ECE 3075, Fall 2008

Homework #2

Due Friday September 5, in class

Reading: C&M Chapter 2

Using you class notes, prepare a 1-2 paragraph summary of what we talked about in class the first week. I do not want just a bulleted list of topics, I want you to use complete sentences and establish context (Why is what we have learned relevant? How does it connect with other things you have learned here or in other classes?). The more insight you give, the better. Turn in your summary on a separate sheet of paper with your solutions to the problems below.

1. Bob goes to the doctor to get tested for a rare disease (only one person out of every 50,000 suffers from this affliction). The test is fairly accurate: in the case where you have indeed contracted this disease it returns 'negative' only 0.01% of the time, in the case where you have not contracted the disease, it returns 'positive' only 0.05% of the time.

   Bob is disheartened when the test comes back positive. But what is the probability that he has actually contracted the disease?

2. A transmitter sends bits to a receiver across a noisy channel. The model for the noise is simple: regardless of whether a one or a zero was transmitted, the channel *flips* the bit with probability $p = 0.05$. Furthermore, assume that the noise acts on each of the transmitted bits independently.

   In an attempt to combat the noise, the transmitter adopts an n-fold repetition strategy — it transmits each message bit n times. For example, with $n = 5$, the transmitted sends 00000 to convey a message bit of 0, and it sends 11111 to convey a message bit of 1.

   (a) Suppose $n = 5$ and the receiver observes 00010. Compare the probability that the message bit was 1 to the probability that it was 0. In other words, compare $\Pr[\text{msg} = 1|00010 \text{ observed}]$ to $\Pr[\text{msg} = 0|00010 \text{ observed}]$. Assume that 0 and 1 are a priori equally likely, that is $\Pr[\text{msg} = 0] = \Pr[\text{msg} = 1] = 0.5$.

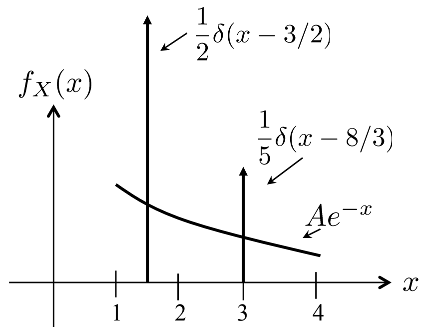
   (b) Based on (a), a reasonable decoding strategy at the receiver is *majority rules* — decide 0 if there are more zeros than ones received, and decide 1 otherwise (we will assume n is odd to avoid ambiguities). Find the probability that this strategy results in a decoding error with $n = 5$ and $p = 0.05$.

   (c) With $p = 0.05$, how large must n be in order for the majority-rules decoder to achieve a probability of error that is less than 10^{-5}?

3. C& M, Problem 1-10.9. (This problem requires MATLAB.)

4. Suppose that a random variable X has probability density function $f_X(x)$ shown below. (The $\delta(\cdot)$ are Dirac delta functions.)

   (a) Compute the value of A.



(b) Compute $\Pr[1 \le X \le 2]$.

(c) Compute $\Pr[5/2 \le X \le 4]$.

(d) Find an explicit expression for the cdf $F_X(x)$ and sketch it.

5. Let the continuous random variable X have a two-sided exponential distribution:

$$f_X(x) = \frac{1}{2}e^{-|x|}.$$

We are interested in recording the value of X, but only have a budget of 3 bits with which to do so. To do this, a *quantizer* creates the discrete random variable $X_{\text{quant}} = q(X)$, where

$$q(x) = \begin{cases} -7/2 & x \le -3 \\ -5/2 & -3 \le x < -2 \\ -3/2 & -2 \le x < -1 \\ -1/2 & -1 \le x < 0 \\ 1/2 & 0 \le x < 1 \\ 3/2 & 1 \le x < 2 \\ 5/2 & 2 \le x < 3 \\ 7/2 & x \ge 3 \end{cases}.$$

The value of X_{quant} is then encoded into a bitstream of length 3: 000 for $X_{\text{quant}} = -7/2$, 001 for $X_{\text{quant}} = -5/2$, ..., 110 for $X_{\text{quant}} = 5/2$, 111 for $X_{\text{quant}} = 7/2$.

(a) Calculate and sketch the pmf, pdf, and cdf of X_{quant}.

(b) What is the probability that the second bit is a '1', given that the first bit is a '0'?

(c) Design a 3 bit quantizer (a function $q(x)$ that takes 8 different values) for this pdf such that each of the 8 bitstreams is equally likely.