ECE 3075, Fall 2008

Homework #10

Due Wednesday November 26, in class

Reading: Cooper and McGillem, Chapter 7

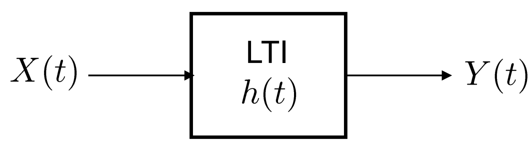
Using you class notes, prepare a 1-2 paragraph summary of what we talked about in class in the last week. I do not want just a bulleted list of topics, I want you to use complete sentences and establish context (Why is what we have learned relevant? How does it connect with other things you have learned here or in other classes?). The more insight you give, the better. Turn in your summary on a separate sheet of paper with your solutions to the problems below.

1. Suppose that a continuous-time WSS process $X(t)$ has spectrum

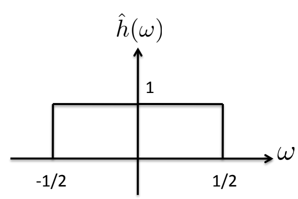
$$\hat{S}_X(\omega) = \begin{cases} 1 - |\omega| & |\omega| < 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) What is the power $\mathrm{E}[|X(t)|^2]$ of this process?

   (b) Now suppose we pass $X(t)$ through an LTI system

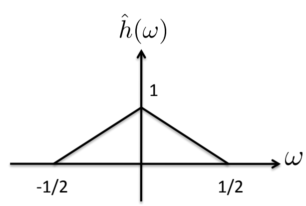


   with frequency response shown below



   What is the power of the output $Y(t)$?

   (c) Now suppose we pass $X(t)$ through an LTI system with this frequency response:



   Now what is the power of the output $Y(t)$?

2. Let $X_c(t)$ be a WSS continuous-time random process with

$$R_{X_cX_c}(\tau) = \delta(\tau)$$

Suppose we pass $X_c(t)$ through the filter in part 1b to get $Y_c(t)$ and then sample the output to get the discrete-time process $Y_d[n] = Y_c(nT_s)$.

(a) Write the autocorrelation function of $Y_d[n]$ in terms of the acf for $Y_c(t)$.

(b) For what values of T_s are the samples uncorrelated (i.e. $R_{Y_dY_d}[\ell] = \delta[n]$)?

3. Let $X_1(t),\ X_2(t)$ be independent, zero mean, continuous-time WSS random processes with

$$\begin{aligned} R_{X_1X_1}(\tau) &= e^{-|\tau|}, \\ R_{X_2X_2}(\tau) &= e^{-2|\tau|}. \end{aligned}$$

Set

$$Z(t) = X_1(t) + X_2(t).$$

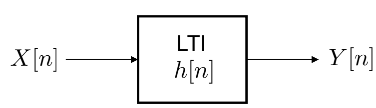
(a) What is the autocorrelation function $R_{ZZ}(\tau)$ of $\{Z(t)\}_{t\in\mathbb{R}}$?

(b) What is the spectral density $\hat{S}_Z(\omega)$ of $\{Z(t)\}_{t\in\mathbb{R}}$?

(c) What is the power $\mathrm{E}[|Z(t)|^2]$?

(d) Now suppose we pass $Z(t)$ through the LTI system described in 1b. What is the power of the output $Y(t)$?

4. Let $X[n]$ be white noise with power 1. Suppose we put $X[n]$ through an LTI filter



whose input-output relationship is described by the difference equation

$$Y[n] = \frac{1}{3}Y[n-1] + X[n].$$

(a) What is the impulse response $h[n]$ of this LTI system?

(b) What is the spectral density $\hat{S}_Y(\omega)$ of the output?

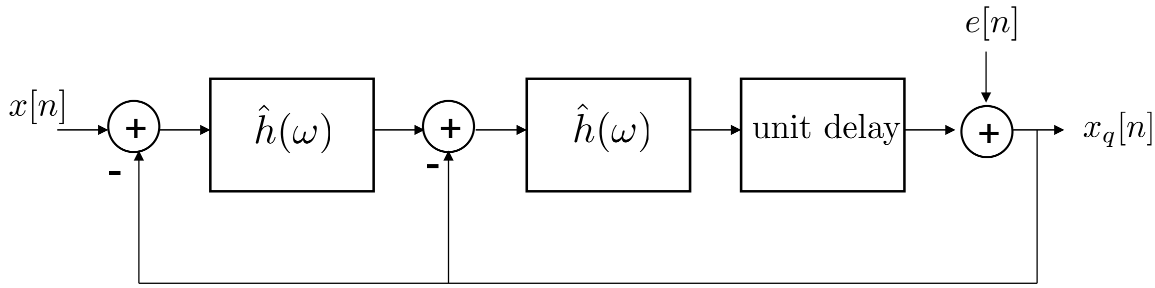
(c) Does an LTI system exist such that when I give $Y[n]$ as input, the output is white noise? If so, what is its impulse response?

(d) When in general, given a WSS process $Y[n]$ with a certain autocorrelation structure, is it possible to find such a "whitening" system?

5. A second-order Sigma-Delta quantizer can be modeled as shown below.
The $x[n]$ are exact samples of a bandlimited signal, $x_q[n]$ are the quantized samples, and $e[n]$ is a realization of a white noise random process $E[n]$ with power $\Delta^2/12$. The filters above both have frequency response

$$\hat{h}(\omega) = (1 - e^{-j\omega})^{-1},$$

and recall that the frequency response for a unit delay is simply $e^{-j\omega}$.



(a) The discrete time Fourier transform $\hat{x}_q(\omega)$ of the output will have the form

$$\hat{x}_q(\omega) = \hat{g_x}(\omega)\hat{x}(\omega) + \hat{g_e}(\omega)\hat{e}(\omega).$$

Find $\hat{g_x}(\omega)$ and $\hat{g_e}(\omega)$.

(b) Argue that we can model the entire quantization process as $x_q[n] = x[n-1] + e_2[n]$ where $e_2[n]$ is a realization of a non-white (colored) random process $E_2[n]$. Compute and plot the power spectrum of $E_2[n]$. From this, calculate the power of $E_2[n]$.

(c) Suppose that the $x[n]$ are oversampled by a factor of M, meaning that $\hat{x}(\omega) = 0$ for $\pi/M < |\omega| \leq \pi$. To take advantage of this, we pass $x_q[n]$ through an ideal lowpass filter with frequency response

$$\hat{h}_{lp}(\omega) = \begin{cases} 1 & |\omega| \leq \pi/M \\ 0 & \pi/M < |\omega| \leq \pi \end{cases}.$$

Call the output of this filter $\tilde{x}_q[n]$. Argue that we can write $\tilde{x}_q[n] = x[n-1] + \tilde{e}[n]$, where $\tilde{e}[n]$ is again a realization of a colored random process. Show that the power of this process is approximately $\pi^4\Delta^2/(60M^5)$.

(d) For which values of M does this second-order Sigma-Delta quantizer offer an advantage over the first-order system discussed in class?

6. The file `pulsedata.mat` contains two variables: a pulse p of length 300 and a series of $10,000$ observations y. y is mostly noise, but a (scaled and shifted) copy of p is in there somewhere. Write a matlab script to figure out where. Turn in your code, an explanation about what you did and why, and any supporting figures.