Formation, Evolution, and Tuning of Frequency Combs in Microelectromechanical Resonators

Mingyo Park and Azadeh Ansari

***Abstract*— This paper presents the formation, evolution, and tuning of frequency combs in a piezoelectric micromechanical resonator, based on nondegenerate parametric pumping. The frequency combs consist of precisely spaced spectral lines located close to the mechanical resonance modes of a drumhead micro-electromechanical resonator. We investigate the parameter space of different dynamical regimes, wherein induced signal/idler tones break down into combs, and evolve into triangular-envelope shaped spectrums. Furthermore, we discuss the tuning mechanisms of frequency combs and study the dependency of the center frequency of combs and the frequency spacing between the spectral lines on the pump power and frequency. We demonstrate the evolution of combs in a mechanical system with direct electrical excitation and readout. This paper offers ultra-compact (30 μm in diameter), low-power (−13 dBm of threshold power) and highly tunable integrated frequency comb sources as low-cost alternatives to optomechanical frequency combs. [2018-0254]**

***Index Terms*— MEMS frequency combs, duffing nonlinearity, piezoelectric resonator, nondegenerate parametric pumping, mode coupling.**

I. Introduction

THE interaction between optical cavities and mechanical resonators has attracted enormous interest for its outlook in quantum phenomena [1]–[3]. In optomechanical systems, phonons are generated and dissipated through energy exchange due to coupling effects with photons [1], [2]. In such parametrically-amplified systems, optical frequency combs are induced by an external continuous-wave (CW) pump laser coupled to the nonlinear cavity by Kerr nonlinearity [4]. Existence of phononic frequency combs were theoretically predicted in [5] and recently observed experimentally in MEM resonators based on Duffing nonlinearity and mode coupling between a resonance mode and parametrically-excited modes [6], and 1:3 internal resonance mode coupling [7]. Previously, we reported on observation of phononic frequency combs in a piezoelectric circular drumhead resonator using two resonance modes within the same phonon cavity via nondegenerate parametric pumping [8], and presented a novel dual-mode sensor based on the mechanical frequency combs [9]. Such comb sensors provide a low-noise solution to resonance frequency tracking, by eliminating electronics circuitry and feedback loop required to sustain oscillation for frequency-shift detection [9], [10]. The focus of this letter, is investigation of various dynamical regimes of parametric excitation that lead to generation of signal/idler tones, frequency combs and chaotic behavior as the parametric pump power and frequency varies. We observe similar tendencies to temporal dissipative solitons in optomechanical resonators [3], [11]. The generation of frequency combs in micro-resonator based optomechanical systems has been attributed to compensation of dispersive behavior of phonons with cubic nonlinearities [1]. In our mechanical system, the cubic nonlinearity required for generation of combs, is captured by the Duffing term in the equation of motion, described in Eq. 1.

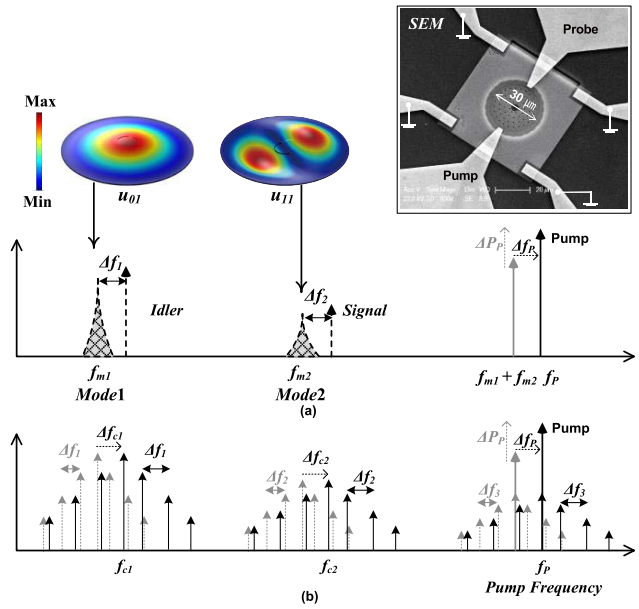

Fig. 1. (a) The comb generation using nondegenerate parametric pump frequency at sum of two mechanical modes, corresponding to u_{01} and u_{11}. Sets of combs are observed; two sets are placed close to the signal/idler frequency and one centered at the pump frequency. On the top right, SEM image of a standalone AlN-on Si micromechanical resonator is shown. (b) By tuning pump amplitude and frequency, the center frequency of combs and Δf_p can be detuned.

II. Dynamical Regimes of Parametric Instability

A. Mechanical Mode Coupling and Duffing Nonlinearity

The concept of parametric amplification has been widely used in various contexts, such as microwave amplifiers and optomechanical oscillators [12]. In this work, we use nondegenerate parametric pumping for energy transfer from pump to the mechanical resonance modes. Fig.1(a) shows the device geometry and displacement mode shapes of two flexural resonance modes. When a single-tone electric pump is applied at a frequency close to the sum of two mechanical modes ($\omega_p = \omega_{m1} + \omega_{m2} + \delta_p$), two frequency tones—idler and signal— are induced close to the two mechanical modes,

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The authors are with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332 USA (e-mail: m.park@gatech.edu).

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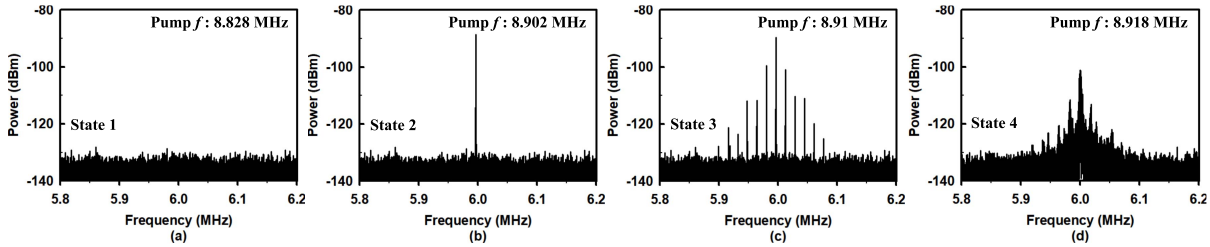

Fig. 2. Different dynamical regimes depicting the sequential evolution of power spectrum as the parametric pump frequency increases from (a) to (d), while the pump power is kept constant at 7 dBm (500 mV amplitude). (a) No parametric excitation is observed. (b) Single tone generated close to u_{11} resonance mode of the circular membrane. This state exists at pump frequencies ranging from 8.83 MHz to 8.902 MHz. (c) The tone breaks down into several equidistanced spectral lines (frequency combs), pump frequency ranging from 8.904 MHz to 8.916 MHz, resembling the spectrum of temporal solitary waves in optomechanical systems. (d) Triangular-envelope spectrum persists for a frequency range of 106 kHz bandwidth from 8.918 MHz to 9.024 MHz.

where δ_p denotes the pump frequency detuning from the sum of mechanical modes. Following the notation described in [2], the equation of motion of the signal and idler modes in presence of Duffing nonlinearity (β) are captured by Eq. 1 (a,b), when a single pump signal with frequency of ω_p and amplitude of Γ is applied to the resonator. The effective mass of both modes is set to unity for simplicity.

$$\ddot{x}_1 + \frac{\omega_{m1}}{Q_1}\dot{x}_1 + \omega_{m1}^2 x_1 + \beta_1 \omega_{m1}^2 x_1^3 = \Gamma x_2 cos\omega_p t, \quad (1.a)$$

$$\ddot{x}_2 + \frac{\omega_{m2}}{Q_2}\dot{x}_2 + \omega_{m2}^2 x_2 + \beta_2 \omega_{m2}^2 x_2^3 = \Gamma x_1 cos\omega_p t, \quad (1.b)$$

where $x_{1,2}$ are the mode displacements, corresponding to u_{01} and u_{11} modes of circular membrane shown in Fig. 1(a). Mode 1 has a resonance frequency of $\omega_{m1} = 2\pi \times 2.9$ MHz with quality factor (Q_1) of 7500 and mode 2 resonates at $\omega_{m2} = 2\pi \times 6$ MHz with Q_2 of 4200 [9]. The details of the solutions to the parametrically-coupled equations of motion are described in [2]. In this mechanical system, the motion of the first mode induces stress that shifts the frequency of the phonon cavity, creating sidebands at $\omega_{m1} + \omega_{m2}$. The parametric modal coupling is given by the stress-induced change in the phonon-cavity frequency for a given displacement of each mode [2]. For certain (Γ and δ_p) combinations, the solutions to the coupled equations of motion yield a train of equidistant spectral lines.

B. Frequency Comb Formation and Evolution

In this section, we investigate four different dynamic states of parametric excitation close to mechanical resonance modes of a drumhead circular MEM resonator. Fig.2 (a)-(d) depicts the continuous sequel of parametric excitation in a strongly driven nonlinear MEM resonator. At a constant pump power 7 dBm, pump frequency is swept with 2 kHz steps, starting at 8.828 MHz, where no signal/idler tone is observed. The spectrum evolves into formation of a single tone (signal/idler), combs, and chaotic behavior. Fig.3 (a) shows the parameter map, corresponding to these four dynamic states as the pump power and frequency is detuned (Γ and δ_p). The four different states are described below:

State 1 (No Parametric Excitation): Corresponds to dark blue area in parametric tongue and marks the beginning and end of the evolution sequence, shown in Fig.2 (a).

State 2 (Single Tone Formation): Corresponds to dark red area in parametric tongue and is located at the mechanical resonance mode u_{11}, or the signal mode, shown in Fig.2 (b).

State 3 (Frequency Comb Formation): Corresponds to green area in parametric tongue. Under certain combination of pump amplitude and frequency, frequency combs with equidistant lines are generated, as shown in Fig.2 (c). The center frequency of the comb is close to u_{11} mechanical mode and can be tuned by detuning pump frequency and amplitude.

State 4 (Triangular-Envelope Formation): Corresponds to pink area in parametric tongue. We observed a triangular shaped envelope formation in the power spectrum evolution, depicted in Fig.2 (d) that maintains for a large frequency range. Fig.3 (b) shows the amplitude of parametrically-excited tone or center frequency of comb depending on the combination of pump frequency and amplitude. The threshold power for parametric oscillation is -14.5 dBm and the minimum power required to generate frequency combs is only -13 dBm, which corresponds to 50 mV of threshold pump amplitude. This marks the lowest threshold power for excitation of mechanical combs compared to 100 mV and 600 mV values reported in [6] and [7] respectively. Furthermore, the center frequency power of the combs and signal tone is limited by the insertion loss of the resonator and impedance mismatch. This can be improved by arraying resonators to reduce the motional impedance, as well as using an optimal impedance termination.

III. Frequency Comb Tuning Mechanisms

Using nondegenerate parametric amplification, three sets of frequency combs were observed; two sets were located close to the two mechanical resonance modes (u_{01} and u_{11}) and one centered at the pump frequency (f_p) as described in Fig.1 (a). Fig.1 (b) shows the comb measurement scheme, highlighting tuning of center frequencies (f_{c1} and f_{c2}) and frequency spacings (Δf_1 and Δf_2) of the two sets of combs located close to the mechanical modes by detuning the pump frequency and power.

A. Center Frequency Tuning

Fig.4 (a) shows the dependency of center frequency of signal/idler as the pump amplitude is swept. The center frequencies of combs f_{c1} and f_{c2} increase/decrease simultaneously such that $f_{c1} + f_{c2} = f_p$ condition is satisfied [6], verifying parametric amplification criteria. It is observed that the sum of the slopes of the center frequency versus pump amplitude add up to zero, as expected.

The center frequency of the combs is also sensitive to pump frequency. As shown in Fig.4 (b), the comb center frequencies

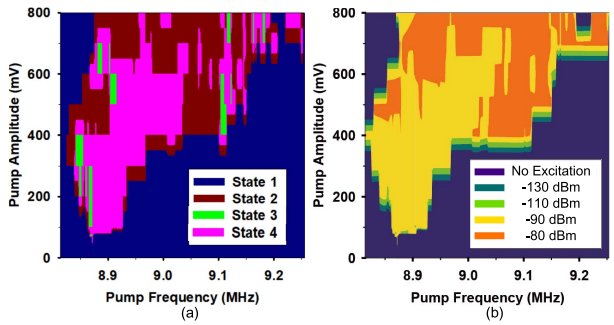

Fig. 3. Parametric tongues: (a) Different dynamical states in the parameter map of detuning pump frequency and amplitude: no parametric excitation (blue), excited single tone (red), triangular-envelope shape (pink) and combs (green). (b) Map of induced single-tone power (signal) or power of center frequency of the frequency combs by detuning pump frequency / amplitude.

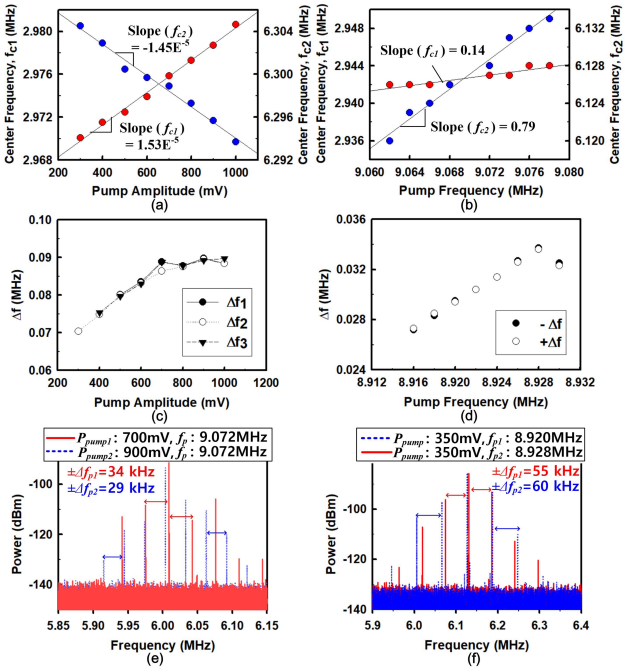

Fig. 4. Frequency tuning mechanisms: (a) At constant pump frequency, center frequency (f_{c1}, Idler) and (f_{c2}, Signal) shift by pump amplitude. (b) At constant pump amplitude, f_{c1} and f_{c2} shift by pump frequency. (c) Tuning of frequency spacing Δf with pump amplitude. (d) Tuning of Δf with pump frequency. The measured frequency comb spectrums: (e) at constant pump frequency and (f) at constant pump power.

(f_{c1} and f_{c2}) increase, as the pump frequency increases at a constant amplitude with a linear trend. The slopes of the lines add up to one, verifying that the comb center frequencies f_{c1} and f_{c2} are tuned by pump frequency when detuned around the sum of the two mechanical modes. The proposed MEM frequency combs have highly tunable characteristic with center frequency tuning ratio of 77% depending on pump frequency.

B. Frequency Spacing Tuning

Fig.4 (c), shows Δf_1 and Δf_2 versus pump amplitude at a constant pump frequency of 9.3 MHz. As increasing pump amplitude with 100 mV steps, Δf_1 and Δf_2 increase equally. It is worth noting that in this regime Δf_1 and Δf_2 are locked to Δf_3, which is the frequency spacing of the comb located at the pump frequency, *i.e.* $\Delta f_1 = \Delta f_2 = \Delta f_3$. The measured frequency spacings for all the three combs vary from 70 kHz to 90 kHz with similar trends. Fig.4 (d) shows Δf_1 and Δf_2 versus pump frequency at a constant pump amplitude of 700 mV. As pump frequency is increased with 2 kHz steps, Δf_1 and Δf_2 increase with linear trend. The combs exhibit equal left/right ($-\Delta f/+\Delta f$) frequency spacings from the center frequency, verifying the equidistance nature of the spectral lines. Fig.4 (e), (f) show the measured power spectrum of combs depending on detuning pump frequency and amplitude. Fig.4 (e) shows the blue and red combs with different frequency spacing when pump amplitude varies at a constant pump frequency. Fig.4 (f) depicts frequency spacing tuning of blue and red comb spectrums when pump frequency is detuned at a constant pump amplitude.

IV. Conclusion

This work reports on different dynamic regimes of parametric excitation and tuning mechanisms of frequency combs in a standalone piezoelectric MEM resonator via nondegenerate parametric pumping. Cubic nonlinearity due to geometrical Duffing effect along with stress-induced mode coupling in an Aluminum Nitride (AlN) circular MEM resonator, induce various parametrically-excited regimes. Signal/idler generation, comb formation, and triangular-envelope shaped spectrum are experimentally observed, which resemble soliton-behavior in optomechanical microresonators. This work offers small footprint, low-cost and highly-tunable alternative to optomechanical combs; that operates with fully-integrated piezoelectric transducers in a mechanical system and exhibits low threshold power of only −13 dBm for comb generation.

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