe traversing of different vibration modes in the event of an emergency shutdown of the machinery. In this case the ‘‘control effort’’ penalty matrix $[R]$ could be increased to penalize correction magnitude. Figure 5 compares multiple-plane optimal control results for both zero and non-zero $[R]$ matrices.

Note that the penalized steady-state control effort (balance correction magnitude) is reduced to about one half of the nonpenalized control effort. This optimal trade-off obviously also results in higher steady-state vibration error magnitudes.

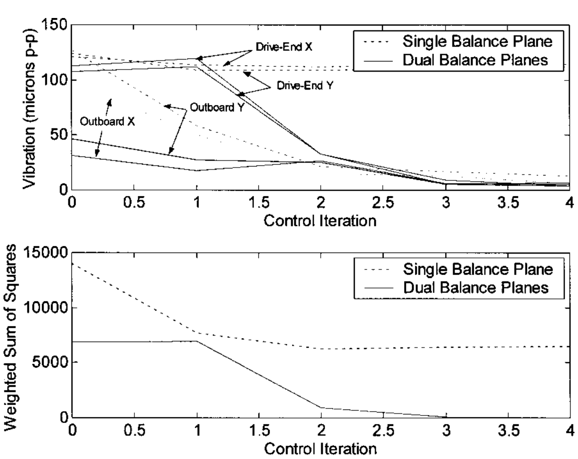

Fig. 4 Comparitive results of single-plane and dual-plane active balancing

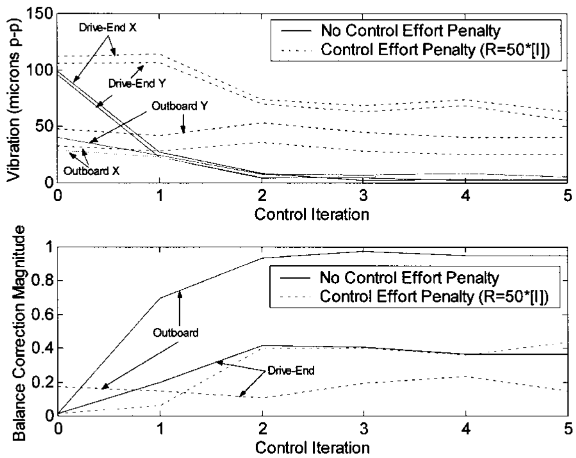

Fig. 5 Optimal control results with and without control effort penalty

Comparative Results of Control With Inaccurate Influence Matrix Estimate. A significant test of the optimal control robustness was accomplished by providing an erroneous influence matrix estimate to the controller. Figure 6 shows a comparison of optimal control results for three control scenarios when the initial influence matrix estimate was moderately inaccurate. Each term of the influence matrix estimate for the test was approximately 0.6 times the magnitude and rotated 70 deg from the corresponding term in the actual matrix shown in Table 1. Figure 6(*a*) shows the unstable performance resulting from traditional WLS Error control under this condition. Figure 6(*b*) shows how the control was stabilized using a non-zero $[R]$ matrix. Thirdly, Fig. 6(*c*) illustrates control stabilization using a non-zero $[S]$ matrix. Figure 6(*d*) shows the WSS error performance of each type of optimal control. During all the tests the control was turned off upon convergence, or divergence as in the case of Fig. 6(*a*). Error sensors all received equal weighting during the tests and the $[R]$ and $[S]$ matrices were either diagonal with the value shown on the diagonal or zero matrices if not noted.

Because of the inaccurate influence matrix estimate, the conventional WLS control was unstable as shown in Fig. 6(*a*). Note that because the control was not stable, Eqs. (11) and (12) predicting steady-state control values did not apply. The active balancing devices were both saturated (outputting maximum correction possible), providing the only limit to the vibration increasing further. Such high vibration values represented potentially damaging levels for typical production machines. The oscillations in vibration levels after saturation were caused by the oscillating correction phase angle. If the balancer outputs had not been saturated, the vibration would have continually increased.

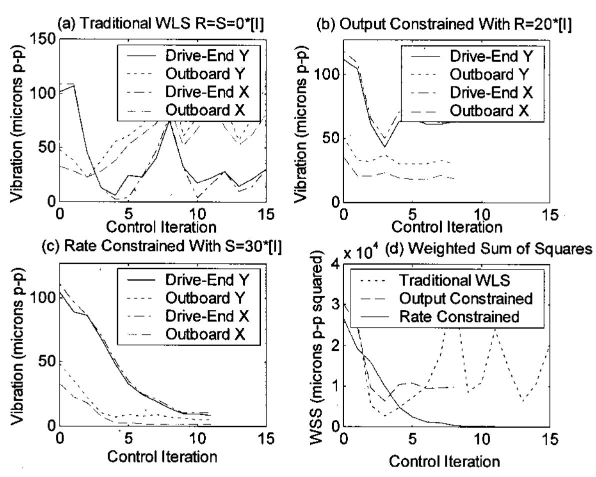

Fig. 6 Optimal control results for inaccurate influence matrix estimate

Adding a nonzero penalty on control effort to the objective performance function had the positive effect of producing stable control as seen in Fig. 6(*b*). Furthermore, since the criterion of Eq. (13) was met by using the control effort penalty, WSS error was reduced from the uncontrolled value as shown in Fig. 6(*d*). However, the controlled WSS error values were not as low as those for the control rate penalized case.

The use of a nonzero control rate of change penalty term in the objective performance function also resulted in stable control as illustrated in Fig. 6(*c*). The control convergence rate was slower than for the control effort penalized case. But, the control rate penalized case provided very good steady-state WSS control.

For stable control, the stability metric α of Eq. (10) must be below the value one. Using the actual influence measured matrix shown in Table 1 for $[C]$ in Eq. (10), the stability metric for the traditional WLS control had a computed value close to two, indicating instability. By adding nonzero penalty matrices $[R]$ and $[S]$, the stability criterion value dropped below one to 0.62 and 0.93 respectively and the control converged in both cases.

Summary and Conclusions

Multiple-plane active balancing is necessary when the effect of more than one vibrational mode is significant at any operating speed of interest and a single balance plane cannot be placed in the same axial plane as the major residual unbalance(s).

An optimal control law was derived based on the influence coefficient approach so that the active balancing system minimized a quadratic objective performance function. The objective performance function includes weighting matrices for steady-state residual error signals, control effort and control rate-of-change. The latter two terms provide a means for implementing more cautious control such as required in many critical industrial continuous processes. Furthermore, they were shown to have a beneficial effect on the stability robustness of the optimal control.

A stability criterion was derived for the optimal control that indicates the optimal control is unconditionally stable when the estimated influence matrix is sufficiently accurate. A control effort penalty matrix can be chosen to provide stable (bounded) steady-state control regardless of the influence matrix estimation accuracy. When a certain criterion is met, this control effort penalty is also guaranteed to reduce steady-state controlled vibration. However, this approach limits the minimum attainable steady-state vibration levels. Therefore, it should be used only when additional considerations exist to motivate the need for extra caution in the control.

If $\mathrm{Re}([\hat{C}]^*[Q][C])$ is positive definite, and both $[\hat{C}]$ and $[C]$ are full rank, then a nonzero real control rate of change penalty matrix can be selected so that the control is stable. This approach necessarily reduces the rate of control convergence. The steady-state WSS error performance, however, will be much better than if a control error penalty term was used to ensure control stability. Therefore, the control rate penalty should be used as the first choice for enhancing control stability robustness in the absence of other considerations.

Experimental results verified the analytical stability and performance analyses for the optimal control. The optimal control strategies presented here could be applied to a wide variety of constant-speed industrial machinery to ensure stable minimization of harmful vibration regardless of unknown or time-varying dynamics.

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