ECE 3077, Summer 2014

Homework #8

Due Thursday July 24, in class

1. Using you class notes, prepare a 1–2 paragraph summary of what we talked about in class in the last week. I do not want just a bulleted list of topics, I want you to use complete sentences and establish context (Why is what we have learned relevant? How does it connect with other things you have learned here or in other classes?). The more insight you give, the better.

2. A factory produces X_n gadgets on day n, where X_n are independent and identically distributed random variables with mean 5 and variance 8.

   (a) Find an approximation to the probability that the number of gadgets produced in 100 days is less than 440?

   (b) Find (approximately) the largest value of n such that

   $$P(X_1 + X_2 + \cdots + X_n \geq 200 + 4n) \leq 0.05.$$

   (c) Let N be the first day on which the total number of gadgets produced exceeds 1000. Calculate an approximation to the probability that $N \geq 220$.

3. Let $X_1, Y_1, X_2, Y_2, \ldots$ be independent random variables uniformly distributed on $[0, 1]$. Let

   $$W = \frac{(X_1 + X_2 + \ldots + X_{16}) - (Y_1 + Y_2 + \ldots + Y_{16})}{16}.$$

   Find a numerical approximation to

   $$\mathrm{P}\left(|W - \mathrm{E}[W]| < 0.001\right).$$

   [Hint: What are the mean and variance of $X_1 - Y_1$?]

   [Hint: The condition $|W - \mathrm{E}[W]| < 0.001$ is equivalent to W falling in which interval?]

4. In this problem, you will write MATLAB code that takes a set of data $\underline{\mathrm{X}}$ of length N and an $0 < \alpha < 1$, and then returns the sample mean $\hat{\Theta}_N$ along with two numbers A and B so that $[A, B]$ is a $1 - \alpha$ confidence interval.

   You will write three m-files for the following three scenarios. In all of them, you can assume that the estimation error $\hat{\Theta}_N - \theta$ is a normal random variable (which of course has zero mean).

   (a) Known variance. Suppose you have explicit knowledge of $\mathrm{var}(\hat{\Theta}_N - \theta)$. Your m-file should start with

   ```
   function [thetan,A,B] = confint_var(X, v, alpha)
   ```

(The argument `v` above the the known variance.) You will probably find the `erfinv.m` and/or `norminv.m` MATLAB functions useful. Run your code for $\alpha = 0.1$ and $\alpha = 0.05$ on the data in `hw8p4a.mat`. This file contains 213 realizations of a normal random variable with unknown mean and variance $v = 6$.

(b) Unknown variance, large sample size. Here you do not have explicit knowledge of the variance, but you have enough samples that you feel you can estimate it reliably. Your m-file should start with

```
function [thetan,A,B] = confint_novar(X, alpha)
```

Run your code for $\alpha = 0.1$ and $\alpha = 0.05$ on the data in `hw8p4b.mat`, which contains 176 realizations of a normal random variable with unknown mean and unknown variance.

(c) Unknown variance, small sample size. Here you do not have explicit knowledge of the variance, nor are you confident that you can estimate it reliably from the amount of data that you have. Your m-file should start with

```
function [theatn, A, B] = confint_novar_small(X, alpha)
```

I have provided the function `student_t_inv.mat` which can evaluate the inverse of the Student's t cdf.

Run your code for $\alpha = .08$ and $\alpha = 0.035$ on the data in `hw8p4c.mat` which contains 8 realizations of a normal random variable with unknown mean and unknown variance.

For all parts above, turn in your code and the calculated answers.

5. Please fill out the CIOS survey:
http://www.cetl.gatech.edu/cios