Poloidal rotation and density asymmetries in a tokamak plasma with strong toroidal rotation

W. M. Stacey, Jr.
Fusion Research Center and Nuclear Engineering Program, Georgia Institute of Technology, Atlanta, Georgia 30332

A neoclassical model for calculating poloidal rotation and poloidal density asymmetries in a tokamak plasma with $v_\phi \sim v_{th}$ and $E/B_\theta \sim O(1)$ is developed. Application is made to the analysis of a deuterium plasma with a dominant carbon impurity. The dependences of the results on collisionality, the anomalous radial deuterium flux and the toroidal rotation speed are evaluated. The implications of the calculated poloidal velocities and density asymmetries for the magnitude of the gyroviscous torque are discussed.

I. INTRODUCTION

Poloidal rotation and poloidal density asymmetries are topics of intrinsic interest in tokamak research which have received increased attention recently because of their apparent linkage to confinement properties. These two quantities are coupled via particle and momentum conservation requirements, which makes it natural to consider them together.

The neoclassical theory[1,2] of poloidal rotation was developed from drift kinetic theory under the standard ordering assumptions

$$\delta \equiv \frac{\rho}{L} \ll 1, \quad \frac{E}{B} = O(\delta), \quad \frac{\partial}{\partial t} = O(\delta^2), \quad \frac{v}{v_{th}} = O(\delta),$$

where ρ is the gyroradius, L is a characteristic gradient scale length, E and B are the electric and magnetic fields, respectively, and v and v_{th} are the average and thermal velocities, respectively. The resulting expression for the poloidal rotation velocity v_θ depends upon the collisionality parameter $v_* \equiv v/\omega$, where v is the collision frequency and $\omega \equiv v_{th}/L$ is the thermal transient frequency. In the large aspect ratio case,[1] with $\epsilon \equiv r/R$,

$$v_\theta = \frac{k}{eB} \frac{\partial T}{\partial r}, \tag{1}$$

where

$$k = \begin{cases} 1.17 & \text{for} \quad \epsilon^{-3/2} > v_*^{-1}, \\ -0.5 & \text{for} \quad 1 \ll v_*^{-1} \ll \epsilon^{-3/2}, \\ -2.1 & \text{for} \quad v_*^{-1} \ll 1. \end{cases} \tag{2}$$

Subsequent authors[3-6] have used the Hirshman–Sigmar moment approach[7] to obtain

$$v_\theta = KB_\theta/n \tag{3}$$

where K is a quantity constant on a flux surface which is evaluated from momentum balance, and n is the particle density. These authors have all noted that the poloidal velocity is different for the impurity and main ion species.

We note that in many present tokamaks with unbalanced neutral beam injection (NBI), the average toroidal rotation velocity is such that $v/v_{th} \sim O(1)$ and that this condition implies[8] $E/B \sim O(1)$. Thus, the standard neoclassical expression, Eqs. (1) and (2), may not pertain to tokamak plasmas with unbalanced NBI. References[3-6] did not invoke the standard neoclassical ordering arguments.

Recently, the observation[9-11] of changes in poloidal rotation accompanying the L to H transition has stimulated several new neoclassical investigations of poloidal rotation driven by a balance between ion-orbit-loss-induced torque and magnetic pumping in the edge region,[12] by an anomalous thermal conductivity that decreases with E_r shear,[13] and by the poloidal asymmetry of an anomalously large radial particle flux acting on the inertial term.[14]

In addition to edge-localized modes (ELM's), large poloidal asymmetries in impurity ion densities have been observed in ELM-free discharges in a number of tokamaks.[15-19] Neoclassical formulations have been developed for poloidal impurity density asymmetries driven by friction,[20-21] poloidal electric fields,[20,22] inertia,[23] NBI momentum input,[3,4] and asymmetric impurity sources.[24] Reasonable agreement[17,25] between theory and experiment was obtained in two collisional tokamak plasmas in which friction was the dominant driving force.

In this paper, we are interested in tokamak plasmas with strong, unbalanced NBI. Two notable characteristics have been observed in such plasmas: (1) $v_\phi/v_{th} \sim O(1)$ for ions, implying $E/B_\theta \sim O(1)$; and (2) anomalously large radial particle fluxes, which exert a torque to drive poloidal rotation. Our approach will be to take the experimentally observed values of toroidal rotation and radial particle flux as given, then to work out the implications, via particle and momentum balance, for the poloidal rotation and poloidal density asymmetries. Since previous work suggests that the results will depend on collision regime, we will take the collisionality v_* as a third parameter in our study. We will use the Hirshman–Sigmar moment approach,[7] in which kinetic theory effects are included in a fluid formalism via the definition of friction and viscosity coefficients. The calculational model for poloidal rotation and poloidal density asymmetries is an extension of the model of Ref. 6 to arbitrary collisionality.

Gyroviscous momentum transport is intrinsically related to poloidal density asymmetries and poloidal rotation, which determine the poloidal asymmetry factor $\tilde{\theta}$,

which in turn sets the magnitude of the gyroviscous torque. Momentum confinement properties calculated with the gyroviscous theory and the assumption $\tilde{\theta} \sim O(1)$ have agreed well in magnitude and, with a few exceptions, in parameter dependence with experiment in a number of tokamaks.[6,26–29] In the one case[6] in which $\tilde{\theta}$ was calculated, the agreement with experiment was to within a factor of 2, although the separation-of-variables approximation used in the model was not supported by the poloidal density asymmetry calculation.

The paper is organized as follows. A neoclassical model for the calculation of poloidal rotation and poloidal density asymmetries is derived in Sec. II. The poloidal asymmetry factor of the gyroviscous theory is defined in terms of the poloidal density, velocity, and potential asymmetries in Sec. III. The calculational models are employed to analyze a two-species, ion-impurity plasma in Sec. IV. The results are summarized in Sec. V.

II. THEORY OF POLOIDAL ROTATION AND POLOIDAL DENSITY ASYMMETRIES

A. Basic equations and ordering

The poloidal rotation velocity and poloidal density asymmetries can be determined from the fluid particle and momentum balance equations with kinetic theory effects included via the viscosity and friction in the manner of Hirshman–Sigmar.[7] (We use zero-heat-flux models for the friction and viscosity; use of finite-heat-flux models would require also the fluid energy and energy flux balance equations.) In the steady-state, and in the absence of particle sources, these equations are

$$\nabla \cdot (n_j \mathbf{v}_j) = 0, \tag{4}$$

$$n_j m_j (\mathbf{v}_j \cdot \nabla \mathbf{v}_j) + \nabla p_j + \nabla \cdot \pi_j + e_j n_j \nabla \Phi + n_j e_j \mathbf{v}_j \times \mathbf{B}$$
$$= \mathbf{R}_j + \mathbf{M}_j, \tag{5}$$

where n_j, $\mathbf{v}_j$, m_j, e_j, p_j, and π_j are the density, velocity, mass, charge, pressure and viscosity tensor, respectively, for species j; $\mathbf{R}_j$ is the collisional friction; and $\mathbf{M}_j$ is the rate of momentum input to species j (e.g., from collisions with energetic beam ions or from a parallel electric field). Here, Φ is the electrostatic potential.

We consider an ordering $v_{\phi j}/v_{\mathrm{th}\,j} \sim O(1)$, consistent with toroidal rotation near the ion sonic speed, $E/B_\theta \sim O(1)$, $\beta \equiv B_\theta / B < 1$, and $\delta_\theta \equiv r_{L\theta}/r \equiv m v_{\mathrm{th}}/eB_\theta r \ll 1$. In this ordering, Eq. (4) can be solved for the poloidal velocity ($\equiv$ speed)

$$v_{\theta j}(r,\theta) = \frac{K_j(r) B_\theta(r,\theta)}{n_j(r,\theta)}, \tag{6}$$

where we use a large-aspect-ratio (n,θ,ϕ) orthogonal toroidal coordinate system. The quantity K_j is a surface function which must be evaluated, as subsequently discussed. The radial component of Eq. (5) can be solved to obtain

$$v_{\phi j}(r,\theta) = \frac{v_{\theta j}(r,\theta)}{\beta} - \frac{1}{B_\theta(r,\theta)} \frac{\partial \Phi(r,\theta)}{\partial r} + O(\delta_\theta v_{\mathrm{th}\,j}). \tag{7}$$

The $O(\delta_\theta v_{\mathrm{th}\,j})$ term involves radial pressure gradients [which appear to leading order when the usual $v_{\phi j}/v_{\mathrm{th}\,j} \sim O(\delta_{\theta j})$ ordering is employed] and inertial terms.

We introduce the thermal transit frequency

$$\omega_j \equiv v_{\mathrm{th}\,j}/qR, \tag{8}$$

and the collisionality parameters

$$\nu_{jk}^* \equiv \nu_{jk}/\omega_j, \tag{9}$$

where q is the safety factor, R is the major radius, and $v_{\mathrm{th}\,j}$ is the thermal speed, and ν_{jk} is the collision frequency between species j and k.

B. Constitutive relations

We neglect heat flux effects relative to particle flow effects, in order not to unnecessarily complicate the formalism.

We use a simple Lorentz model for the friction:

$$\mathbf{R} = -n_j m_j \sum_{k \neq j} \nu_{jk}(\mathbf{v}_j - \mathbf{v}_k), \tag{10}$$

valid when $m_j \gg m_k$.

For the viscosity, we use the Braginskii decomposition of the stress tensor into parallel ($\parallel$), perpendicular ($\perp$) and gyroviscous (GV) components, in toroidal geometry. The poloidal projection of $\nabla \cdot \pi$ is dominated by the $\parallel$ component, and can be written[6,30]

$$\hat{n}_\theta \cdot \nabla \cdot \pi^{\parallel} = \frac{\eta_{0j}}{r} \left\{ \frac{v_{\theta j}}{r} \left[\epsilon \cos\theta - \epsilon^2 \sin^2\theta + \epsilon \left(\frac{\partial \ln \eta_{0j}}{\partial \theta} \right. \right. \right.$$
$$\left. + \frac{1}{3} \frac{\partial \ln n_j}{\partial \theta} \right) \sin\theta - \frac{2}{3} \frac{\partial}{\partial \theta} \frac{\partial \ln n_j}{\partial \theta}$$
$$\left. + \frac{2}{3} \frac{\partial \ln n_j}{\partial \theta} \left(\frac{\partial \ln n_j}{\partial \theta} - \frac{\partial \ln \eta_{0j}}{\partial \theta} \right) \right]$$
$$\left. + \frac{A_j}{q} \left(4\epsilon \sin\theta - \frac{\partial \ln \eta_{0j}}{\partial \theta} - \frac{\partial}{\partial \theta} (\ln A_j) \right) \right\}, \tag{11}$$

where

$$A_j \equiv \frac{1}{RB_\theta} \frac{\partial}{\partial \theta} \left(\frac{\partial \Phi}{\partial r} \right) \tag{12}$$

and a rational approximation to the parallel viscosity coefficient valid in all collisional regimes is given by[7]

$$\eta_{0j} \equiv \frac{n_j m_j v_{\mathrm{th}\,j} qR \epsilon^{-3/2} \nu_{jj}^*}{(1 + \epsilon^{-3/2} \nu_{jj}^*)(1 + \nu_{jj}^*)} \equiv n_j m_j v_{\mathrm{th}\,j} qR f_j(\nu_{jj}^*). \tag{13}$$

[We note that the viscosity coefficient given by Eq. (13) is based on the bounce frequency and transient frequency corresponding to the $m = 1$ mode. Since these frequencies differ for $m \neq 1$ modes, different viscosity coefficients would be necessary if this formalism were used for an analysis of $m \neq 1$ modes. In this paper, we restrict our attention to $m = 1$ modes.]

C. Model for poloidal velocity and density asymmetries

Equations for the poloidal velocity and the poloidal density asymmetries are developed by representing $B(r,\theta) = B(r)/(1+\epsilon\cos\theta)$ and

$$x(r,\theta) = \bar{x}(r)[1 + \epsilon\tilde{x}^s\sin\theta + \epsilon\tilde{x}^c\cos\theta], \tag{14}$$

where $x = n$, Φ, using the forms for the poloidal and toroidal velocities given by Eqs. (6) and (7), and taking the flux surface average of the poloidal projection of the momentum balance equation, Eq. (5), with the weighting functions $\xi = 1$, $\sin\theta$, $\cos\theta$. The resulting equations are as follows.

1. $\xi = 1$

$$\left[q^2 f_j \left(1 + \frac{5}{6}\tilde{n}_j^c + \frac{2}{3}\tilde{n}_j^s + \frac{1}{3}(\tilde{n}_j^s)^2 + \frac{1}{3}(\tilde{n}_j^c)^2 + \frac{1}{2}\tilde{\Phi}^s\tilde{n}_j^s \right. \right.$$

$$\left. + \frac{1}{2}\tilde{\Phi}^c(5 + \tilde{n}_j^c) \right) + \sum_{k\neq j}\bar{\nu}_{jk}^* - q^2\hat{v}_{\phi j}(\tilde{n}_j^s + \tilde{\Phi}^s) \Big] \hat{v}_{\theta j}$$

$$= \left(\hat{M}_{\theta j} + \hat{v}_{rj} + \sum_{k\neq j}\bar{\nu}_{jk}^* \sqrt{\frac{m_j}{m_k}}\hat{v}_{\theta k} - q^2(\hat{v}_{\phi j})^2\tilde{\phi}^s \right.$$

$$\left. - \frac{1}{2}q^2 f\hat{p}_{\phi j}[\tilde{\Phi}^s\tilde{n}_j^s + \tilde{\Phi}^c(5 + \tilde{n}_j^c)] \right). \tag{15}$$

2. $\xi = \sin\theta$

$$\left[\left(\frac{2}{3}f_j - \beta^2\sum_{k\neq j}\bar{\nu}_{jk}^* \right)\hat{v}_{\theta j} \right]\tilde{n}_j^s - \left(\frac{1}{2} \right)\tilde{n}_j^c$$

$$= -(\hat{v}_{\phi j})^2 + \frac{1}{2}\hat{\Phi}_j\tilde{\Phi}^c - \beta^2\sum_{k\neq j}\bar{\nu}_{jk}^*\hat{v}_{\theta j}\tilde{n}_k^s. \tag{16}$$

3. $\xi = \cos\theta$

$$\left[\left(\frac{2}{3}f_j - \beta^2\sum_{k\neq j}\bar{\nu}_{jk}^* \right)\hat{v}_{\theta j} \right]\tilde{n}_j^c + \left(\frac{1}{2} \right)\tilde{n}_j^s$$

$$= -f\hat{p}_{\theta j} - \frac{1}{2}\hat{\Phi}_j\tilde{\Phi}^s + \beta^2\sum_{k\neq j}\bar{\nu}_{jk}^*\left(\hat{v}_{\theta j} - \sqrt{\frac{m_j}{m_k}}\hat{v}_{\theta k} \right)$$

$$- \beta^2\sum_{k\neq j}\bar{\nu}_{jk}^*\hat{v}_{\theta j}\tilde{n}_k^c - \beta^2\hat{v}_{rj}, \tag{17}$$

where

$$\hat{v}_{\theta j} \equiv \frac{\bar{v}_{\theta j}}{\beta v_{\text{th }j}}, \quad \hat{v}_{\phi j} \equiv \frac{\bar{v}_{\phi j}}{v_{\text{th }j}}, \quad \hat{v}_{rj} \equiv \frac{\bar{v}_{rj}}{\beta\delta_{\theta j}v_{\text{th }j}},$$

$$\hat{M}_{\theta j} \equiv \frac{\bar{M}}{\bar{n}_j m_j \omega_j \beta v_{\text{th }j}}, \quad \hat{\Phi}_j \equiv \frac{e_j}{m_j}\frac{\bar{\Phi}}{v_{\text{th }j}^2}. \tag{18}$$

Equations (15) determine the mean poloidal velocities, $\bar{v}_{\theta j}$, or the surface constants, $K_j = \bar{n}_j\bar{v}_{\theta j}/B_\theta$. The terms on the right-hand side (rhs) may be regarded as driving the poloidal velocity; the five driving terms arise from poloidal *momentum* input, radial particle flow, collisional friction,

inertia and viscosity, respectively. The terms on the left-hand side (lhs) may be regarded as damping the poloidal velocity; the three damping terms arise from viscosity, collisional friction and inertia, respectively.

Equations (16) and (17) determine the poloidal density asymmetries $\tilde{n}_j^c$ and $\tilde{n}_j^s$. (We note that the terms containing β^2 are, in general, smaller than the other terms.) The terms on the rhs of these equations may be regarded as driving terms for density asymmetries; these driving terms arise from inertia $[(\hat{v}_{\phi j})^2]$, poloidal potential asymmetries ($\tilde{\Phi}^{c/s}$), friction ($\bar{\nu}_{jk}^*$), viscosity (f_j), and radial particle flow ($\hat{v}_{rj}$).

Equations (15)–(17) are essentially neoclassical. However, rather than use neoclassical theory to evaluate the $\hat{v}_{rj}$, we take note of the anomalously large radial particle flows that have been observed in experiments with strong NBI and incorporate them into our calculation via the $\hat{v}_{rj}$. Because there are no data on any poloidal variation of radial particle fluxes, we will take $\hat{v}_{rj}$ as uniform over a flux surface.

Equations (15)–(17) are coupled and nonlinear in the unknowns $\hat{v}_{\theta j}$, $\tilde{n}_j^c$, and $\tilde{n}_j^s$. They must be solved numerically.

D. Electrostatic potential and radial electric field

The poloidal component of the electron momentum balance and charge neutrality can be used to obtain

$$\tilde{\Phi}^{s/c} = \left(\frac{T_e}{e\bar{\Phi}} \right)\tilde{n}_e^{s/c} = \left(\frac{T_e}{e\bar{\Phi}} \right)\sum_j^{\text{ions}} Z_j\frac{\bar{n}_j}{\bar{n}_e}\tilde{n}_j^{s/c}, \tag{19}$$

which relates the poloidal potential asymmetries to the poloidal density asymmetries.

The flux surface average of Eq. (7) yields a relationship among the radial electric field and the mean rotation velocities:

$$\bar{E}_r \equiv -\frac{\partial\bar{\Phi}}{\partial r} = \bar{B}_\theta\left(\bar{v}_{\phi j} - \frac{\bar{v}_{\theta j}}{\beta} \right). \tag{20}$$

If we were carrying out a purely first-principles calculation, the sum over species of the flux surface average of the toroidal component of the momentum balance equation could be solved for E_r or $\bar{v}_{\phi j}$, explicitly,[3,4] and the solution would be completely determined. However, our purpose in this calculation is to take $\bar{v}_{\phi j}$ as known from experiment. Then, Eqs. (15)–(17) and (20) completely determine $\bar{v}_{\theta j}$, $\tilde{n}_j^c$, $\tilde{n}_j^s$, and E_r.

We note that if the summed toroidal momentum balance equations were solved for E_r the solution would have two roots,[3] introducing the possibility of bifurcation of the solutions for E_r and, via Eq. (20), for $\bar{v}_{\theta j}$. Such a possibility is of interest relative to the sharp change in v_θ observed in conjunction with an L to H transition, but its investigation is beyond the scope of this paper.

III. GYROVISCOUS TORQUE

The total (summed over ion species) gyroviscous torque[30] can be written

$$\sum_{j}^{\text{ions}} \langle R^2 \nabla \phi \cdot \nabla \cdot \pi_j^{gv} \rangle \simeq \sum_{j}^{\text{ions}} \frac{\tilde{\theta}_j G_j}{2} \frac{\bar{v}_{\phi j}}{\bar{R}} \frac{\bar{n}_j m_j T_j}{e \bar{B} z_j}$$

$$\simeq \frac{m_D \bar{n}_e \bar{v}_\phi T_{\text{ion}}}{2e\bar{B}R} \sum_{j}^{\text{ions}} \frac{\bar{n}_j \tilde{\theta}_j G_j}{\bar{n}_e}$$

$$\equiv \frac{m_D \bar{n}_e \bar{v}_\phi T_{\text{ion}}}{2e\bar{B}R} \left(\frac{\tilde{\theta} G}{z} \right)_{\text{eff}}, \tag{21}$$

when $m_j \simeq z_j m_D$ and common $\bar{v}_\phi$ and T_{ion} are assumed for all species, where

$$G_j \equiv -\frac{r}{\bar{\eta}_{4j}\bar{v}_{\phi j}} \frac{\partial(\bar{\eta}_{4j}\bar{v}_{\phi j})}{\partial r}, \tag{22}$$

$$\tilde{\theta}_j \equiv (4+\tilde{n}_j^c)\tilde{v}_{\phi j}^s + (1-\tilde{v}_{\phi j}^c)\tilde{n}_j^s$$

$$= (4+\tilde{n}_j^c)[-\hat{v}_{\theta j}(\hat{v}_{\phi j})^{-1}(\tilde{\Phi}^s+\tilde{n}_j^s)+\tilde{\Phi}^s]$$

$$+\tilde{n}_j^s[\hat{v}_{\theta j}(\hat{v}_{\phi j})^{-1}(2+\tilde{\Phi}^c+\tilde{n}_j^c)-\tilde{\Phi}^c]. \tag{23}$$

The $\tilde{v}_{\phi j}^{s/c}$ were determined in terms of the $\tilde{n}_j^{s/c}$ by using Eq. (7), and the $\tilde{\Phi}^{s/c}$ can be similarly determined from Eq. (19), so that $\tilde{\theta}_j = \tilde{\theta}_j(\tilde{n}_j^{s/c})$.

When the radial profiles have the form $x(r) = \bar{x}_0[1-(r/a)^2]^{\alpha_x}$, Eq. (22) becomes

$$G(r) = \frac{2(r/a)^2(\alpha_n + \alpha_v + \alpha_T)}{[1-(r/a)^2]}. \tag{24}$$

IV. ANALYSIS OF AN ION-IMPURITY PLASMA

In order to study the implications of the equations given in the previous two sections for poloidal velocity and poloidal asymmetries, we consider the relatively simple, but representative, case of a deuterium (i) plasma with a carbon impurity (I) concentration $\alpha = n_I Z_I^2/n_i = 1.8$, corresponding to $Z_{\text{eff}} = 2.15$. Such a model is representative of many tokamak experiments. In this model

$$\frac{\bar{n}_i}{n_e} = \frac{Z_I - Z_{\text{eff}}}{Z_I - 1}, \quad \frac{\bar{n}_I}{\bar{n}_e} = \frac{Z_{\text{eff}} - 1}{Z_I(Z_I - 1)},$$

$$\alpha \equiv \frac{\bar{n}_I Z_I^2}{\bar{n}_i} = \frac{(Z_{\text{eff}} - 1)Z_I}{Z_I - Z_{\text{eff}}} \tag{25}$$

and

$$\tilde{v}_{ii}^* = \frac{\tilde{v}_{II}^*}{\alpha Z_I^2}, \quad \tilde{v}_{Ii}^* = \left(\frac{\tilde{v}_{II}^*}{\alpha}\right)\sqrt{\frac{m_I + m_i}{2m_i}},$$

$$\tilde{v}_{iI}^* = \alpha\tilde{v}_{ii}^*\sqrt{\frac{m_i + m_I}{2m_I}}. \tag{26}$$

We choose parameters representative of $(r/a) = 0.5$: $q = 2.0$, $\beta = 0.1$, $T_{\text{ion}}/T_e = 1-3$. We set $\hat{M}_\theta = 0$, corresponding to the absence of poloidal momentum input in most tokamak experiments. We will consider values of the radial deuterium velocity $\hat{v}_{ri}$, and of the toroidal rotation velocity $\hat{v}_\phi$ in the range 0–1, corresponding to experimental observation. We will consider a range of collisionality $0.1 < \tilde{v}_{II}^* \leqslant 500$, corresponding to the range of this parameter in

present and past tokamak experiments. The parameter $e\Phi/T_e$ is set equal to unity, as suggested by measurements in the Impurity Studies Experiment[31] (ISX-B) with a strongly rotating plasma.

A. Collisionality dependence

The collisionality determines the relative importance of the friction term and the relative importance of the ion and impurity viscosity terms. The magnitude of the friction term increases linearly with interspecies collisionality ν_{jk}^*. The magnitude of the viscous force varies with ν_{jj}^* as

$$f_j(\nu_{jj}^*) = \frac{\epsilon^{-3/2}\nu_{jj}^*}{(1+\epsilon^{-3/2}\nu_{jj}^*)(1+\nu_{jj}^*)}. \tag{27}$$

This function increases with ν_{jj}^* up to $\nu_{jj}^* \sim O(1)$, then decreases with ν_{jj}^* for $\nu_{jj}^* > 1$. We will characterize our calculations with the impurity self-collisionality parameter ν_{II}^*. The corresponding $\nu_{ii}^* \simeq \nu_{jj}^*/Z_I^{3/2} \simeq 0.07 \nu_{II}^*$.

The poloidal velocities, density asymmetries, and poloidal asymmetry factors are plotted versus ν_{II}^* in Fig. 1 for $(q = 2.5, \beta = 0.1, e\Phi/T_e = 1.0, \hat{v}_\phi = 0.5, \hat{v}_{ri} = 0.0, r/a = 0.5)$ and $\alpha_n = \alpha_T = \alpha_v = 1.0$. The viscous damping of the carbon poloidal rotation decreases for $\nu_{II}^* \gtrsim 1.0$, resulting in an increase of $|\hat{v}_{\theta I}|$ with ν_{II}^*. The viscous damping of the deuterium ions increases with $\nu_{ii}^* \simeq 15 \nu_{ii}^*$ up to about $\nu_{II}^* = 10.0$, then decreases with ν_{II}^*, resulting in the dependence of $|\hat{v}_{\theta i}|$ on ν_{II}^* shown in Fig. 1(a). Because the viscous damping of the deuterium is greater than that of the carbon, over the range of ν_{II}^* considered, $|\hat{v}_{\theta I}| > |\hat{v}_{\theta i}|$ except for $\nu_{II}^* \simeq 0.1$ where the viscous damping is comparable for both species. In general, $0.15 (B_\theta/B)v_{\text{th } j} \leqslant v_{\theta j} \leqslant (B_\theta/B)v_{\text{th } j}$, with the larger values corresponding to large collisionality. The poloidal rotation is in the negative B_θ direction for both species.

The in–out poloidal density asymmetry $(\tilde{n}_j^c)$ is insensitive to collisionality, for $\nu_{II}^* < 100$; for $\nu_{II}^* \gtrsim 100$, frictional forces act to reduce the difference between $\tilde{n}_i^c$ and $\tilde{n}_j^c$ via the poloidal variation in the electrostatic potential. The up-down density asymmetries are sensitive to the value of ν_{II}^*.

The individual poloidal asymmetry factors for deuterium and carbon, $\tilde{\theta}_j$, and the effective composite asymmetry factor

$$\left(\frac{\tilde{\theta} G}{Z}\right)_{\text{eff}} = \left(\frac{\bar{n}_i}{\bar{n}_e}\tilde{\theta}_i + \frac{\bar{n}_I}{\bar{n}_e}\tilde{\theta}_I\right)G$$

$$\simeq (0.7\tilde{\theta}_i + 0.05\tilde{\theta}_I)\left(\frac{2(r/a)^2(\alpha_n + \alpha_T + \alpha_v)}{[1-(r/a)^2]}\right) \tag{28}$$

are plotted in Fig. 1(c). The major contribution to the effective composite asymmetry factor, hence to the gyroviscous torque, is made by the deuterium, not the carbon as previously conjectured.[29]

Equation (21) has successfully predicted the momentum confinement properties of the Joint European Torus[29] (JET) and of the Tokamak Fusion Test Reactor[6] (TFTR) when $(\tilde{\theta}G/z)_{\text{eff}} \simeq 0.1$ was used, and has predicted the mo-
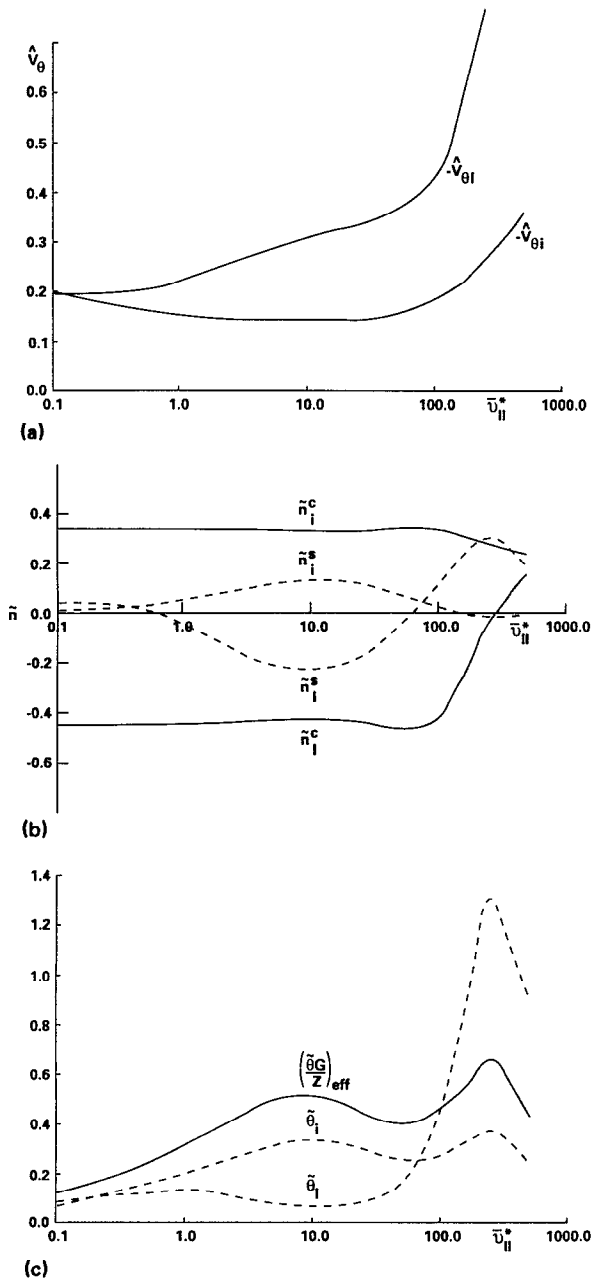
FIG. 1. Dependence on collisionality $\nu_{II}^* \equiv \nu_{II} \nu_{\text{th } I}/qR$ $(q=2.0, \beta=0.1,$ $\alpha=1.8, \hat{v}_{\phi I} \equiv v_{\phi I}/v_{\text{th } I}=0.5, T_{\text{ion}}/T_e=2.0, r/a=0.5)$: (a) normalized poloidal rotation $\hat{v}_{\theta j} \equiv (v_{\theta j}/v_{\text{th } j})$ (B/B_θ); (b) normalized density asymmetry $\tilde{n} \equiv (n-\bar{n})/\epsilon$; (c) poloidal asymmetry factors of the gyroviscous theory.

mentum confinement properties of ISX-B and of the Princeton Large Torus[26] (PLT) when $(\theta G/z)_{\text{eff}} \simeq 0.4$ was used. The collisionality of carbon in the TFTR and JET experiments was $\nu_{II}^* \simeq 0.1$, while in PLT and ISX-B, 1.0 $\lesssim \nu_{II}^* \lesssim 10.0$ (assuming that the dominant contribution to Z_{eff} was from carbon or oxygen). The results shown in Fig. 1(c) suggest that $(\theta G/z)_{\text{eff}}$ would be larger in ISX-B and PLT than in JET and TFTR, and, in fact, support the choices $(\theta G/z)_{\text{eff}} \simeq 0.4$ and 0.1, respectively. This point will be investigated in more detail and reported in the future.

B. Toroidal rotation dependence

The results in Fig. 1 were all calculated for $\bar{v}_{\phi I}=0.5$ $v_{\text{th } I}$ [and $\bar{v}_{\phi i} = \bar{v}_{\phi I} = 0.5\sqrt{(m_i/m_I)}v_{\text{th } i}$]. The magnitude of $\bar{v}_{\phi j}$ determines the magnitude of the inertial term, the effects of which propagate through the calculation. In order to illustrate the sensitivity of the results to the toroidal rotation, we have repeated the calculation shown in Fig. 1 for a fixed $\nu_{II}^* = 0.5$ and varied $\bar{v}_{\phi I}$ over the range 0.1 $v_{\text{th } I} \leqslant \bar{v}_{\phi I} \leqslant 0.7 v_{\text{th } I}$. (We had problems with the convergence of our numerical procedure for $\bar{v}_{\phi I} \gtrsim 0.7$ $v_{\text{th } I}$.) The results are plotted in Fig. 2.

The quadratic dependence of the inertial driving term in Eq. (15) on $\bar{v}_{\phi j}$ resulted in the strongly increasing dependence of $|\bar{v}_{\theta j}|$ on $\bar{v}_{\phi j}$ shown in Fig. 2(a); and the similar dependence in Eq. (16) accounts for the strong dependence of $\tilde{n}_j^c$ shown in Fig. 2(b). The dependence of $\tilde{n}_j^s$ on $\bar{v}_{\phi j}$ is indirect through the viscous term in Eq. (17), with $\bar{v}_{\theta j}=\bar{v}_{\theta j}(\bar{v}_{\phi j})$, and less dramatic.

The poloidal asymmetry factors increase strongly with $\bar{v}_{\phi I}$, except that $\tilde{\theta}_I$ saturates because of compensating effects. Again, the effective composite parameter $(\tilde{\theta}G/z)_{\text{eff}}$ is determined primarily by the deuterium ions. An implication of this strong dependence of $(\tilde{\theta}G/z)_{\text{eff}}$ on $\bar{v}_{\phi I}$ is that toroidal rotation velocities should saturate with increasing NBI.

These calculations were repeated for $\nu_{II}^* = 100$. The results for $\bar{v}_{\theta j}$ and $\tilde{n}_I^s$ were similar to those shown in Fig. 2. However, $\tilde{n}_I^s$ became positive for $\hat{v}_{\phi I} > 0.35$, with the consequence that $\tilde{\theta}_I$ exhibited a variation more like that of $\tilde{\theta}_i$. The magnitude of $(\tilde{\theta}G/z)_{\text{eff}}$ was about two times that shown in Fig. 2(c), and the qualitative dependence was similar.

The calculations shown in Fig. 2 were also repeated for $\bar{v}_{\phi j} < 0$ (i.e., for counter injection). The results were the "mirror image" of those shown in Fig. 2, to within 10% in magnitude. The $\bar{v}_{\theta j}$ were now positive, the signs of the $\tilde{n}_j^c$ were unchanged and those of the $\tilde{n}_j^s$ were reversed, and the signs of the $\tilde{\theta}_j^c$ and $(\tilde{\theta}G/z)_{\text{eff}}$ were reversed. [Note that a negative $\bar{v}_\phi$ and a negative $(\tilde{\theta}G/z)_{\text{eff}}$ still produce a positive torque and an outward angular momentum flux.]

C. Radial particle flux dependence

The calculations of Figs. 1 and 2 were made with $\hat{v}_{rj} \simeq 0$, corresponding to the radial particle fluxes predicted by neoclassical theory. However, large anomalous radial electron fluxes are characteristic of tokamaks,[32,33] with v_{re} in the range 1–10 m/sec not being uncommon. Since there is no internal source of impurities and Z_{eff} is frequently almost constant, we will use $\hat{v}_{ri} \simeq \hat{v}_{re}$ and $\hat{v}_{rI} \simeq 0$ in our calculations. With reference to Eq. (18), for $\beta=0.1$, $\delta_\theta \simeq 10^{-3}$, and $v_{\text{th } i} \simeq 5 \times 10^5$ m/sec, $\bar{v}_{ri}=10$ m/sec corresponds to $\hat{v}_{re} \simeq 0.2$. It is possible that the anomalous forces that produce the measured anomalous v_{re} should be included in Eqs. (15)–(17); this can be represented[6] as an effective reduction of $\hat{v}_{ri}$. Thus, we repeat the previous calculations for $\bar{v}_{\phi I}=0.5$ $v_{\text{th } I}$, $\nu_{II}^* = 0.5$, $\hat{v}_{rI}=0$ and $0 \leqslant \hat{v}_{ri} \leqslant 1.0$. The results are shown in Fig. 3.
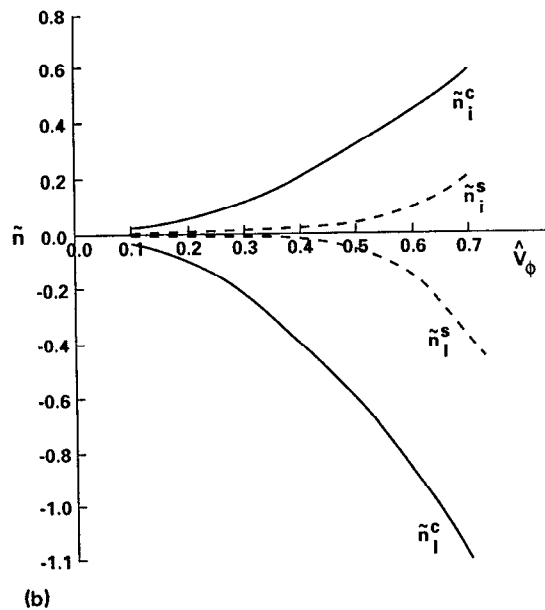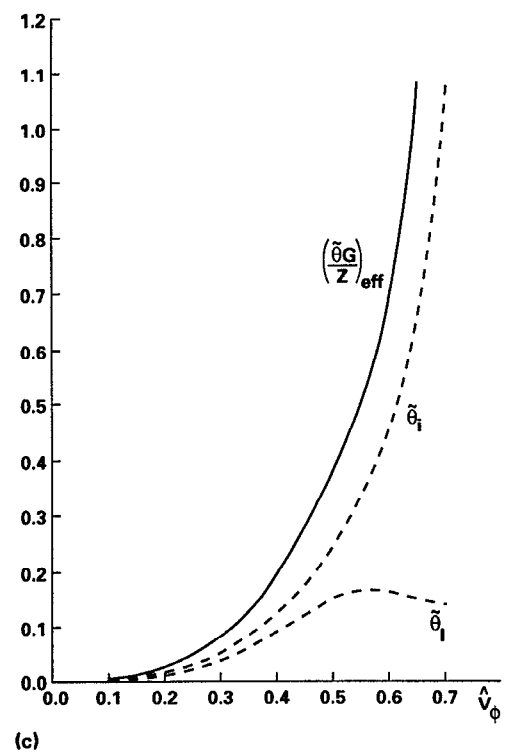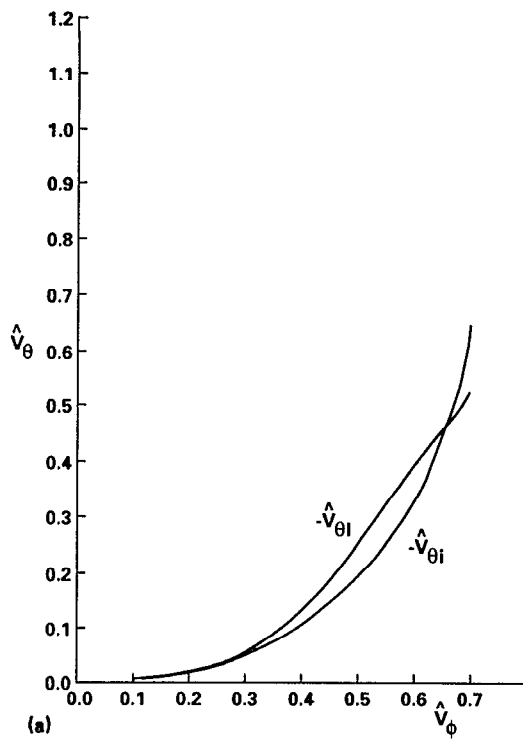
(a)

(b)

(c)

FIG. 2. Dependence on toroidal rotation $\hat{v}_{\phi I} \equiv v_{\phi I}/v_{\text{th}\,I}$ ($q=2.0$, $\beta=0.1$, $\alpha=1.8$, $v_{II}^* = 0.5$, $T_{\text{ion}}/T_e=2.0$, $r/a=0.5$): (a) normalized poloidal rotation $\hat{v}_{\theta j} \equiv (v_{\theta j}/v_{\text{th}\,j})\,(B/B_\theta)$; (b) normalized density asymmetry $\tilde{n} \equiv (n - \bar{n})/\epsilon$; (c) poloidal asymmetry factors of the gyroviscous theory.

A positive (radially outward) $\hat{v}_{ri}$ contributes a positive contribution to an otherwise negative (B_θ direction) $\bar{v}_{\theta i}$—see Eq. (15). For sufficiently large $\hat{v}_{ri}$, this contribution causes $\hat{v}_{\theta i}$ to change sign. Here, $\hat{v}_{\theta I}$ depends on $\hat{v}_{ri}$ through its coupling to $\hat{v}_{\theta i}$.

The in–out density asymmetries, $\tilde{n}_j^c$, are independent of $\hat{v}_{ri}$, but the up–down density asymmetries, $\tilde{n}_j^s$, change sign and then increase rapidly in magnitude when $\hat{v}_{ri} \gtrsim 0.1$.

Both $\tilde{\theta}_i$ and $\tilde{\theta}_I$ decrease with increasing $\hat{v}_{ri}$ approaching 0.1. Then, $\tilde{\theta}_i$ increases again for $\hat{v}_{ri} \gtrsim 0.4$, while $\tilde{\theta}_I$ becomes negative (indicating inward momentum transport by the impurities). The effective composite asymmetry factor $(\tilde{\theta}G/z)_{\text{eff}}$, decreases with $\hat{v}_{ri}$ up to $\hat{v}_{ri} \approx 0.3$–0.4, then increases rapidly. The implication is that increasing radial particle fluxes act to diminish the outward momentum

transport up to $\hat{v}_{ri} \approx 0.3$–0.4, then act to substantially enhance it for $\hat{v}_{ri} \gtrsim 0.4$.

V. SUMMARY

A neoclassical model has been developed for the calculation of poloidal rotation and poloidal density asymmetries in a tokamak plasma with strong unbalanced NBI leading to toroidal rotation $v_\phi \sim v_{\text{th}}$.

Poloidal rotation and poloidal density asymmetries have been calculated for main (deuterium) and impurity (carbon) ions in a two-species specialization of the model. The predicted poloidal rotation is negative (B_θ direction), different for carbon and deuterium, and in the range $0.1 \lesssim (|v_{\theta j}|/v_{\text{th}\,j})/(B/B_\theta) \lesssim 0.4$ for parameters character-
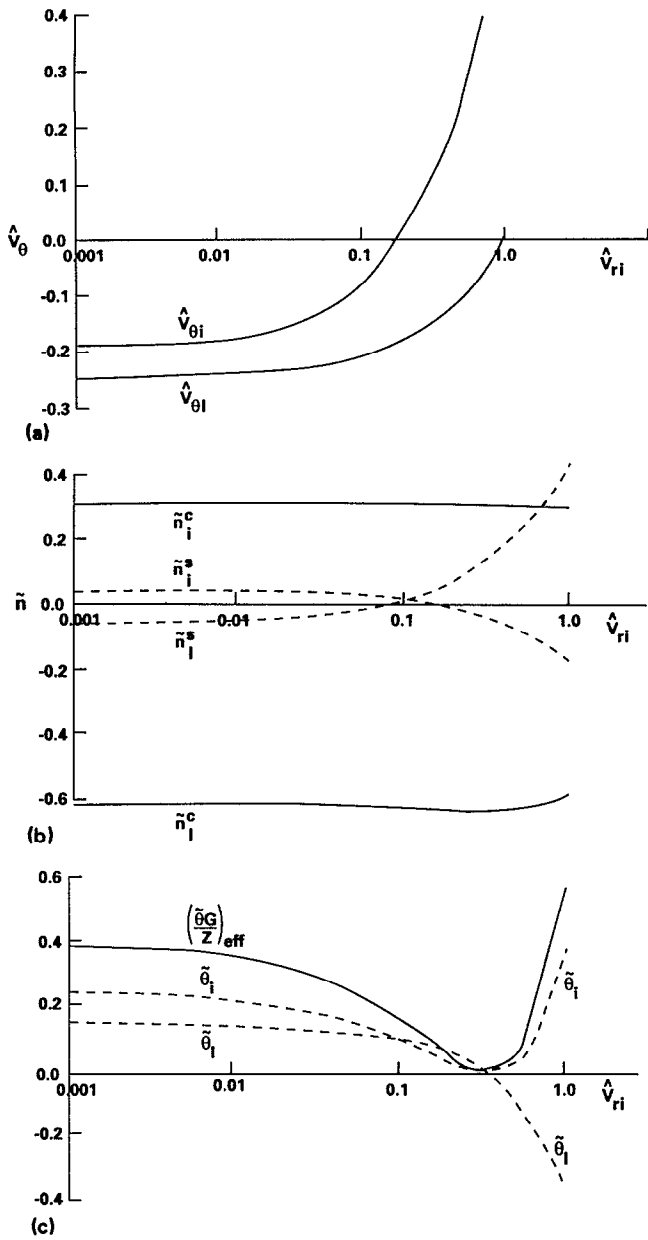
FIG. 3. Dependence on radial deuterium flux $\hat{v}_{ri} \equiv v_{ri}/(\beta v_{th} \delta_{\theta i})$ ($q=2.0$, $\beta=0.1$, $\alpha=1.8$, $\hat{v}_{\phi I} \equiv v_{\phi I}/v_{th\,I} = 0.5$, $v_{II}^* = 0.5$, $T_{ion}/T_e=2.0$, $r/a=0.5$): (a) normalized poloidal rotation $\hat{v}_{\theta j} \equiv (v_{\phi j}/v_{th\,j})\,(B/B_\theta)$; (b) normalized density asymmetry $\tilde{n} \equiv (n-\bar{n})/\epsilon$; (c) poloidal asymmetry factors of the gyroviscous theory.

the gyroviscous theory for momentum transport was evaluated using the results of these poloidal rotation and poloidal density asymmetry calculations. This factor increases with increasing toroidal rotation and has a complex dependence on both collisionality and the radial deuterium flux. The calculations broadly support the values of the composite effective poloidal asymmetry factor that have been assumed in the successful comparisons of the gyroviscous theory with experimental momentum confinement properties in ISX-B, PLT, Doublet III (D-III), TFTR, and JET.

ACKNOWLEDGMENTS

[1]R. D. Hazeltine, Phys. Fluids 17, 961 (1974).
[2]F. L. Hinton and M. N. Rosenbluth, Phys. Fluids 16, 836 (1973).
[3]W. M. Stacey and D. J. Sigmar, Phys. Fluids 27, 2078 (1984).
[4]W. M. Stacey, A. W. Bailey, D. J. Sigmar, and K. C. Shaing, Nucl. Fusion 25, 463 (1985).
[5]Y. B. Kim, P. H. Diamond, and R. J. Groebner, Phys. Fluids B 3, 2050 (1991).
[6]See National Technical Information Service Document No. PB92177187 (W. M. Stacey, "Analysis of a Dedicated Rotation Experiment in TFTR," Georgia Tech report GTFR-100, 1992). Copies may be ordered from the National Technical Information Service, Springfield, VA 22161; also G. Pautasso, Ph.D. thesis, Georgia Institute of Technology, 1991.
[7]S. P. Hirshman and D. J. Sigmar, Nucl. Fusion 21, 1079 (1981).
[8]F. L. Hinton and S. K. Wong, Phys. Fluids 28, 3082 (1985).
[9]R. J. Groebner, K. H. Burrell, and R. P. Seraydarian, Phys. Rev. Lett. 64, 3015 (1990).
[10]F. Wagner, F. Ryter, A. R. Field, G. Fussmann, J. V. Hofmann, M. E. Manso, O. Vollmer, R. Buchse, G. Dodel, A. Eberhagen, M. Endler, W. Engelhardt, H. U. Fahrgach, O. Gehre, J. Gernhardt, L. Giannone, O. Gruber, H. J. Hartfuss, W. Hermann, E. Holzhauer, A. Kallenbach, O. Kardaun, F. Karger, O. Kluber, M. Kornherr, K. Lackner, R. Lang, J. Matias, H. M. Mayer, K. McCormick, V. Mertens, E. R. Muller, H. D. Murmann, J. Neuhauser, H. Niedermeyer, W. Poschenrieder, A. Rudyj, U. Schneider, R. Schubert, G. Siller, E. Simmet, F. X. Soldner, A. Stabler, K. H. Steuer, U. Stroth, N. Tsois, H. Verbeek, and H. Zohm, Plasma Physics and Controlled Fusion Research, Proceedings of the 13th International Conference, Washington (International Atomic Energy Agency, Vienna, 1991), p. 277.
[11]Y. Miura, H. Aikawa, K. Hoshino, S. Kasai, T. Kawakami, H. Kawashima, H. Maeda, T. Matsuda, M. Mori, K. Odajima, H. Ogawa, S. Sengoku, M. Shimada, T. Shoji, N. Suzuki, H. Tamai, S. Tsuji, T. Yamamoto, T. Yamauchi, T. Fujita, A. M. Howald, A. W. Hyatt, K. Ida, A. W. Leonard, N. Ohyabu, S. Gunji, T. Hamano, K. Hasegawa, A. Honda, I. Ishibori, Y. Kashiwa, M. Kazawa, K. Kikuchi, F. Okano, E. Sato, N. Seki, T. Shibata, T. Shiina, K. Suzuki, S. Suzuki, T. Tokutake, and S. Uno, in Ref. 10, p. 325.
[12]K. C. Shaing and E. C. Crume, Phys. Rev. Lett. 63, 2369 (1990).
[13]F. L. Hinton, Phys. Fluids B 3, 696 (1991).
[14]A. B. Hassam, T. M. Antonsen, Jr., J. F. Drake, and C. S. Liu, Phys. Rev. Lett. 66, 309 (1991).
[15]J. L. Terry, S. L. Allen, H. W. Moos, R. K. Richards, and E. S. Marmar, Phys. Rev. Lett. 39, 1615 (1977); S. L. Allen, H. W. Moos, R. K. Richards, J. L. Terry, and E. S. Marmar, Nucl. Fusion 21, 251 (1981).
[16]See National Technical Information Service Document No. PPPL-1430 (S. Suckewer, E. Hinnov, and J. Schivell, "Rapid Scanning of Spatial Distribution of Spectral Line Intensities in PLT Tokamak," Princeton report PPPL-1430, 1978). Copies may be ordered from the National Technical Information Service, Springfield, VA 22161.

izing present tokamaks. The poloidal rotation generally increases with increasing toroidal rotation, decreases with increasing radial deuterium flux, and varies nonmonotonically with collisionality. In–out poloidal density asymmetries vary from $\sim 0.1\epsilon$ to $\sim 1.0\epsilon$ for carbon (and about half of this magnitude for deuterium) as the carbon toroidal rotation speed increases from $\sim 0.2\,v_{th\,I}$ to $\sim 0.7\,v_{th\,I}$, independent of collisionality or radial deuterium flux. Up–down poloidal density asymmetries vary from ~ 0.0 to $\sim 0.4\epsilon$ for carbon (and half this for deuterium) over the same range of carbon rotation speeds, for $v_{II}^* = 0.5$, and are quite sensitive to collisionality and radial deuterium flux.

The effective composite poloidal asymmetry factor of

[17]K. Brau, S. Suckewer, and S. K. Wong, Nucl. Fusion **23**, 1657 (1983).

[18]P. Smeulders, Nucl. Fusion **26**, 267 (1986).

[19]K. W. Wenzel, Bull. Am. Phys. Soc. **34**, 2153 (1989).

[20]C. S. Chang and R. D. Hazeltine, Nucl. Fusion **20**, 1397 (1980).

[21]C. T. Hsu and D. J. Sigmar, Plasma Phys. Controlled Fusion **32**, 499 (1990).

[22]R. D. Hazeltine and A. A. Ware, Phys. Fluids **19**, 1163 (1976).

[23]K. H. Burrell, T. Ohkawa, and S. Wong, Phys. Rev. Lett. **47**, 511 (1981).

[24]W. M. Stacey, Nucl. Fusion **27**, 1213 (1987).

[25]See National Technical Information Service Document No. PB9016891 (D. J. Sigmar, G. S. Lee, C. T. Hsu, K. W. Wenzel, E. Sindoni, F. Troyon, and J. Vaclavik, "Nonlinear Collisional Impurity Transport Including Bifurcations and Coupling to Rippling Mode," MIT/PFC report PFC/JA-90-28, 1990). Copies may be ordered from the National Technical Information Service, Springfield, VA 22161.

[26]W. M. Stacey, C. M. Ryu, and M. A. Malik, Nucl. Fusion **26**, 293 (1986).

[27]M. Bitter, V. Arunasalam, C. Barnes, M. G. Bell, S. Bosch, N. L. Bretz, R. Budny, C. E. Bush, D. L. Dimock, H. F. Dylla, P. C. Efthimion, R. J. Fonck, E. Fredrickson, H. P. Furth, G. Gammel, R. J. Goldston, B. Grek, L. R. Grisham, G. Hammett, K. Hattori, R. J. Hawryluk, H. W. Hendel, K. W. Hill, E. Hinnov, T. Hirayama, R. A. Hulse, H. Hsuan, K. P. Jaehnig, D. Jassby, F. C. Jobes, D. W. Johnson, L. C. Johnson, R. Kaita, R. Kamperschroer, S. J. Kilpatrick, R. J. Knize, H. Kugel, P. H. LaMarche, B. LeBlanc, R. Little, D. M. Manos, D. K. Mansfield, S. McDermott, K. McGuire, D. H. McNeill, D. M. Meade, S. S. Medley, D. R. Mikkelsen, W. Morris, D. Mueller, E. B. Nieschmidt, T. Nishitani, D. K. Owens, H. Park, A. T. Ramsey, M. H. Redi, J. Rice, A. L. Roquemore, P. H. Rutherford, T. Saito, G. Schilling, J. Schivell, G. L. Schmidt, S. D. Scott, J. C. Sinnis, F. J. Stauffer, B. C. Stratton, G. D. Tait, Y. Takase, G. Taylor, J. R. Timberlake, H. H. Towner, M. Ulrickson, S. von Goeler, R. Wieland, M. Williams, K. L. Wong, S. Yoshikawa, K. M. Young, M. C. Zarnstorff, and S. J. Zweben, Plasma Phys. Controlled Fusion **29**, 1234 (1987).

[28]K. H. Burrell, R. J. Groebner, H. St. John, and R. P. Seraydarian, Nucl. Fusion **28**, 3 (1988).

[29]W. M. Stacey, Nucl. Fusion **31**, 31 (1991).

[30]W. M. Stacey and D. J. Sigmar, Phys. Fluids **28**, 2800 (1985).

[31]G. A. Hallock, J. Mathew, W. C. Jennings, R. L. Hickok, and R. C. Isler, Phys. Rev. Lett. **56**, 1248 (1986).

[32]M. Zarnstorff, V. Arunasalam, M. G. Bell, C. W. Barnes, M. Bitter, H.-S. Bosch, N. L. Bretz, R. Budny, C. E. Bush, A. Cavallo, T. K. Chu, S. A. Cohen, P. Colestock, S. L. Davis, D. L. Dimock, H. F. Dylla, P. C. Efthimion, A. B. Ehrhardt, R. J. Fonck, E. Fredrickson, H. P. Furth, G. Gammel, R. J. Goldston, G. Greene, B. Grek, L. R. Grisham, G. Hammett, R. J. Hawryluk, H. W. Hendel, K. W. Hill, E. Hinnov, J. Hosea, R. B. Howell, H. Hsuan, R. A. Hulse, K. P. Jaehnig, A. C. Janos, D. Jassby, F. Jobes, D. W. Johnson, L. C. Johnson, R. Kaita, C. Kieras-Phillips, S. J. Kilpatrick, V. A. Krupin, P. H. LaMarche, B. LeBlanc, R. Little, A. I. Lysoyvan, D. M. Manos, D. K. Mansfield, E. Mazzucato, R. T. McCann, M. P. McCarthy, D. C. McCune, K. McGuire, D. H. McNeill, D. M. Meade, S. S. Medley, D. R. Mikkelsen, R. Motley, D. Mueller, J. A. Murphy, Y. Murakami, E. B. Nieschmidt, D. K. Owens, H. Park, A. T. Ramsey, M. H. Redi, A. L. Roquemore, P. H. Rutherford, T. Saito, N. R. Sauthoff, G. Schilling, J. Schivell, G. L. Schmidt, S. D. Scott, J. C. Sinnis, J. Stevens. J. D. Strachan, B. C. Stratton, W. Stodiek, G. D. Tait, G. Taylor, J. R. Timberlake, H. H. Towner, M. Ulrickson, S. von Goeler, R. Wieland, M. Williams, J. R. Wilson, K.-L. Wong, S. Yoshikawa, K. M. Young, and S. J. Zweben, *Plasma Physics and Controlled Nuclear Fusion Research*, Proceedings of the 12th International Conference, Nice (International Atomic Energy Agency, Vienna, 1989), p. 183.

[33]A. J. Wootton, B. A. Carreras, H. Matsumoto, K. McGuire, W. A. Peebles, C. P. Ritz, P. W. Terry, and S. J. Zweben, Phys. Fluids B **2**, 2879 (1990).

3309   Phys. Fluids B, Vol. 4, No. 10, October 1992

W. M. Stacey, Jr.   3309