

ECE 3075, Fall 2008

Homework #1

Due Friday August 29, in class

Reading: Cooper&McGillem, Chapter 1

Using your class notes, prepare a 1-2 paragraph summary of what we talked about in class the first week. I do not want just a bulleted list of topics, I want you to use complete sentences and establish context (Why is what we have learned relevant? How does it connect with other things you have learned here or in other classes?). The more insight you give, the better. Turn in your summary on a separate sheet of paper with your solutions to the problems below.

1. Suppose that r people are in a room together.
 - (a) Find an expression for the probability that at least two people share a birthday. Ignore leap years, and assume that birthdays are evenly spread throughout the 365 days in a year.
 - (b) What is the smallest value of r such that this probability is greater than $1/2$?
2. Alice's cooler contains 10 lemonades and 4 sprites. Bob's cooler contains 6 lemonades and 7 sprites. Unfortunately, the two coolers look identical:



- (a) Suppose Bob picks one drink at random from a randomly chosen cooler. Find the probability that the chosen cooler was Bob's given that the chosen drink was a lemonade.
 - (b) Now suppose Bob picks three drinks out of one cooler at the same time, and observes that they are all lemonades. Then he chooses a fourth drink out of the same cooler. Calculate the probability that this fourth drink is a lemonade.
3. C&M 1-7.1.
 4. C&M 1-7.5.
 5. C&M 1-8.3.
 6. You are dealt (in a fair manner) 5 cards from a single deck.
 - (a) What is the probability that your hand contains no spades?
 - (b) What is the probability that your hand contains **exactly** 1 spade?
 - (c) Exactly 2 spades?
 - (d) Exactly 3 spades?

- (e) Exactly 4 spades?
 - (f) 5 spades?
7. You are playing 5 card draw, and you have made it your mission to get a spade flush¹. After you receive your 5 cards, you discard all of the ones which are not spades, then replenish your hand from the 47 cards not dealt to you. (For example, if you receive 2 spades and 3 not-spades, then you discard the 3 not-spades and draw 3 cards from the deck.) What is the probability that you will have a spade flush after the draw?
Hint: Combine the law of total probability with your answer to number 6 above.
8. (**Extra credit.**) Now you are on a mission to get any flush, no matter the suit. The essential strategy is to look at what you've been dealt, figure out which suit you have the most of, discard everything not in this suit, and draw. What is the probability that you will get a flush using this strategy? Note: you cannot simply multiply your answer to number 7 by 4, as there are overlapping cases now.
Hint: You need a smart way of partitioning the possible hands after the deal (for the spade flush, the partition is given by the subproblems in problem 6). A good partition might have six possibilities with labels $(2,1,1,1)$, $(2,2,1)$, $(3,1,1)$, $(3,2)$, $(4,1)$, and (5) .

¹For those that don't know, a "flush" means all five cards you have are the same suit.