Diversity in Synchronization for Scheduled OFDM Time-Division Cooperative Transmission

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***Abstract*—An energy efficient synchronization method is presented for an OFDM-based Time Division Cooperative Transmission (TDCT) system for the purpose of range extension. The proposed algorithm operates on a novel preamble consisting of two OFDM symbols. In TDCT, copies of the packet are transmitted through different time slots. Exploiting dependence between the synchronization parameters of the different TDCT copies, the approach achieves diversity gain in all estimated synchronization parameters. Since the design and algorithm pertain only to the preamble, the method can be applied to decode-and-forward as well as amplify-and-forward TDCT schemes. This algorithm also offers a significant performance improvement to the single-input-and-single-out (SISO) communication system. Through computer simulation, we show that our novel TDCT approach achieves synchronization in the context of range extension, avoiding the need for more preamble energy.**

***Index Terms*—OFDM synchronization, cooperative transmission, diversity gain, range extension**

I. Introduction

In cooperative transmission (CT), multiple radios in a network transmit copies of the same message through differently fading multipath channels, and a receiver combines the copies in the physical layer. The cooperatively transmitting nodes form a virtual array, from which the receiver can derive diversity and array gains [1]. These gains can be used to increase reliability, reduce transmit power, or extend range. In particular, range extension can overcome shadowing and path loss that would otherwise partition the network. CT range extension can benefit many types of wireless networks. For example, it can increase the two-hop coverage area of a single access point [2]. CT can be performed concurrently (CCT) or in different time slots; we call the time-slotted version "time-division CT" (TDCT). This paper treats TDCT with the objective of range extension.

In the context of range-extension, TDCT has some advantages over CCT. Neither transmitter-side channel state information nor phase coherency across cooperating transmitters are required by TDCT; conversely, they are required in the coherent form of CCT, also known as coherent beamforming [3]. Thus, all the cooperators must be recruited prior to a CCT transmission, whereas for TDCT, they can be recruited incrementally, as needed [4]. To achieve range extension, transmitters should use the same transmit power as in the conventional single-input-and-single-output (SISO) case. This leads to another advantage of TDCT: its interference range is the same as SISO interference, whereas CCT interference range is larger due to the high power emitted concurrently.

Orthogonal Frequency Division Multiplexing (OFDM) techniques have been employed intensively for their robustness and high spectral efficiency in the frequency selective fading channel. Synchronization is a big issue for any OFDM based system for it is widely known that symbol timing offsets larger than the cyclic prefix (CP) will introduce inter-symbol-interference (ISI), and carrier frequency offsets (CFOs) will introduce inter-carrier-interference (ICI). Sampling frequency offsets are also estimated in OFDM receivers, however, we assume these offsets are zero in this paper.[1]

In the existing literature, there are several studies on OFDM-based CCT synchronization. In [5],[6], multiple offset estimates are based on the combined preamble, which is problematic in a TDCT system, because the timing for each operating link is not known a priori. To the best knowledge of the authors, there is no synchronization scheme designed specifically for OFDM-based TDCT.

A simple approach is to increase the preamble energy and apply a SISO OFDM synchronization scheme to each copy and then do the combination. The performance of the SISO schemes will be used as benchmarks for the TDCT performance and as a basis for our novel techniques. The conventional SISO OFDM synchronization schemes are preamble-based [7], [8], [9], [10]. The method proposed by Schmidl and Cox [7] is the most popular preamble-based scheme. It uses two OFDM symbols as the preamble for both timing and frequency synchronization. The timing and fractional CFO estimations are done based on two identical halves of the first OFDM symbol in the time domain, while the integer CFO is computed with the help of second OFDM symbol in frequency domain. However, because of the CP, the timing metric has a plateau which makes the timing synchronization fallible. To eliminate the plateau, Park et al. [8] designed a new repeated-conjugated-symmetric sequence, which makes the timing metric have a sharp peak. However, because of the special structure of the preamble sequence, the timing metric has side lobes which can still disturb the timing synchronization. The CAZAC (Constant Amplitude Zero Auto Correlation) sequence, first used for OFDM synchronization

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[1] An offset is the difference between the receiver's notion of a synchronization parameter (e.g. OFDM symbol start time, carrier frequency, sample frequency) and the ideal value from the signal being received.

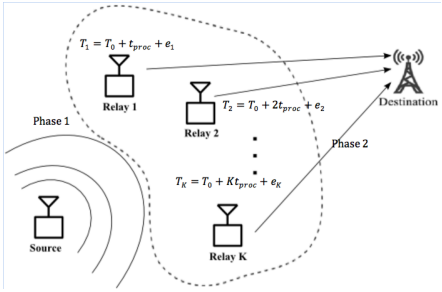


Fig. 1. Illustration of two-hop TDCT system.

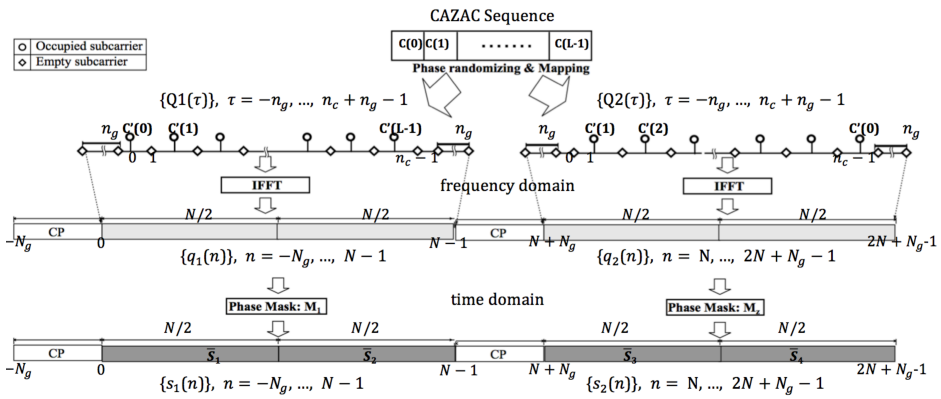


Fig. 2. Illustration of proposed preamble structure for $D = 1$.

by Ren et al. [9], has a constant envelope and a good property of self-correlation (i.e., high value at zero lag and low value at any other lag). It performs very well in the additive white gaussian noise (AWGN) channel, but the performance degrades significantly in the Rayleigh fading channel [11]. In [10], an OFDM symbol using a CAZAC sequence weighted by a pseudo noise sequence serves as the preamble for timing and frequency synchronization. The timing metric has a sharp peak without any side lobe, however, the integer CFO estimation based on cross-correlation between the received frequency domain preamble and the local training sequence doesn't work when there is phase rotation on the received frequency domain preamble caused by a fractional timing offset. Thus, development of an energy efficient algorithm is still an open problem for PHY layer design of the OFDM-based TDCT.

In this paper, we propose a synchronization scheme for timing and frequency offset estimation for OFDM-TDCT at the destination receiver, with range extension in the multipath fading channel, based on a preamble of only two OFDM symbols. In conventional decode-and-forward relays, the offsets learned by the relay upon reception are forgotten upon transmission. In contrast, in our proposed method, the offsets are kept, making the offsets observed at the destination from different relays correlated. Thus, in effect, the destination estimates the offsets relative to the source, based on a combination of the individual offset estimates from each relay. The symbol timing synchronization scheme has two stages (coarse and fine timing) and achieves diversity gain for symbol timing estimation and packet detection. The fractional and integer CFO estimation schemes achieve partial diversity gain. Therefore, the proposed scheme is able to operate at low SNR, no matter how many cooperating transmitters are involved. Moreover, by thresholding the fine timing metric, our algorithm reduces the computing complexity in the receiver while still maintaining the best performance by dynamically selecting only qualified cooperating links with good CSI.

II. System Model

We consider a half-duplex time-division cooperative communication system with one source node, $\mathbf{S}$, a relay cluster of K cooperating relay nodes $\{R_1, R_2, ..., R_K\}$, and a destination node, $\mathbf{D}$, as shown in Fig.1.We assume direct communication between source and destination is not available. There are two phases of transmission to achieve the communication between source and destination. In the first phase, the source, $\mathbf{S}$, broadcasts the message to potential relay nodes. All the relay nodes that correctly decode the packet from the source node will participate in the second phase, keeping the same offsets for transmission that they learned in reception. A quasi-static multipath fading channel is assumed.

A. Pre-synchronization

In this work, we focus on the synchronization at the destination receiver during the second phase, assuming pre-synchronization was done within the relay cluster before retransmission.With timing pre-synchronization [12], the kth relay is scheduled to transmit in the kth time slot as $T_k = T_0 + kt_{proc} + e_k$, where T_0 is the transmitting time of the source node, and also the time that the source packet arrives at the antennas of the relay nodes; t_{proc} is a period of time designed so that the transmit first-in-first-out buffer will be full when the transmission starts. The timing pre-sychronization error, e_k, is modeled as a zero mean Gaussian random variable with variance δ_e^2, denoted as $e_k \sim \mathcal{N}(0, \delta_e^2)$.

For pre-synchronization of frequency [13], each relay node estimates the CFO relative to the source node, and compensates it to any data it plans to transmit during the second phase. Therefore, after pre-synchronization, the CFO between kth relay node, R_k, and destination, $\mathbf{D}$, f_{kd}, is modeled as $f_{kd} = f_{sd} + e_\epsilon$, where f_{sd} represents the CFO between source and destination and $e_\epsilon \sim \mathcal{N}(0, \delta_\epsilon^2)$ is the frequency pre-synchronization error.

Consider a normalized time offset ε_k with respect to sampling period T_s and the normalized frequency offset ω_k with respect to subcarrier spacing, f_s. Then, $\omega_k = \frac{f_{kd}}{f_s} = \mu_k + \nu_k$, where μ_k and ν_k are the fractional CFO and integer CFO, respectively, of the kth relay node relative to the destination. Therefore, the received signal at the destination during the kth time slot can be expressed as $r^k(n) = e^{j2\pi\omega_k \frac{n}{N}} \sum_{g=0}^{G} h^k(g) s^k(n - g - \varepsilon_k) + z^k(n)$, where $s^k(n)$ is the source signal; $\mathbf{h}^k = [h^k(0), \ldots, h^k(G-1)]^T$ and $z^k(n)$ are channel impulse responses with G as the number of resolvable paths and additional white Gaussian noise (AWGN) with variance σ_N^2, respectively.

III. Proposed Method

We first establish some notation. Small letters represent time domain signals, and capital letters indicate frequency

domain signals. † indicates complex conjugate. $*$ indicates the convolution operation. $< \mathbf{s}_i, \mathbf{s}_j >= \sum_{l=0}^{L-1} s_i s_j$ indicates the inner product between two sequences $\mathbf{s}_i$ and $\mathbf{s}_j$ with length L.

A. Preamble Design

The proposed preamble structure in both time and frequency domains is illustrated in Fig.2. The two used OFDM symbols are constructed based on a CAZAC sequence $\{C(i)\}$ with length L. According to [14], the CAZAC sequence can be chosen as: $C(i) = e^{j\pi p i^2/L}$, where p is the positive integer and relative-prime to L; we choose p=L-1. The good properties of CAZAC sequences are: $\|C(i)\| = 1, and \sum_{i=0}^{L-1} C(i)C^*(i+k)$ equals L if $k = 0$, but is zero otherwise.

To construct the preamble in the frequency domain, we first randomize the phase of the CAZAC sequence to get the intermediate sequence $C'(i) = C(i)e^{j2\pi r_i}, i = 0, \ldots, L-1$, where $r_i \sim U[0,1]$ is uniformly distributed over the interval [0,1]. The purpose of randomizing the phase is to enlarge the difference of the target metric for integer CFO estimation between correct estimation and false estimation, which will be discussed in detail in the section below.

For the sake of convenience, we assume that the subcarrier index starts at $-n_g$, where n_g is the length of a guard interval of null tones reserved on both sides of an OFDM symbol in case of preamble aliasing due to integer CFO [7]. The n_c subcarriers in the middle part are used for preamble design. Therefore, the subcarrier index in frequency domain is then indicated by $\tau \in \{-n_g, \ldots, n_c + n_g - 1\}$, and the total length of the OFDM symbol in the frequency domain is $n_c + 2n_g$, as shown in Fig.2. To construct the preamble in the frequency domain, the sequence $\{C'(i)\}$ is mapped to all even subcarriers on the first OFDM symbol, and is mapped to all odd subcarriers on the second OFDM symbol, after a left circular shift of D steps ($1 \le D \le L-1$ $D = 1$ in Fig.2). Therefore, the preamble in the frequency domain, consisting of two OFDM symbols, $Q_1(\tau)$ $Q_2(\tau)$ for $\tau = 0, \ldots, n_c - 1$, are

$$Q_1(\tau) = \begin{cases} C'(\frac{\tau}{2}), & modulo(\tau, 2) = 0 \\ 0, & modulo(\tau, 2) = 1 \end{cases}, \quad Q_2(\tau) = \begin{cases} C'(\frac{\tau-1}{2} + D), & modulo(\tau, 2) = 1 \\ 0, & modulo(\tau, 2) = 0 \end{cases}. \tag{1}$$

The two OFDM symbols of the preamble in the frequency domain are designed with the relationship as $Q_1(\tau)$ is the result of right circular shift of $Q_2(\tau)$ by 2D-1 steps, which provides the ideal structure in the time domain for timing synchronization, as we will see in a later section.

The preamble in the time domain is constructed in two steps. First, the constructed preamble in the frequency domain is converted into the time domain signal through the inverse discrete Fourier transform (IDFT) and a cyclic prefix with length N_g is attached in front to avoid ISI. For the sake of convenience, we assume that the time domain preamble starts at discrete index zero and the attached cyclic prefix has negative time indices as shown in Fig.2. So the length of the OFDM symbol in the time domain is $N + N_g$. N is the IDFT/DFT length, which equals $n_c + 2n_g$. The two OFDM preamble symbols are denoted as $\{q_1(n)\}$, $n = -N_g, \ldots, N-1$, and $\{q_2(n)\}, n = N, \ldots, 2N + N_g - 1$, respectively.

Because of the special structure of the preamble in the frequency domain, $\{q_1(n)\}, \{q_2(n)\}$ have a repetition property,

$$\begin{cases} q_1(n) = q_1(n + \frac{N}{2}), & n = 0, \ldots, \frac{N}{2} - 1 \\ q_2(n) = -q_2(n + \frac{N}{2}), & n = N + N_g, \ldots, \\ & N + N_g + \frac{N}{2} - 1. \end{cases} \tag{2}$$

In addition, the two preamble sequences also satisfy $q_1(n) = q_2(n)e^{-j2\pi(n-1)\frac{2D-1}{N}}$.

In the second stage, to avoid the timing ambiguity caused by the cyclic prefix, we use two phase masks $M_j, j = 1, 2$, which are constructed as $M_j(n) = e^{j2\pi m_n^j}$, for $n = 0, \ldots, N-1$, where, $m_n^j \sim U[0,1]$ are uniformly distributed random variables over the interval $[0,1]$. The phase masks M_j are constructed with a similar repetition property as: $M_j(n) = M_j(n + \frac{N}{2}), for$ $n = 0, \ldots, \frac{N}{2} - 1, j = 1, 2$.

We then modulate the two preamble sequences $\{q_1(n)\}$ and $\{q_2(n)\}$ with the two phase masks M_1 and M_2, respectively. The two time domain preambles become

$$\begin{cases} s_1(n) = q_1(n)M_1(n), n = 0, \ldots, N-1 \\ s_1(n - N_g) = q_1(n - N_g), n = 0, \ldots, N_g - 1 \\ s_2(n + N + N_g) = q_2(n + N + N_g)M_2(n), \\ \qquad n = 0, \ldots, N-1 \\ s_2(n + N) = q_2(n + N), n = 0, \ldots, N_g - 1. \end{cases} \tag{3}$$

As we can see from Eq.(3), the phase mask operates on only N out of $N + N_g$ samples, to avoid a plateau in the timing metric [7], [8].

Because of the repetition property of the phase masks, the time domain preamble still keeps the same repetition property shown in Eq.(2). For TDCT relaying, all relay nodes use the same preamble structure; the preamble for kth relay node is denoted as $s^k(n) = \{s_1(n), s_2(n)\}$.

B. Symbol Timing Estimation

The goal of symbol timing estimation is to estimate the normalized time offset ε_k for each relay and destination link k, so that the start of the packet (SOP) of each relay is identified.

The symbol timing estimation in the proposed method is obtained through two steps: (1)coarse timing estimation with the goal of detecting the packet and estimating the coarse SOP, and (2)fine timing estimation with the goal of estimating the SOP with high accuracy.

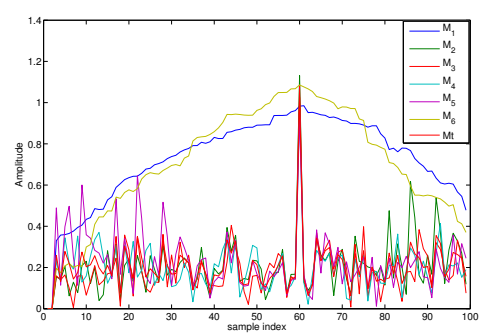


Fig. 3. The six coarse timing metric in Eq.(7) under multipath fading channel with SNR=10dB and SOP=60.

1) Coarse timing estimation: Because of the repetition property of the preamble, we keep processing the received signal, $r^k(n)$, as four segments, each with length $\frac{N}{2}$. The four segments for the kth relay link received at the receiver are defined as: $\bar{s}_1^k(i) = r^k(n+i), \bar{s}_2^k(i) = r^k(\frac{N}{2} + n + i), \bar{s}_3^k(i) =$

$r^k(N+N_g+n+i), \bar{s}_4^k(i) = r^k(N+N_g+\frac{N}{2}+n+i), for\ i = 0,\ldots,N/2$.. We compute six correlation metrics among the four segments as

$$\begin{aligned} &P_j^k(n) =< (\hat{\mathbf{s}}_x^k(n))^\dagger, \hat{\mathbf{s}}_y^k(n) >, j = 1,..,6, \\ &with\ x \in \{1,2,3,4\}, y \in \{2,3,4\}, \end{aligned} \tag{4}$$

where, $x = 1, y = 2,3,4; x = 2, y = 3,4; x = 3, y = 4$ corresponds to j=1,..,6, respectively. $\hat{\mathbf{s}}_1^k(n) =< \bar{\mathbf{M}}_1^\dagger, \bar{\mathbf{s}}_1^k(n) >$ $, \hat{\mathbf{s}}_2^k(n) =< \bar{\mathbf{M}}_1^\dagger, \bar{\mathbf{s}}_2^k(n) >, \hat{\mathbf{s}}_3^k(n) =< \bar{\mathbf{M}}_2^\dagger, \bar{\mathbf{s}}_3^k(n) >, \hat{\mathbf{s}}_4^k(n) = - < \bar{\mathbf{M}}_2^\dagger, \bar{\mathbf{s}}_4^k(n) >$, are the received four preamble segments with phase masks removed, as $\bar{\mathbf{M}}_1 = [M_1(0),\ldots,M_1(\frac{N}{2}-1)]$ is the first half of the phase mask M_1, and $\bar{\mathbf{M}}_2 = [M_2(0)e^{j2\pi\frac{2D-1}{N}0},\ldots,M_2(\frac{N}{2}-1)e^{j2\pi\frac{2D-1}{N}(\frac{N}{2}-1)}]$, is the first half of the phase mask M_2 with phase rotation of $e^{j2\pi\frac{2D-1}{N}i}$ on the ith element. The purpose of the rotation on the phase in Segments 3 and 4 is to cancel the difference relative to Segments 1 and 2, so that all the six correlation metrics in Eq.(4) reach the maximal value at the SOP point.

For simplicity in description of the scheme, we derive the correlation metrics assuming only one resolvable path for each relay link (our simulation uses multiple paths). The jth correlation metric for the kth relay link at the SOP can be expressed as

$$\begin{aligned} &P_j^k(\varepsilon_k) = \textstyle\sum_{i=0}^{N/2-1} \Big\{ \left|h^k\right|^2 e^{j2\pi\omega_k\frac{N}{2}} \left|s^k(i)\right|^2 \\ &+(z^k(\varepsilon_k+i))^\dagger(h^k e^{j2\pi\omega_k\frac{i+N/2}{N}} s^k(i+\tfrac{N}{2})) \\ &+(h^k e^{j2\pi\omega_k\frac{i}{N}} s^k(i))^\dagger(z^k(\varepsilon_k+i+\tfrac{N}{2})) \\ &+(z^k(\varepsilon_k+i))^\dagger z(\varepsilon_k+i+\tfrac{N}{2})\Big\}, for\ j = 1,\ldots,6. \end{aligned} \tag{5}$$

Except for the first term, all the other terms in Eq.(5) have expectation of zero, therefore, the expectation of the correlation metric conditioned on h^k is

$$\begin{aligned} E\{P_j^k(\varepsilon_k)\} &= \left|h^k\right|^2 e^{j\pi\omega_k\frac{N}{2}} \sum_{i=0}^{N/2-1} \left\{\left|s^k(i)\right|^2\right\} \\ &= \left|h^k\right|^2 e^{j\pi\omega_k\frac{N}{2}} E_0, \end{aligned} \tag{6}$$

where $E_0 = \sum_{i=0}^{N/2-1}\left\{\left|s^k(i)\right|^2\right\}$ is half the energy of one preamble sequence, which, as shown in Eq.(**??**), is the same for all links.

To normalize the correlation metric, we define a normalizing factor as $\hat{V}^k(n) = \frac{1}{4}\sum_{i=0}^{N/2-1}\Big(|r^k(n)|^2 + |r^k(n+\frac{N}{2})|^2 + |r^k(n+N+N_g)|^2 + |r^k(n+N+N_g+\frac{N}{2})|^2\Big)$.

Similar to [7], we can compute six normalized timing metrics as $M_j^k(n) = \frac{P_j^k(n)}{\hat{V}^k(n)}, for\ j = 1,...,6$. It is obvious that the timing metric $M_j^k \in (0,1]$ and achieves the maximum value 1 for the ideal channel at the SOP. As shown in Fig.3, because of the phase masks and symmetric property of the proposed preamble structure, all the timing metrics have peak value at the SOP point except for M_1^k and M_6^k, which have plateaus because of the phase masks being removed automatically during the CP period.

We then propose the coarse timing metric for kth relay link based on four timing metrics as

$$C^k(n) = \frac{\hat{P}^k(n)}{V^k(n)}, \tag{7}$$

where, $\hat{P}^k(n) = \frac{1}{4}\sum_{j=2}^{5}\left|P_j^k(n)\right|$. As shown in Fig.3, the coarse timing metric has a sharp peak value at SOP.

Once the coarse timing metric $C^k(n)$ exceeds a pre-defined threshold $T_c \in (0,1)$, the coarse timing for kth relay link during the kth time slot can be estimated at the destination by searching $\tilde{\varepsilon}_k = \arg\max_{n\in\mathcal{N}}\{C^k(n)\}$, where $\mathcal{N}$ is a set of adjacent sample points that exceed the threshold T_c. T_c is a system parameter which determines the packet detection rate (PDR). The higher T_c is, the lower PDR is achieved.

For TDCT relaying, as long as the destination detects at least one of the relayed packets, we say the destination receiver detects the packet from the source successfully. We define the packet detection rule for TDCT as

$$\begin{cases} C^{k_{opt}}(\tilde{\varepsilon}_{k_{opt}}) > T_c, & Packet\ detected \\ o.w. & Packet\ missed, \end{cases} \tag{8}$$

where, $k_{opt} = \arg\max_k\{\hat{P}^k(\tilde{\varepsilon}_k)\}$.

If the k_{opt} packet is detected, and the receiver can decode the k_{opt} from the header, then assuming the relays transmit consecutively (this is the scheduling aspect); the receiver can find the other copies at time indexes $\bar{\varepsilon}_k = \tilde{\varepsilon}_{k_{opt}} + (k - k_{opt})t_{proc}$, for $k = 1,\ldots,K$. Since both the numerator and denominator of the metric in Eq.(7) are weighted by $|h^k|^2$, there is a benefit from diversity, because noise will have the lowest degradation on the strongest channel.

2) Fine timing estimation: We aim to improve the accuracy of timing estimation in the second stage of timing estimation by exploring the information in both the time and the frequency domains. The fine timing metric $F^k(n)$ for the kth relay link is defined over a searching window centered at the updated coarse timing $\bar{\varepsilon}_k$ as

$$F^k(n) = w_t^k \bar{P}_t^{\,k}(n) + w_f^k \bar{P}_f^{\,k}(n), n \in [\bar{\varepsilon}_k - W_F, \bar{\varepsilon}_k + W_B],$$

where, W_F and W_B are user-defined parameters of the forward and backward window sizes. We use $W_B = W_F = N_g$ for simulation in Section IV; $\bar{P}_t^{\,k}(n) = \frac{\hat{P}^k(n)-min(\hat{P}^k)}{max(\hat{P}^k)-min(\hat{P}^k)}$ is the normalized time correlation metric of $\hat{P}^k(n)$ over the searching window $[\bar{\varepsilon}_k - W_F, \bar{\varepsilon}_k + W_B]$. $\bar{P}_f^{\,k}(n) = \frac{U^k(n)-min(U^k)}{max(U^k)-min(U^k)}$ is the normalized frequency metric of $U^k(n)$ over the searching window, which is defined in Subsection D. The w_t^k and w_f^k are the combining weights. Since the peak value indicates the correct timing point, we propose to compute the weights dynamically based on the instantaneous peak-to-average-ratio (PAR) of sequences $\{\bar{P}_t^{\,k}(n)\}$ and $\{\bar{P}_f^{\,k}(n)\}$ over the searching window, respectively,

$$w_t^k = \frac{PAR_t^k}{PAR_t^k + PAR_f^k}, w_f^k = \frac{PAR_f^k}{PAR_t^k + PAR_f^k}, \tag{9}$$

where $PAR_t^k = \frac{\max\{\bar{P}_t^{\,k}(n)\}}{\frac{1}{W_B+W_F+1}\sum_{n=\bar{\varepsilon}_k-W_F}^{\bar{\varepsilon}_k+W_B}\bar{P}_t^{\,k}(n)}, PAR_f^k = \frac{\max\{\bar{P}_f^{\,k}(n)\}}{\frac{1}{W_B+W_F+1}\sum_{n=\bar{\varepsilon}_k-W_F}^{\bar{\varepsilon}_k+W_B}\bar{P}_f^{\,k}(n)}$.

Eventually, the fine timing for the kth relay link is estimated by finding the maximum value of the fine timing metric as

$$\hat{\varepsilon}_k = \arg\max_{n\in[\bar{\varepsilon}_k-W_F,\bar{\varepsilon}_k+W_B]}\{F^k(n)\}. \tag{10}$$

To enhance the selection diversity and save receiver energy, we can select the qualified links with good channel condition by pre-defining a fine timing threshold T_f. The destination will process only the signals from relay node k, if $F^k(\hat{\varepsilon}_k) \geq T_f$.

C. Fractional CFO Estimation

According to [7], the fractional CFO can be estimated based on the angle of the correlation metric in the time domain as $\frac{\angle P_j^k(\hat{\varepsilon}_k)}{\pi}$. Because of the special structure of our preamble with phase masks, we propose to use only the first and last correlation metrics P_1^k, and P_6^k for fractional CFO estimation as the phase masks are removed automatically when computing P_1^k, and P_6^k as long as the timing error is less than the length of the CP period.

Instead of estimating the fractional CFO for each relay link separately, we propose a combined fractional CFO estimation scheme to utilize the correlation among all copies to (this is where low CFO pre-synchronization error makes a difference) to achieve diversity gain. The combined fractional CFO can be written as

$$\hat{\mu}_{comb} = c_1 \frac{\angle P_1^{comb}}{\pi} + c_2 \frac{\angle P_6^{comb}}{\pi}, k = 1, \ldots, K, \quad (11)$$

where, $P_1^{comb} = \sum_{k\in\mathcal{K}} P_1^k(\hat{\varepsilon}_k), P_6^{comb} = \sum_{k\in\mathcal{K}} P_6^k(\hat{\varepsilon}_k)$, $c_1 = \frac{\left|P_1^{comb}\right|}{\left|P_1^{comb}\right|+\left|P_6^{comb}\right|}, c_2 = \frac{\left|P_6^{comb}\right|}{\left|P_1^{comb}\right|+\left|P_6^{comb}\right|}$. $\mathcal{K}$ is a set of "qualified" relay links whose fine timing metric $F^k(\hat{\varepsilon}_k)$ exceed the threshold T_f.

Based on the autocorrelation metric in Eq.(6) and the fact that all the transmitters use the same preambles, the combined autocorrelation metric has the conditional expectation value

$$E\{P_1^{comb}|h^k\} = E\{P_6^{comb}|h^k\} = \\ = E\{\sum_{k\in\mathcal{K}} \left|h^k\right|^2 e^{j\pi\omega_k \frac{N}{2}} E_0\} \simeq e^{j\pi\omega_0 \frac{N}{2}} E_0 \sum_{k\in\mathcal{K}} \left|h^k\right|^2, \quad (12)$$

where $\omega_k = \omega_0 + \epsilon_k$ with ω_0 as the normalized fractional CFO between source and destination, and ϵ_k as the frequency pre-synchronization error. For TDCT, when SNR is very low, the pre-synchronization error, which is a function of the SNRs at the relays, is assumed in this paper to be small enough to be ignored [12].

As we can see in Eq.(12), the combined fractional CFO estimation metric is weighted by the sum of magnitude squared channel gains, $\left|h^k\right|^2$, which offers the diversity gain similar to maximum ratio combing (MRC).

D. Integer CFO Estimation

The integer CFO ν_k is estimated based on the preamble in the frequency domain. The received preamble of the two OFDM symbols in the frequency domain for the kth relay link with fractional CFO μ_k compensated can be expressed as

$$R_j^k(\tau) = \bar{R}_j^k * M_j^F + Z_j^k(\tau), j = 1, 2, \quad (13)$$

where $\bar{R}_j^k(\tau) = H^k(\tau)Q_j(\tau - \nu_k)$, for $\tau = -n_g, \ldots, n_g + n_c - 1$, and $\mathbf{H}^k = [H^k(-n_g), \ldots, H^k(n_g + n_c - 1)]^T$, $Z_j^k(\tau)$ are frequency channel response and an AWGN respectively. M_j^F is the Discrete Fourier Transform (DFT) of the phase mask M_j.

Given the DFT of the conjugated phase masks, $\bar{M}_j^F = DFT(M_j^\dagger)$, the frequency domain preamble without phase masks can be recovered by convolution in the frequency domain,

$$\hat{R}_j^k = R_j^k * \bar{M}_j^F = \bar{R}_j^k(\tau) + \bar{Z}_j^k(\tau) \quad (14)$$

where, $\bar{Z}_j^k(\tau) = Z_j^k(\tau) * \bar{M}_j^F$.

Then, the integer CFO estimation is based on the frequency metric $U^k(\tau)$, which is computed as

$$U^k(\tau) = |\sum_{i=0}^{n_c-1} G^k(\tau + i)^\dagger O(i)|, \ for \ \tau \in [-n_g, n_g], \quad (15)$$

where, $G^k(\tau + i) = (\hat{R}_1^k(\tau + i))^\dagger \hat{R}_2^k(\tau + i + 1), O(i) = Q_1(i)^\dagger Q_2(i + 1)$.

When we assume the frequency channel response for adjacent subcarriers is the same ($H^k(i) \cong H^k(i + 1)$), the expectation of metric U^k is

$$E\left[U^k(\tau)\right] = \quad (16)$$
$$\begin{cases} \sum_{i=0}^{n_c-1} \left|H^k(\nu_k + i)\right|^2 \left|Q_1(i)\right|^2 \left|Q_2(i+1)\right|^2, & \tau = \nu_k \\ \delta_{\tau-\nu_k}^2 \sum_{i=0}^{n_c-1} \left|H^k(\tau + i)\right|^2, & o.w. \end{cases}$$

where, $\delta_\Delta = E[\sum_{i=0}^{n_c-1} Q_1(i + \Delta)^\dagger Q_1(i)] = E[\sum_{i=0}^{n_c-1} Q_2(i + \delta)^\dagger Q_2(i)]$. Since, $\{Q_1(i)\}, \{Q_2(i)\}$ are constructed from the sequence $\{C'(i)\}$ with randomized phase, we expect the δ_Δ to be very small when $\Delta \neq 0$.

The integer CFO for the kth relay node could be estimated by searching the maximum value of the target metric $\hat{\nu}_k = \underset{\tau\in[-n_g,n_g]}{\arg\max} \left\{U^k(\tau)\right\}$. However, instead of doing this, we propose a combined integer CFO estimation metric as

$$\hat{\nu}_{comb} = \underset{\tau\in[-n_g,n_g]}{\arg\max} \left\{U^{comb}(\tau)\right\}, \quad (17)$$

where, $U^{comb}(\tau) = \left(\sum_{k\in\mathcal{K}} U^k(\tau)\right)^\dagger O(\tau)$. $\mathcal{K}$ is the set of "qualified" relay links.

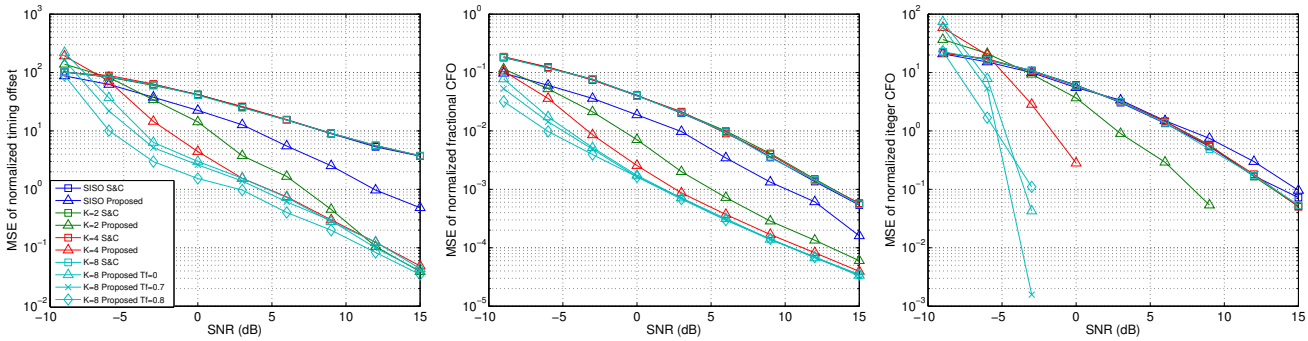
The expectation of the combined frequency correlation metric at correct integer CFO, $\nu_k = \nu_0$, is

$$E\left\{U^{comb}(\nu_0)\right\} = \\ \sum_{i=0}^{\tau_c-1} \left|(Q_1(i))\right|^2 \left|Q_2(i + D)\right|^2 \left\{\sum_{k\in\mathcal{K}} \left|H^k(\nu_0 + i)\right|^2\right\}, \quad (18)$$

where ν_0 is the integer CFO between source and destination. As shown in Eq.(18), we observe that the combined correlation result is weighted by $\sum_{k\in\mathcal{K}} |H(\nu_k + i)|^2$. Therefore, the combined integer CFO estimation, $\hat{\nu}_{comb}$, based on the combined frequency metric, also achieves diversity gain.

IV. Simulation Results

Monte Carlo simulations are performed to simulate the proposed synchronization algorithm for both TDCT relaying and conventional SISO relaying. Of 128 subcarriers, 112 are used for preamble design. The CP length normalized to the sampling duration is eight. The CFO is set to be 2.2×subcarrier spacing for all relay links, and $t_{proc} = 100T_s$. We evaluate the MSE performance of symbol timing and CFO estimation under the frequency-selective channel with the exponential power delay profile model in [15] with a



(a) Symbol timing (b) Fractional CFO (c) Integer CFO

Fig. 4. Simulation result of MSE performance under multipath fading channel with pre-synchronization errors.

sampling period of $T_s = 10^{-7}s$ and a RMS delay spread of $50ns$. More than 10,000 trials were simulated. We set the variance of pre-synchronization frequency and timing error to be $\delta_\epsilon^2 = 0.029$ppm and $\delta_e^2 = 50$ppm, respectively. The MSE for offset estimation $x \in (\varepsilon, \mu, \nu)$ is computed as $MSE_x = E\{\frac{1}{|\mathcal{K}|}\sum_{k\in\mathcal{K}}(\hat{x}^k - x^k)^2\}$, where, $|\mathcal{K}|$ is the number of "qualified" relay links used for synchronization and decoding. We considered different fining timing thresholds, $T_f = 0, 0.7, 0.8$ when $K = 8$, and $T_f = 0$ for $K = 1, 2, 4$.

The MSE performance of both timing and frequency estimation with number of cooperating relays $K \in \{1, 2, 4, 8\}$ is shown in Fig.4. As we can see, compared with the S&C method, for SISO link (K=1), the proposed method has about a 5dB improvement on symbol timing estimation, and has about 3dB improvement on fractional CFO estimation. That's because the timing estimation of S&C suffers from the plateau caused by CP, and the proposed algorithm has timing and fractional CFO estimation averaged over 4 and 2 metrics with phase masks, respectively. Also, when the number of cooperating relays increases, the proposed method achieves diversity gain (as evidenced by slope changes) on symbol timing and fractional CFO at low SNR, and diversity gain on integer CFO estimation for all SNR, while the performance of S&C's method stays the same as SISO relaying. We also observe that we lose the diversity gain for timing estimation and fractional CFO starting from $K = 4$ at relatively high SNR. This happens because, for the simulation, we use only the fine timing threshold, T_f, for $K = 8$. Therefore, when the SNR is large, the timing estimation is determined mostly by the fine timing metric without diversity because of $T_f = 0$. For K=8, as SNR grows, eventually, all copies get selected, then the diversity gain goes away, too.

For fractional CFO estimation, as we mentioned before, the error induced by the second and third terms in Eq.(12) is larger than the error induced by noise at high SNR. Therefore, we lose diversity generally. However, what matters for range extension is the diversity performance at low SNR, and as long as the error is small enough it won't affect the decoding diversity performance for TDCT.

We also observe that we are able to keep good synchronization performance while increasing the fine timing threshold for $K = 8$ in Fig.4. In this way, we can lower the processing energy at the destination. Because the selected "qualified" relay links are decreased when the fine timing threshold increases; we did not show it here due to the page limitation.

V. Conclusion

In this paper, a method for time and frequency synchronization for OFDM-based TDCT that achieves diversity gain is proposed for range extension applications. Based on computer simulation of MSE, all three synchronization parameters show evidence of diversity gain at low SNR.

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